DIFFERENTIATION

NM – MITHIBAI PAST PAPER QUESTIONS

STUDENTS ARE REQUESTED TO TAKE A PRINT OF THIS FILE & THEN PRACTISE

01. $y = (2x^3 - 7)^5 \cdot \log(\tan x)$

STEP 1 :

$\frac{d}{dx}\log(\tan x)$

$= \frac{1}{\tan x} \cdot \frac{d}{dx}\tan x$

$= \frac{1}{\tan x} \cdot \sec^2 x$

$= \frac{1}{\frac{\sin x}{\cos x}} \cdot \frac{1}{\cos^2 x}$

$= \frac{1}{\sin x \cdot \cos x}$

$= \frac{2}{2\sin x \cdot \cos x}$

$= \frac{2}{\sin 2x} \quad = 2 \cdot \operatorname{cosec} 2x$

STEP 2 : $\frac{d}{dx}(2x^3 - 7)^5$

$= 5(2x^3 - 7)^4 \cdot \frac{d}{dx}(2x^3 - 7)$

$= 5(2x^3 - 7)^4 \cdot 6x^2$

$= 30x^2 \cdot (2x^3 - 7)^4$

STEP 3 :

$y = (2x^3 - 7)^5 \cdot \log(\tan x)$

$\frac{dy}{dx} = (2x^3 - 7)^5 \frac{d}{dx}\log(\tan x) + \log(\tan x) \frac{d}{dx}(2x^3 - 7)^5$

$= (2x^3 - 7)^5 \cdot 2\operatorname{cosec}2x + \log(\tan x) \cdot 30x^2 \cdot (2x^3 - 7)^4$

$= (2x^3 - 7)^5 \cdot 2\operatorname{cosec}2 + 30x^2 \cdot (2x^3 - 7)^4 \cdot \log(\tan x)$

$= (2x^3 - 7)^4 \left[2(2x^3 - 7)\operatorname{cosec}2x + 30x^2 \cdot\cdot \log(\tan x) \right]$

02. $y = \frac{\sin(4x^2 - 3)}{(x^2 - 2)^4}$

STEP 1 :

$\frac{d}{dx}\sin(4x^2 - 3)$

$= \cos(4x^2 - 3) \cdot \frac{d}{dx}(4x^2 - 3)$

$= \cos(4x^2 - 3) \cdot 8x \quad = 8x \cdot \cos(4x^2 - 3)$

STEP 2

$\frac{d}{dx}(x^2 - 2)^4$

$= 4(x^2 - 2)^3 \cdot \frac{d}{dx}(x^2 - 2)$

$= 4(x^2 - 2)^3 \cdot 2x \quad = 8x\,(x^2 - 2)^3$

STEP 3

$y = \frac{\sin(4x^2 - 3)}{(x^2 - 2)^4}$

$\frac{dy}{dx} = \frac{(x^2-4)^4 \frac{d}{dx}\sin(4x^2-3) - \sin(4x^2-3)\frac{d}{dx}(x^2-4)^3}{\left[(x^2 - 2)^4\right]^2}$

$= \frac{(x^2-4)^4 \cdot 8x \cdot \cos(4x^2-3) - \sin(4x^2-3) \cdot 8x(x^2 - 2)^3}{(x^2 - 2)^8}$

ARRANGING THE TERMS

$= \frac{8x \cdot (x^2-4)^4 \cdot \cos(4x^2-3) - \cdot 8x(x^2 - 2)^3 \sin(4x^2-3)}{(x^2 - 2)^8}$

$= \frac{8x(x^2 - 2)^3 \left[(x^2-4) \cdot \cos(4x^2-3) - \sin(4x^2-3) \right]}{(x^2 - 2)^8}$

$= \frac{8x\,(x^2-4) \cdot \left[\cos(4x^2-3) - \sin(4x^2-3) \right]}{(x^2 - 2)^5}$

03. $y = \dfrac{x.\cos^2 x}{(1+x)^3}$

STEP 1 :

$\dfrac{d}{dx} x.\cos^2 x$

$= x \dfrac{d}{dx}\cos^2 x + \cos^2 x . \dfrac{d}{dx} x$

$= x.2\cos x \; \dfrac{d}{dx}\cos x \; + \cos^2 x . 1$

$= x.\, 2\cos x\,(-\sin x) + \cos^2 x$

$= -x . 2 \sin x \cos x \; + \cos^2 x$

$= \cos^2 x - x.\sin 2x$

STEP 2 :

$\dfrac{d}{dx} (1+x)^3$

$= 3(1+x)^2 \; \dfrac{d}{dx}(1+x)$

$= 3(1+x)^2$

STEP 3 :

$y = \dfrac{x.\cos^2 x}{(1+x)^3}$

$$\frac{dy}{dx} = \frac{(1+x)^3 \dfrac{d}{dx} x.\cos^2 x \; - x.\cos^2 x \dfrac{d}{dx}(1+x)^3}{\left[(1+x)^3\right]^2}$$

$$= \frac{(1+x)^3 (\cos^2 x - x.\sin 2x) - x.\cos^2 x . 3(1+x)^2}{(1+x)^6}$$

ARRANGING THE TERMS

$$= \frac{(1+x)^3 (\cos^2 x - x.\sin 2x) - 3(1+x)^2.x.\cos^2 x}{(1+x)^6}$$

$$= \frac{(1+x)^2 \left[(1+x)(\cos^2 x - x.\sin 2x) - 3x.\cos^2 x\right]}{(1+x)^6}$$

$$= \frac{(1+x)(\cos^2 x - x.\sin 2x) - 3x.\cos^2 x}{(1+x)^4}$$

04. $y = \dfrac{\log(\cos 5x)}{x^2 + 3x - 1}$

STEP 1 :

$\dfrac{d}{dx} \log(\cos 5x)$

$= \dfrac{1}{\cos 5x} \; \dfrac{d}{dx} \cos 5x$

$= \dfrac{1}{\cos 5x} . (-\sin 5x) . \dfrac{d}{dx} 5x$

$= \dfrac{1}{\cos 5x} . (-\sin 5x) . 5$

$= -5 . \tan 5x$

STEP 2 :

$y = \dfrac{\log(\cos 5x)}{x^2 + 3x - 1}$

dy/dx =

$$\frac{(x^2+3x-1) \dfrac{d}{dx} \log(\cos 5x) - \log(\cos 5x) \dfrac{d}{dx}(x^2+3x-1)}{(x^2+3x-1)^2}$$

$$= \frac{(x^2+3x-1)(-5.\tan 5x) - \log(\cos 5x) . (2x+3)}{(x^2+3x-1)^2}$$

$$= \frac{-5(x^2+3x-1).\tan 5x - (2x+3). \log(\cos 5x)}{(x^2+3x-1)^2}$$

05. $y = \dfrac{x^4 + 4^x}{8 + \sin x}$

$$\frac{dy}{dx} = \frac{(8 + \sin x) \dfrac{d}{dx}(x^4+4^x) - (x^4 + 4^x) \dfrac{d}{dx}(8+\sin x)}{(8 + \sin x)^2}$$

$$= \frac{(8 + \sin x)(4x^3 + 4^x.\log^4) + (x^4 + 4^x).\cos x}{(8 + \sin x)^2}$$

06. $y = \log(\sin e^x) + \sqrt{5 + x^6} \,.\, \sec x$

STEP 1 :

$\frac{d}{dx}\log(\sin e^x)$

$= \frac{1}{\sin e^x} \frac{d}{dx}\sin e^x$

$= \frac{1}{\sin e^x} . \cos e^x . \frac{d}{dx} e^x$

$= \frac{1}{\sin e^x} . \cos e^x . e^x$

$= e^x . \cot e^x$

STEP 2 :

$\frac{d}{dx}\sqrt{5 + x^6} . \sec x$

$= \sqrt{5 + x^6} . \frac{d}{dx}\sec x + \sec x \frac{d}{dx}\sqrt{5 + x^6}$

$= \sqrt{5+x^6} . \sec x.\tan x + \sec x \frac{1}{2\sqrt{5+x^6}} \frac{d}{dx}(5+x^6)$

$= \sqrt{5+x^6} . \sec x.\tan x + \sec x \frac{1}{2\sqrt{5+x^6}} 6x^5$

$= \sqrt{5+x^6} . \sec x.\tan x + \sec x \frac{3x^5}{\sqrt{5+x^6}}$

$= \sec x \left(\sqrt{5+x^6} . \tan x + \frac{3x^5}{\sqrt{5+x^6}} \right)$

STEP 3 :

$y = \log(\sin e^x) + \sqrt{5 + x^6} . \sec x$

$\frac{dy}{dx} = e^x.\cot e^x + \sec x \left(\sqrt{5+x^6} . \tan x + \frac{3x^5}{\sqrt{5+x^6}} \right)$

07. $y = \frac{\sec^3 x}{e^{4x}.(1+x)^5}$

STEP 1 :

$\frac{d}{dx}\sec^3 x = 3\sec^2 x . \frac{d}{dx}\sec x$

$= 3\sec^2 x . \sec x . \tan x$

$= 3\sec^3 x.\tan x$ x

STEP 2 :

$\frac{d}{dx} e^{4x}.(1+x)^5$

$= e^{4x}. \frac{d}{dx}(1+x)^5 + (1+x)^5 . \frac{d}{dx} e^{4x}$

$= e^{4x}. 5(1+x)^4 \frac{d}{dx}(1+x) + (1+x)^5 . e^{4x} \frac{d}{dx} 4x$

$= e^{4x}. 5(1+x)^4 + (1+x)^5 . e^{4x} . 4$

$= 5.e^{4x}. (1+x)^4 + 4.e^{4x}. (1+x)^5$

$= e^{4x}. (1+x)^4 \left(5 + 4.(1+x)\right)$

$= e^{4x}. (1+x)^4 \ (9 + 4x)$

STEP 3 :

$\frac{dy}{dx} = \frac{e^{4x}.(1+x)^5 \frac{d}{dx}\sec^3 x - \sec^3 x \frac{d}{dx} e^{4x}.(1+x)^5}{\left(e^{4x}.(1+x)^5\right)^2}$

$= \frac{e^{4x}.(1+x)^5 \ 3\sec^3 x\tan x - \sec^3 x . e^{4x}. (1+x)^4 \ (9+4x)}{\left(e^{4x}.(1+x)^5\right)^2}$

$= \frac{3e^{4x}.(1+x)^5 \sec^3 x\tan x - e^{4x}.(1+x)^4 (9+4x).\sec^3 x}{\left(e^{4x}\right)^2 (1+x)^{10}}$

$= \frac{e^{4x}.(1+x)^4\sec^3 x \left[3(1+x)\tan x - (9+4x)\right]}{\left[e^{4x}\right]^2 (1+x)^{10}}$

$= \frac{\sec^3 x \left[3(1+x)\tan x - (9+4x)\right]}{e^{4x}. (1+x)^6}$

08. $y = \sin^3 3x \cdot e^{\sqrt{x}} + \log \frac{x+1}{\sqrt{x^2+1}}$

STEP 1

$\frac{d}{dx} \sin^3 3x \cdot e^{\sqrt{x}}$

$= \sin^3 3x \cdot \frac{d}{dx} e^{\sqrt{x}} + e^{\sqrt{x}} \frac{d}{dx} \sin^3 3x$

$= \sin^3 3x \cdot e^{\sqrt{x}} \frac{d}{dx} \sqrt{x} + e^{\sqrt{x}}\, 3\sin^2 3x \frac{d}{dx} \sin 3x$

$= \sin^3 3x \cdot e^{\sqrt{x}} \frac{1}{2\sqrt{x}} + e^{\sqrt{x}}\, 3\sin^2 3x \cdot \cos 3x \frac{d}{dx} 3x$

$= \sin^3 3x \cdot e^{\sqrt{x}} \frac{1}{2\sqrt{x}} + e^{\sqrt{x}}\, 3 \sin^2 3x \cdot \cos 3x \;\; 3$

$= \frac{e^{\sqrt{x}} \cdot \sin^3 3x.}{2\sqrt{x}} + 9\, e^{\sqrt{x}} \sin^2 3x \cdot \cos 3x$

$= e^{\sqrt{x}} \cdot \sin^2 3x \left(\frac{\sin 3x}{2\sqrt{x}} + 9. \cos 3x \right)$

STEP 2 :

$\frac{d}{dx} \log \frac{x+1}{\sqrt{x^2+1}}$

$= \frac{d}{dx} \left[\log (x+1) - \log \sqrt{x^2+1} \right]$

$= \frac{d}{dx} \left[\log (x+1) - \log (x^2+1)^{1/2} \right]$

$= \frac{d}{dx} \left[\log (x+1) - \frac{1}{2} \log (x^2+1) \right]$

$= \frac{1}{x+1} \frac{d}{dx} (x+1) - \frac{1}{2} \frac{1}{x^2+1} \frac{d}{dx} (x^2+1)$

$= \frac{1}{x+1} - \frac{1}{2} \frac{1}{x^2+1} 2x$

$= \frac{1}{x+1} - \frac{x}{x^2+1}$

STEP 3 :

$\frac{dy}{dx} = e^{\sqrt{x}} \cdot \sin^2 3x \left(\frac{\sin 3x}{2\sqrt{x}} + 9\cos 3x \right) + \frac{1}{x+1} - \frac{x}{x^2+1}$