PERMUTATION - 03

01. a number of 4 different digits is formed by using the digits 1, 2 , 3 , 4 , 5 , 6 ,7 , 8 in all possible ways . Find how many numbers are greater than 3000

thousand place can be filled by any one of the digits 3 , 4 , 5 , 6 , 7 , 8 in $^6P_1$ ways

Having done that the remaining 3 places can be filled by any 3 of the remaining 7 digits in $^7P_3$ ways

By fundamental principle of Multipliation ,

Total numbers formed = $^6P_1 \times {}^7P_3$

= 6 x 7 x 6 x 5 = 1260

02 . a number of 4 different digits is to be formed by using the digits 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 , 9 . Find how many of them are

a) greater than 4000

thousand place can be filled by any one of the digits 4 , 5 , 6 , 7 , 8 , 9 in $^6P_1$ ways

Having done that the remaining 3 places can be filled by any 3 of the remaining 8 digits in $^8P_3$ ways

By fundamental principle of Multipliation ,

Total numbers formed = $^6P_1 \times {}^8P_3$

= 6 x 8 x 7 x 6 = 2016

b) divisible by 2

unit place can be filled by any one of the digits 2 , 4 , 6 , 8 in $^4P_1$ ways

Having done that the remaining 3 places can be filled by any 3 of the remaining 8 digits in $^8P_3$ ways

By fundamental principle of Multipliation ,

Total numbers formed = $^4P_1 \times {}^8P_3$

= 4 x 8 x 7 x 6 = 1344

c) divisible by 5

unit place can be filled by digit '5' in 1 way

Having done that the remaining 3 places can be filled by any 3 of the remaining 8 digits in $^8P_3$ ways

By fundamental principle of Multipliation ,

Total numbers formed = $1 \times {}^8P_3$

= 8 x 7 x 6 = 336

03. How many 5 different digit numbers can be formed with digits 2 , 3 , 5 , 7 , 9 which are

a) greater than 30000

thousand place can be filled by any one of the digits 3 , 5 , 7 , 9 in $^4P_1$ ways

Having done that the remaining 4 places can be filled by remaining 4 digits in $^4P_4$ = 4!ways

By fundamental principle of Multipliation ,

Total numbers formed = $^4P_1 \times 4!$ = 4 x 24 = 96

b) less than 70000

thousand place can be filled by any one of the digits 2 , 3 , 5 in $^3P_1$ ways

Having done that the remaining 4 places can be filled by remaining 4 digits in $^4P_4$ = 4!ways

By fundamental principle of Multipliation ,

Total numbers formed = $^3P_1 \times 4!$ = 3 x 24 = 72

b) between 30000 & 90000

thousand place can be filled by any one of the digits 3 , 5 , 7 in $^3P_1$ ways

Having done that the remaining 4 places can be filled by remaining 4 digits in $^4P_4$ = 4!ways

By fundamental principle of Multipliation ,

Total numbers formed = $^3P_1 \times 4!$ = 3 x 24 = 72

04. how many different digit numbers can be formed between 100 and 1000 using 0 , 1 , 3 , 5 and 7 which is not divisible by 5

unit place can be filled by any one of digits 1 , 3 & 7 in $^3P_1$ ways

Having done that ,

Hundreds place can be filled by any one the remaining 3 digits ('0' excluded) in $^3P_1$ ways

Having done that , tens place can then be filled by any one of the remaining 3 digits in $^3P_1$ ways

By fundamental principle of Multipliation ,

Total numbers formed = $^3P_1 \times {}^3P_1 \times {}^3P_1$ = 3 x 3 x 3 = 27

05. How many different digit numbers are formed between 7000 and 8000 using 0 , 1 , 3 , 5 , 7 and 9 which are divisible by 5

thousand place can be filled by digit '7' in 1 way

Having done that , units place can be filled by any one of the digits 0 , 5 in $^2P_1$ ways

Having done that ,

remaining 2 places can be filled by any 2 of the remaining 4 digits in $^4P_2$ ways

By fundamental principle of Multipliation ,

Total numbers formed = $1 \times {}^2P_1 \times {}^4P_2$

= 1 x 2 x 4 x 3 = 24

06. how many even numbers of four digits can be formed using digits 0 , 1 , 2 , 3 , 4 , 5 and 6 , no digit being used more than once

Case 1 : Numbers ending with '0'

Unit place can be filled by digit '0' in one way

Having done that the remaining 3 places can be filled by any 3 of remaining 6 digits in $^6P_3$ ways

By fundamental principle of Multipliation ,

Numbers formed = $1 \times {}^6P_3$ = $6 \times 5 \times 4$ = 120

Case 2 : Numbers ending with '2 , 4 , 6'

Unit place can be filled by any one of digits 2 , 4 , 6 in $^3P_1$ ways

Having done that ,

Thousand place can be filled by any one the remaining 5 digits ('0' excluded) in $^5P_1$ ways

Having done that the remaining 2 places can be filled by any 2 of remaining 5 digits in $^5P_2$ ways

By fundamental principle of Multipliation ,

Numbers formed = $^3P_1 \times {}^5P_1 \times {}^5P_2$ = $3 \times 5 \times 5 \times 4$ = 300

By fundamental principle of **ADDITION**

Total numbers formed = 120 + 300 = 420

07. how many 5 different digit numbers can be formed with digits 0 , 1 , 3 , 5 , 6 , 8 and 9 divisible by 5

Case 1 : Numbers ending with '0'

Unit place can be filled by digit '0' in one way

Having done that the remaining 4 places can be filled by any 3 of remaining 6 digits in $^6P_4$ ways

By fundamental principle of Multipliation ,

Numbers formed = $1 \times {}^6P_4$ = $6 \times 5 \times 4 \times 3$ = 360

Case 2 : Numbers ending with '5'

Unit place can be filled by digit '5' in 1 ways

Having done that ,

ten Thousand place can be filled by any one the remaining 5 digits ('0' excluded) in $^5P_1$ ways

Having done that the remaining 3 places can be filled by any 3 of remaining 5 digits in $^5P_3$ ways

By fundamental principle of Multipliation ,

Numbers formed = $1 \times {}^5P_1 \times {}^5P_3$ = $1 \times 5 \times 5 \times 4 \times 3$ = 300

By fundamental principle of **ADDITION**

Total numbers formed = 360 + 300 = 660

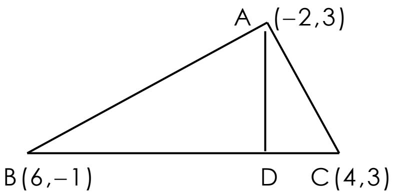
STRAIGHT LINES

NOV 2015

Find the coordinates of the orthocenter of a triangle whose vertices are

(−2,3) , (6,−1) , (4,3)

ALTITUDE AD



$m_{BC} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{3 + 1}{4 - 6} = \dfrac{4}{-2} = -2$

AD

$m = \dfrac{1}{2}$ (AD ⊥ BC) , A(−2,3)

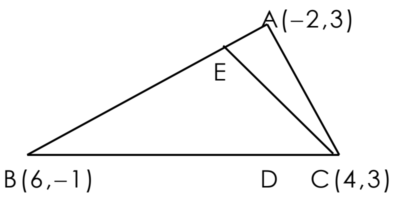
$y - y_1 = m(x - x_1)$

$y - 3 = \dfrac{1}{2}(x + 2)$

$2y - 6 = x + 2$

$x - 2y = -8$

ALTITUDE CE



$m_{AB} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{3 + 1}{-2 - 6} = \dfrac{4}{-8} = -\dfrac{1}{2}$

CE

$m = 2$ (CE ⊥ AB) , C(4,3)

$y - y_1 = m(x - x_1)$

$y - 3 = 2(x - 4)$

$y - 3 = 2x - 8$

$2x - y = 5$

ORTHOCENTER 'H'

$x - 2y = -8$

$2x - y = 5 \quad \times 2$

$x - 2y = -8$

$4x - 2y = 10$

$-3x = -18$

$x = 6$

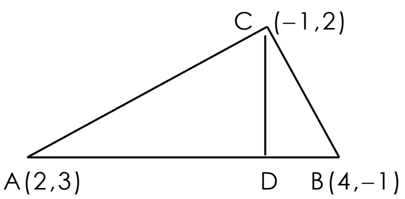
subs in (1)

$y = 7$ H (6,7)

02.

The points A(2,3) , B(4, −1) and C(−1,2) are the vertices of ΔABC . Find the length of the perpendicular from C on AB and hence find then area of Δ ABC

Equation of AB



$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 3 = \frac{-1 - 3}{4 - 2}(x - 2)$

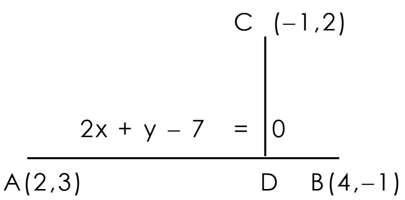
$y - 3 = -2(x - 2)$

$y - 3 = -2x + 4$

$2x + y - 7 = 0$

the length of the perpendicular from C on AB

Height (H)



$H = \left| \frac{2(-1) + 2 - 7}{\sqrt{2^2 + 1^2}} \right|$

$= \left| \frac{-2 + 2 - 7}{\sqrt{5}} \right|$

$= \left| \frac{-7}{\sqrt{5}} \right|$

$= \frac{7}{\sqrt{5}}$

BASE (AB)

$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(2 - 4)^2 + (3 + 1)^2}$

$= \sqrt{4 + 16}$

$= \sqrt{20}$

$= 2\sqrt{5}$

AREA OF (ΔABC)

$= \frac{1}{2} \times \text{BASE} \times \text{HEIGHT}$

$= \frac{1}{2} \times 2\sqrt{5} \times \frac{7}{\sqrt{5}}$

= 7 sq. units