J.K. SHAH CLASSES

MATHEMATICS & STATISTICS

SYJC PRELIUM – 02

DURATION – 3 HR MARKS – 80

NOTES :

1. All questions are compulsory
2. Answers to section I and section II must be written in separate ans. Books
3. Graph paper is compulsory for L.P.P.
4. Logarithm table will be provided on demand
5. Figures to the right indicate full marks
6. Answers to every question must be written on new page
7. Questions from section – I attempted in the answer book of section – II and vice versa will not be assessed/not be given any credit

SECTION - I

Q1. Attempt ANY SIX of the following (12)

01. Write converse and inverse of the following

"If a man is a bachelor then he is unhappy"

SOLUTION

$p \rightarrow q \equiv$ If a man is a bachelor then he is unhappy

CONVERSE : $q \rightarrow p \equiv$ if man is unhappy then he is a bachelor

INVERSE : $\sim p \rightarrow \sim q \equiv$ If a man is not a bachelor then he is happy

02. Discuss the continuity of f at x = 1

$$f(x) = \begin{cases} \dfrac{3 - \sqrt{2x+7}}{x-1}, & x \neq 1 \\ -1/3, & x = 1 \end{cases}$$

SOLUTION

STEP 1 : $\lim_{x \to 1} f(x)$

$$= \lim_{x \to 1} \frac{3 - \sqrt{2x+7}}{x-1} \cdot \frac{3 + \sqrt{2x+7}}{3 + \sqrt{2x+7}}$$

$$= \lim_{x \to 1} \frac{9 - (2x+7)}{x-1} \cdot \frac{1}{3 + \sqrt{2x+7}}$$

$$= \lim_{x \to 1} \frac{9 - 2x - 7}{x-1} \cdot \frac{1}{3 + \sqrt{2x+7}}$$

$$= \lim_{x \to 1} \frac{2 - 2x}{x-1} \cdot \frac{1}{3 + \sqrt{2x+7}}$$

$$= \lim_{x \to 1} \frac{2(1 - x)}{x-1} \cdot \frac{1}{3 + \sqrt{2x+7}}$$

$$= \lim_{x \to 1} \frac{-2(x - 1)}{x-1} \cdot \frac{1}{3 + \sqrt{2x+7}} \quad , \quad x - 1 \neq 0$$

$$= \lim_{x \to 1} \frac{-2}{3 + \sqrt{2x+7}}$$

$$= \frac{-2}{3 + \sqrt{2+7}}$$

$$= \frac{-2}{3+3}$$

$$= -\frac{1}{3}$$

STEP 2 : $f(1) = -1/3$ ………. Given

STEP 3 : $f(1) = \lim_{x \to 1} f(x)$

$\therefore$ f is CONTINUOUS at x = 1

03. Find the value of k if the function

$$f(x) = \begin{cases} \dfrac{(e^x - 1).\sin x}{x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$$

is continuous at x = 0

SOLUTION

STEP 1 : $\lim_{x \to 0} f(x)$

$$= \lim_{x \to 0} \frac{(e^x - 1).\sin x}{x^2}$$

$$= \lim_{x \to 0} \frac{e^x - 1}{x} \quad \frac{\sin x}{x}$$

$$= \log e . (1)$$

$$= 1$$

STEP 2 : Since the f is CONTINUOUS at x = 0

$$f(0) = \lim_{x \to 0} f(x)$$

$$k = 1$$

04. Find the marginal revenue if the average revenue is 45 and elasticity of demand is 5

SOLUTION

$$R_m = R_A\left(1 - \frac{1}{\eta}\right) = 45\left(1 - \frac{1}{5}\right) = 45\left(\frac{4}{5}\right) = 36$$

05. Find $^{dy}/_{dx}$ if $x^3 + y^2 + xy = 7$

SOLUTION

$$x^3 + y^2 + xy = 7$$

$$3x^2 + 2y\frac{dy}{dx} + x\frac{dy}{dx} + y = 0$$

$$(2y + x)\frac{dy}{dx} = -3x^2 - y$$

$$\frac{dy}{dx} = \frac{-3x^2 - y}{x + 2y}$$

06. Find the area bounded by the curve $y = x^4$, x − axis and lines x = 1 and x = 5

SOLUTION

$$A = \int_b^a y \; dx$$

$$= \int_1^5 x^4 \, dx$$

$$= \left[\frac{x^5}{5}\right]_1^5$$

$$= \frac{25}{5} - \frac{1}{5}$$

$$= {}^{24}/_{5} \text{ sq. units}$$

07. Evaluate $\displaystyle\int_{-2}^{3} \frac{dx}{x+5}$

SOLUTION

$$= \left[\log (x+5)\right]_{-2}^{3}$$

$$= \log (3+5) - \log(-2+5)$$

$$= \log 8 - \log 3$$

$$= \log (8/3)$$

08. Evaluate $\displaystyle\int \frac{dx}{16 - 9x^2}$

SOLUTION

$$= \int \frac{1}{4^2 - (3x)^2} \, dx$$

$$= \frac{1}{3}\,\frac{1}{2(4)} \log \left|\frac{4+3x}{4-3x}\right| + c$$

$$= \frac{1}{24} \log \left|\frac{4+3x}{4-3x}\right| + c$$

Q2.(A) Attempt ANY TWO of the following (06)

01. Prove that the following statement is a tautology : $(q \rightarrow p) \vee (p \rightarrow q)$

SOLUTION

p	q	$q \rightarrow p$	$p \rightarrow q$	$(q \rightarrow p) \vee (p \rightarrow q)$
T	T	T	T	T
T	F	T	F	T
F	T	F	T	T
F	F	T	T	T

Since all values are 'T' , the given statement is a TAUTOLOGY

02. Find dy/dx if $y = x^x + 5^x$

SOLUTION

$$y = x^x + 5^x$$

$$y = u + v$$

$$u = x^x$$

taking log

$$\log u = x.\log x$$

$$\frac{1}{u}\frac{du}{dx} = x\frac{d}{dx}\log x + \log x\frac{d}{dx}x$$

$$\frac{du}{dx} = u\left(x\frac{1}{x} + \log x\right)$$

$$\frac{du}{dx} = x^x(1 + \log x)$$

$$v = 5^x$$

$$\frac{dv}{dx} = 5^x.\log 5$$

Hence

$$\frac{dy}{dx} = x^x(1 + \log x) + 5^x\log 5$$

03. Evaluate $\int x \cos^{-1}x \, dx$

SOLUTION

$$= \int \cos^{-1}x . \, x \, dx$$

$$= \cos^{-1}x \int x \, dx - \int \left[\frac{d}{dx} \cos^{-1}x \int x \, dx \right] dx$$

$$= \cos^{-1}x \; . \frac{x^2}{2} - \int \frac{-1}{\sqrt{1 - x^2}} . \frac{x^2}{2} \, dx$$

$$= \frac{x^2}{2} . \cos^{-1}x + \frac{1}{2} \int \frac{x^2}{\sqrt{1 - x^2}.} \, dx$$

$$= \frac{x^2}{2} . \cos^{-1}x - \frac{1}{2} \int \frac{-x^2}{\sqrt{1 - x^2}.} \, dx$$

$$= \frac{x^2}{2} . \cos^{-1}x - \frac{1}{2} \int \frac{1 - x^2 - 1}{\sqrt{1 - x^2}.} \, dx$$

$$= \frac{x^2}{2} . \cos^{-1}x - \frac{1}{2} \int \frac{1 - x^2}{\sqrt{1 - x^2}.} - \frac{1}{\sqrt{1 - x^2}.} \, dx$$

$$= \frac{x^2}{2} . \cos^{-1}x - \frac{1}{2} \int \sqrt{1 - x^2} - \frac{1}{\sqrt{1 - x^2}.} \, dx$$

$$= \frac{x^2}{2} . \cos^{-1}x - \frac{1}{2} \left(\frac{x}{2} \sqrt{1^2 - x^2}. + \frac{1^2}{2} \sin^{-1} \frac{x}{1} - \sin^{-1} \right) x + c$$

$$= \frac{x^2}{2} . \cos^{-1}x - \frac{1}{2} \left(\frac{x}{2} \sqrt{1 - x^2} + \frac{1}{2} \sin^{-1} x - \sin^{-1} \right) x + c$$

$$= \frac{x^2}{2} . \cos^{-1}x - \frac{1}{2} \left(\frac{x}{2} \sqrt{1 - x^2} - \frac{1}{2} \sin^{-1} x \right) + c$$

$$= \frac{x^2}{2} . \cos^{-1}x - \frac{x}{4} \sqrt{1 - x^2} + \frac{1}{4} \sin^{-1} x + c$$

(B) Attempt ANY TWO of the following (08)

01. $f(x) = \dfrac{x^2}{5^x + 5^{-x} - 2}$; $x \neq 0$. Find f(0) if f(x) is CONTINOUS at x = 0

SOLUTION

STEP 1

$$\lim_{x \to 0} f(x)$$

$$= \lim_{x \to 0} \frac{x^2}{5^x + 5^{-x} - 2}$$

$$= \lim_{x \to 0} \frac{x^2}{5^x + \frac{1}{5^x} - 2}$$

$$= \lim_{x \to 0} \frac{x^2}{\frac{(5^x)^2 + 1 - 2.5^x}{5^x}}$$

$$= \lim_{x \to 0} \frac{x^2}{\frac{(5^x - 1)^2}{5^x}}$$

$$= \lim_{x \to 0} \frac{5^x}{\frac{(5^x - 1)^2}{x^2}}$$

$$= \lim_{x \to 0} \frac{5^x}{\left(\frac{5^x - 1}{x}\right)^2}$$

$$= \frac{5^0}{(\log 5)^2}$$

$$= \frac{1}{(\log 5)^2}$$

STEP 2

Since the f(x) is CONTINUOUS at x = 0

$$f(0) = \lim_{x \to 0} f(x)$$

$$= \frac{1}{(\log 5)^2}$$

02. Find the inverse of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{pmatrix}$ using ADJOINT METHOD

SOLUTION

COFACTOR'S

$A_{11} = (-1)^{1+1} \begin{vmatrix} 1 & 5 \\ 4 & 7 \end{vmatrix} = 1(7 - 20) = -13$

$A_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 5 \\ 2 & 7 \end{vmatrix} = -1(7 - 10) = 3$

$A_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix} = 1(4 - 2) = 2$

$A_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 3 \\ 4 & 7 \end{vmatrix} = -1(14 - 12) = -2$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 3 \\ 2 & 7 \end{vmatrix} = 1(7 - 6) = 1$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = -1(4 - 4) = 0$

$A_{31} = (-1)^{3+1} \begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix} = 1(10 - 3) = 7$

$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 3 \\ 1 & 5 \end{vmatrix} = -1(5 - 3) = -2$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = 1(1 - 2) = -1$

COFACTOR MATRIX OF A

$$= \begin{pmatrix} -13 & 3 & 2 \\ -2 & 1 & 0 \\ 7 & -2 & -1 \end{pmatrix}$$

ADJ A

= TRANSPOSE OF THE COFACTOR MATRIX

$$= \begin{pmatrix} -13 & -2 & 7 \\ 3 & 1 & -2 \\ 2 & 0 & -1 \end{pmatrix}$$

|A|

$= 1(7 - 20) - 2(7 - 10) + 3(4 - 2)$

$= 1(-13) - 2(-3) + 3(2)$

$= -13 + 6 + 6$

$= -1$

$A^{-1} = \dfrac{1}{|A|} \cdot \text{adj } A$

$$= \frac{1}{-1} \begin{pmatrix} -13 & -2 & 7 \\ 3 & 1 & -2 \\ 2 & 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 13 & 2 & -7 \\ -3 & -1 & 2 \\ -2 & 0 & 1 \end{pmatrix}$$

03. A manufacturer can sell x items at a price of (280 − x) each . The cost of producing x items is $(x^2 + 40x + 35)$. Find the number of items to be sold so that manufacturer can make maximum profit

SOLUTION

$$R = px$$
$$= (280 - x)x$$
$$= 280x - x^2$$

$$\pi = R - C$$
$$= 280x - x^2 - x^2 - 40x - 35$$
$$= 240x - 2x^2 - 35$$

$$\frac{d\pi}{dx} = 240 - 4x$$

$$\frac{d^2\pi}{dx^2} = -4$$

$$\frac{d\pi}{dx} = 0$$

$$240 - 4x = 0$$
$$x = 60$$

$$\frac{d^2\pi}{dx^2} = -4 < 0$$, Profit is maximum at x = 60

Q3.(A) Attempt ANY TWO of the following (06)

01. if p and q are true statements and r and s are false , find the truth value of the following

$$(p \wedge \sim r) \wedge (\sim q \wedge s)$$

SOLUTION

$$(p \wedge \sim r) \wedge (\sim q \wedge s)$$
$$\equiv (T \wedge \sim F) \wedge (\sim T \wedge F)$$
$$\equiv (T \wedge T) \wedge (F \wedge F)$$
$$\equiv T \wedge F$$
$$\equiv F$$

02. Differentiate e^{4x+5} w.r.t. e^{3x}

SOLUTION

$$u = e^{4x+5}$$

$$\frac{du}{dx} = e^{4x+5} \frac{d}{dx}(4x+5)$$

$$= 4\,e^{4x+5}$$

$$v = e^{3x}$$

$$\frac{du}{dx} = e^{3x} \frac{d}{dx} 3x$$

$$= 3\,e^{3x}$$

$$\frac{du}{dv} = \frac{du/dx}{dv/dx}$$

$$= \frac{4\,e^{4x+5}}{3\,e^{3x}}$$

$$= \frac{4}{3} e^{x+5}$$

03. Evaluate $\int \frac{e^x(1+x)\,dx}{\cos^2(xe^x)}$

SOLUTION

$$xe^x = t$$

$$x \frac{d}{dx} e^x + e^x \frac{d}{dx} x \;.dx = dt$$

$$(x.e^x + e^x.1)\,dx = dt$$

$$e^x(x+1).dx = dt$$

BACK INTO THE SUM

$$= \int \frac{1}{\cos^2 t}\,dt$$

$$= \int \sec^2 t\,dt$$

$$= \tan t + c$$

$$= \tan(xe^x) + c$$

(B) Attempt ANY TWO of the following (08)

01. Evaluate $$\int_{3}^{9} \frac{\sqrt[3]{12 - x}}{\sqrt[3]{x} + \sqrt[3]{12 - x}}\, dx$$

SOLUTION

$$I = \int_{3}^{9} \frac{\sqrt[3]{12 - x}}{\sqrt[3]{x} + \sqrt[3]{12 - x}}\, dx \quad \text{........(1)}$$

USING $$\int_{a}^{b} f(x)dx = \int_{b}^{b} f(a + b - x)\, dx$$

$$I = \int_{3}^{9} \frac{\sqrt[3]{12 - (12 - x)}}{\sqrt[3]{12 - x} + \sqrt[3]{12 - (12 - x)}}\, dx$$

$$I = \int_{3}^{9} \frac{\sqrt[3]{12 - 12 + x}}{\sqrt[3]{12 - x} + \sqrt[3]{12 - 12 + x}}\, dx$$

$$I = \int_{3}^{9} \frac{\sqrt[3]{x}}{\sqrt[3]{12 - x} + \sqrt[3]{x}}\, dx \quad \text{........(2)}$$

(1) + (2)

$$I = \int_{3}^{9} \frac{\sqrt[3]{12 - x} + \sqrt[3]{x}}{\sqrt[3]{12 - x} + \sqrt[3]{x}}\, dx$$

$$2I = \int_{3}^{9} 1\, dx$$

$$2I = \left(x \right)_{3}^{9}$$

$$2I = 9 - 3$$

$$2I = 6 \qquad I = 3$$

02. $x^7.y^9 = (x + y)^{16}$. Show that $\frac{dy}{dx} = \frac{y}{x}$

SOLUTION

$$7 \log x + 9 \log y = 16 \log (x+y)$$

$$\frac{7}{x} + \frac{9}{y}\frac{dy}{dx} = \frac{16}{x+y}\frac{d(x+y)}{dx}$$

$$\frac{7}{x} + \frac{9}{y}\frac{dy}{dx} = \frac{16}{x+y}\left(1 + \frac{dy}{dx}\right)$$

$$\frac{7}{x} + \frac{9}{y}\frac{dy}{dx} = \frac{16}{x+y} + \frac{16}{x+y}\frac{dy}{dx}$$

$$\left(\frac{9}{y} - \frac{16}{x+y}\right)\frac{dy}{dx} = \frac{16}{x+y} - \frac{7}{x}$$

$$\frac{9x + 9y - 16y}{y(x+y)}\frac{dy}{dx} = \frac{16x - 7x - 7y}{(x+y)\,x}$$

$$\frac{9x - 7y}{y}\frac{dy}{dx} = \frac{9x - 7y}{x}$$

$$\frac{dy}{dx} = \frac{y}{x}$$

03. $A = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$, find $(AB)^{-1}$

SOLUTION

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 2+9 & 0+3 \\ 1+6 & 0+2 \end{pmatrix} = \begin{pmatrix} 11 & 3 \\ 7 & 2 \end{pmatrix}$$

$|AB| = 22 - 21 = 1 \neq 0 \therefore (AB)^{-1}$ exist

COFACTORS

$AB_{11} = (-1)^{1+1}(2) = 2$

$AB_{12} = (-1)^{1+2}(7) = -7$

$AB_{21} = (-1)^{2+1}(3) = -3$

$AB_{22} = (-1)^{2+2}(11) = 11$

COFACTOR MATRIX OF AB

$$\begin{pmatrix} 2 & -7 \\ -3 & 11 \end{pmatrix}$$

ADJ AB $= \begin{pmatrix} 2 & -3 \\ -7 & 11 \end{pmatrix}$

$$(AB)^{-1} = \frac{1}{|AB|} \text{ ADJ AB}$$

$$= \begin{pmatrix} 2 & -3 \\ -7 & 11 \end{pmatrix}$$

SECTION - II

Q4. Attempt ANY SIX of the following (12)

01. Two unbiased coins are tossed . If X denotes the number of heads , find the probability distribution of X . Also find E(X)

SOLUTION

exp. : two unbiased coins are tossed , n(S) = 4

r.v. X : number of heads = 0 , 1 , 2

Probability distribution of X →

x	outcomes	p(x)	pi.xi
0	TT	1/4	0
1	HT , TH	2/4	2/4
2	HH	1/4	2/4

$E(X) = \Sigma pi.xi = \frac{4}{4} = 1$

02. if the correlation coefficient between X and Y is 0.8 , what is the correlation coefficient between a) X and 3Y b) X − 5 and Y − 3

SOLUTION

Since correlation coefficient is unaffected by shift of origin and change of scale

a) correlation coefficient between X and 3Y = 0.8

b) correlation coefficient between X − 5 and Y − 3 = 0.8

03. Find the premium on property worth ₹ 12,50,000 at 3% if the property is insured to the extent of 80% of its value

SOLUTION

Property value = ₹ 12,50,000

Insured value $= \frac{80 \times 12,50,000}{100}$

= ₹ 10,00,000

rate of premium = 3%

Premium $= \frac{3}{100} \times 10,00,000$

= ₹ 30,000

04. if the sum of squares of differences of ranks for 10 pairs of observation is 66 , find the rank correlation coefficient

SOLUTION

$$R = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

$$= 1 - \frac{6(66)}{10(99)}$$

$$= 1 - 0.4 = 0.6$$

05. if the present worth of a bill due 6 months hence is ₹ 2500 at 10% p.a. , what is the true discount

SOLUTION

$$T.D. = \text{Interest on PW}$$

$$= 2500 \times \frac{6}{12} \times \frac{10}{100}$$

$$= ₹\ 125$$

06. From the following table , find q0

x	0	1	2	3	4	5
lx	1000	940	780	590	25	0

SOLUTION

$$d_x = l_x - l_{x+1}$$

$$d_0 = l_0 - l_1$$

$$= 1000 - 940$$

$$= 60$$

$$q_x = \frac{d_x}{l_x}$$

$$q_0 = \frac{d_0}{l_0} = \frac{60}{1000} = 0.06$$

07. Compute CDR using the information given below

Age Group (years)	0 – 15	15 – 35	35 – 65	65 and above
Population	9000	25000	32000	9000

Total number of deaths in a year is given to be 900

SOLUTION

$$CDR = \frac{\Sigma D}{\Sigma P} \times 1000 = \frac{900}{75000} \times 1000 = 12 \text{ (deaths per 000)}$$

08. What should be subtracted from each of the numbers 5 , 7 and 10 so that resulting numbers are in continued proportion

SOLUTION

Let the number subtracted be x

As per the given condition

$$\frac{5 - x}{7 - x} = \frac{7 - x}{10 - x}$$

$$(5 - x)(10 - x) = (7 - x)^2$$

$$50 - 15x + x^2 = 49 - 14x + x^2$$

$$x = 1$$

Q5. (A) Attempt ANY TWO of the following (06)

01. an article is marked at ₹ 1500 . A trader allows a discount at 3% and still gains 20% on the cost . Find the cost price of the article

SOLUTION

Marked Price	=	1500
Less Discount		
(@ 3%)	–	45
Net SP	=	1455

$$SP = CP + Profit$$

$$1455 = CP + \frac{20\ CP}{100}$$

$$1455 = CP + \frac{CP}{5}$$

$$1455 = \frac{6}{5}\ CP$$

$$CP = ₹\ 1212.50$$

02. For a binomial distribution n = 6 and p = 0.3 , find the probability of getting 3 successes

SOLUTION

$$X \sim B(6, {}^{3}/_{10})$$

$$P(X = 3) = {}^{6}C_{3}\left(\frac{3}{10}\right)^3\left(\frac{7}{10}\right)^3$$

$$= \frac{20 \times 27 \times 343}{10^6} = 0.18522$$

03. Diet for a sick person must conatin at least 4000 units of vitamins , 50 units of minerals and 1500 calories . Two foods F_1 and F_2 cost ₹ 50 and ₹ 75 per unit respectively . Each unit of food F_1 contains 200 units of vitamins , 1 unit of minerals and 40 calories , whereas each unit of food F_2 contains 100 units of vitamins , 2 unit of minerals and 30 calories . Formulate the above problem as L.P.P. to satisfy the sick persons requirements

SOLUTION

TABULATION

	F_1 x − units	F_2 y − units	Min Requirement
	Contents / unit		
Vitamins	200	100	4000
Minerals	1	2	50
Calories	40	30	1500
Cost/unit	₹ 50	₹ 75	

CONSTRAINTS

01. Since diet of sick person must contain at least 4000 units of vitamins ,

$$200x + 100y \geq 4000$$

02. Since diet of sick person must contain at least 50 units of minerals ,

$$x + 2y \geq 50$$

03. Since diet of sick person must contain at least 1500 units of calories ,

$$40x + 30y \geq 1500$$

04. $x , y \geq 0$

OBJECTIVE FUNCTION

Total cost = $50x + 75y$ (in Rs)

Minimize Z = $50x + 75y$

LPP MODEL : Minimize $Z = 50x + 75y$

Subject to $200x + 100y \geq 4000$, $x + 2y \geq 50$, $40x + 30y \geq 1500$

$x , y \geq 0$

(B) Attempt ANY TWO of the following (08)

01. Two samples from bivariate populations have 15 observations each . The sample means of X and Y are 25 and 18 respectively . The corresponding sum of squares of deviations from means are 136 and 148 . The sum of product of deviations from respective means is 122 . Obtain the equation of line of regression of X on Y

SOLUTION

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$\overline{x} = 25$, $\overline{y} = 18$, $\Sigma(x - \overline{x})^2 = 136$; $\Sigma(y - \overline{y})^2 = 148$, $\Sigma(x - \overline{x})(y - \overline{y}) = 122$

X ON Y

$$bxy = \frac{\Sigma(x - \overline{x})(y - \overline{y})}{\Sigma(y - \overline{y})^2}$$

$$= \frac{122}{148}$$

$$= 0.82$$

LOG CALC
2.0864
−2.1703
AL $\overline{1}.9161$
0.8243

$$x - \overline{x} = bxy\ (y - \overline{y})$$

$$x - 25 = 0.82\ (y - 18)$$

$$x - 25 = 0.82\ y - 14.76$$

$$x = 0.82\ y - 14.76 + 25$$

$$x = 0.82y + 10.24$$

02. Suggest optimum solution to the following assignment problem . Also find the total minimum service time

Counters	Service time (in hrs) Salesman A	B	C	D
W	41	72	39	52
X	22	29	49	65
Y	27	39	60	51
Z	45	50	48	52

SOLUTION

	A	B	C	D
W	2	33	0	13
X	0	7	27	43
Y	0	12	33	24
Z	0	5	3	7

Reducing the matrix using ROW MINIMUM

	A	B	C	D
W	2	28	0	6
X	0	2	27	36
Y	0	7	33	17
Z	0	0	3	0

Reducing the matrix using COLUMN MINIMUM

	A	B	C	D
W	2	28	[0]	6
X	[0]	2	27	36
Y	~~0~~	7	33	17
Z	~~0~~	[0]	3	~~0~~

Assignment Using Single Zero Row Column Method

Assignment INCOMPLETE .

REVISE THE MATRIX

	A	B	C	D
W	2	28	[0]	6
X	[0]	2	27	36
Y	~~0~~	7	33	17
Z	~~0~~	[0]	3	0

Drawing lines to cover all the existing 0's

	A	B	C	D
W	4	28	0	6
X	0	0	25	34
Y	0	5	31	15
Z	2	0	3	0

Reduce all the UNCOVERED elements by its minimum & add the same at intersection

	A	B	C	D
W	4	28	[0]	6
X	~~0~~	[0]	25	34
Y	[0]	5	31	15
Z	2	~~0~~	3	[0]

Reallocation

Since every row & every column has an assigned zero , the ASSIGNMENT PROBLEM is solved

OPTIMAL ASSSIGNMENT : A – Y , B – X , C – W , D – Z

Minimum time = 27 + 29 + 39 + 52 = 147 hrs

03. x : 0 1 2 3 4 5

lx : 1000 850 760 360 25 0 . Complete the life table

AGE x	l_x	$d_x = l_x - l_{x+1}$	$q_x = \frac{d_x}{l_x}$	$p_x = 1 - q_x$	$L_x = \frac{l_x + l_{x+1}}{2}$	T_x	$e_x^0 = \frac{T_x}{l_x}$
0	1000	1000− 850=150	$\frac{150}{1000} = 0.15$	1 − 0.15 = 0.85	$\frac{1850}{2} = 925$	2495	$\frac{2495}{1000} = 2.495$
1	850	850−760 = 90	$\frac{90}{850} = 0.11$	1 − 0.11 = 0.89	$\frac{1610}{2} = 805$	1570	$\frac{1570}{850} = 1.847$
2	760	760−360 = 400	$\frac{400}{760} = 0.53$	1 − 0.53 = 0.47	$\frac{1120}{2} = 560$	765	$\frac{765}{760} = 1.007$
3	360	360 − 25 = 335	$\frac{335}{360} = 0.93$	1 − 0.93 = 0.07	$\frac{385}{2} = 192.5$	205	$\frac{205}{360} = 0.5696$
4	25	25 − 0 = 25	$\frac{25}{25} = 1$	1 − 1 = 0	$\frac{25}{2} = 12.5$	12.5	$\frac{12.5}{25} = 0.5$
5	0	----	----	----	----	----	----

LOG CALCULATIONS FOR 'q_x'

LOG 90 − LOG 850	LOG 400 − LOG 760	LOG 335 − LOG 360
1.9542	2.6021	2.5250
− 2.9294	− 2.8808	− 2.5563
AL $\bar{1}.0248$	AL $\bar{1}.7213$	AL $\bar{1}.9687$
0.1059	0.5264	0.9305

LOG CALCULATIONS FOR 'e_x^0'

LOG 1570 − LOG 850	LOG 765 − LOG 760	LOG 205 − LOG 360
3.1959	2.8837	2.3118
− 2.9294	− 2.8808	− 2.5563
AL 0.2665	AL 0.0029	AL $\bar{1}.7555$
1.847	1.007	0.5696

Q6. (A) Attempt ANY TWO of the following (06)

01. for 50 students of a class the regression equation of marks in Statistics (x) on the marks in a/c (y) is 3y −5x+180 = 0 . The mean of marks of accounts is 44 and variance of marks in Statistics is 9/16th of the variance of marks in accounts . Find mean marks of Statistics and correlation coefficient

SOLUTION

GIVEN : X ON Y : $3y - 5x + 180 = 0$

$$\bar{y} = 44$$

$$\frac{\sigma x^2}{\sigma y^2} = \frac{9}{16}$$

STEP 1

X ON Y : $3y - 5x + 180 = 0$

$$5x = 3y + 180$$

$$x = \frac{3y}{5} + \frac{180}{5}$$

$$bxy = \frac{3}{5}$$

STEP 2

$$bxy = r \cdot \frac{\sigma x}{\sigma y}$$

$$\frac{3}{5} = r \times \frac{3}{4}$$

$$r = \frac{4}{5}$$

STEP 3

Put $y = 44$ in

$$X = \frac{3y}{5} + \frac{180}{5}$$

$$x = \frac{3(44) + 180}{5}$$

$$x = \frac{132 + 180}{5}$$

$$x = \frac{312}{5}$$

$$x = 62.4$$

mean marks in statistics = 62.4

02. the pdf of continuous random variable X is given by

$$f(x) = 2x \quad ; \quad 0 < x < 1$$

$$= 0 \quad ; \quad \text{otherwise}$$

Find $P(1/3 < X < 1/2)$

SOLUTION

$P(1/3 < X < 1/2)$

$$= \int_{1/3}^{1/2} 2x \, dx$$

$$= \left(\frac{2x^2}{2}\right)_{1/3}^{1/2}$$

$$= \left(x^2\right)_{1/3}^{1/2}$$

$$= \left(\frac{1}{4}\right) - \left(\frac{1}{9}\right)$$

$$= \frac{5}{36}$$

03. The time (in hours) required to perform printing and binding operation (in that order) for each book is given in the following table

Job	I	II	III	IV	V
M_1	3	7	4	5	7
M_2	6	2	7	3	4

Find the sequence that minimizes the total elapsed time to complete the work . Also find the minimum elapsed time T and idle time for two machines

SOLUTION

STEP 1

Min time = 2 on job II on M_2

				II

Next Min time = 3 on job I on M_1 , & on job IV on M_2

I			IV	II

Next Min time = 4 on job III on M_1 & on job V on M_2

I	III	V	IV	II

OPTIMAL SEQUENCE

I	III	V	IV	II

STEP 2

Job	I	III	V	IV	II	TOTAL PROCESSING TIME (IN HOURS)
M_1	3	4	7	5	7	= 26
M_2	6	7	4	3	2	= 22

WORK TABLE

JOB	M_1 IN	M_1 OUT	M_2 IN	M_2 OUT	IDLE
I	0	3	3	9	(3)
III	3	7	9	16	
V	7	14	16	20	
IV	14	19	20	23	(3)
II	19	26	26	28	

STEP 3

TOTAL ELAPSED TIME (T) = 28 HRS

IDLE TIME ON M_1 = T − 28 = 2 HRS

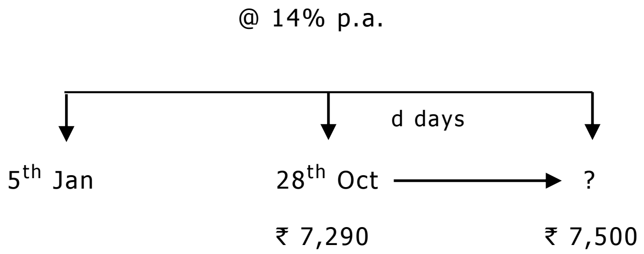
IDLE TIME ON M_2 = T − 22 = 6 HRS

(3 + 3 = 6 , CHECKED)

(B) Attempt ANY TWO of the following (08)

01. a bill of ₹ 7,500 was discounted for ₹ 7290 at a bank on 28th October 2006 . If the rate of interest was 14% p.a. , what is the legal due date

SOLUTION



STEP 1 :

Let Unexpired period = d days

STEP 2 :

B.D. = F.V. – C.V.

= 7,500 – 7,290

= ₹ 210

STEP 3 :

B.D. = Int on F.V. for 'd' days @ 14 % p.a.

$$210 = 7500 \times \frac{d}{365} \times \frac{14}{100}$$

$$d = \frac{210 \times 73}{15 \times 14}$$

d = 73 days

STEP 4 :

Legal Due date

= 28th Oct + 73 days

	OCT		NOV		DEC		JAN
=	3	+	30	+	31	+	9

= 9th January 2007

02. The following data give the marks of 20 students in Mathematics (X) and Statistic (Y) each out of 10 , expressed as (x,y) . Construct ungrouped frequency distribution considering single number as a class. Also prepare marginal distributions

(2,7) ; (3,8) ; (4,9) ; (2,8) ; (2,8) ; (5,6) ; (5,7) ; (4,9) ; (3,8) ; (4,8) ;

(2,9) ; (3,8) ; (4,8) ; (5,6) ; (4,7) ; (4,7) ; (4,6) ; (5,6) ; (5,7) ; (4,6)

SOLUTION

BIVARIATE FREQUENCY DISTRIBUTION TABLE

MARKS IN STATISTICS Y	MARKS IN MATH (X)				
	2	3	4	5	TOTAL
6			|| 2	||| 3	5
7	| 1		|| 2	|| 2	5
8	|| 2	||| 3	|| 2		7
9	| 1		|| 2		3
TOTAL	4	3	8	5	N = 20

MARGINAL FREQUENCY DISTRIBUTION OF X

X	2	3	4	5	TOTAL
F	4	3	8	5	20

MARGINAL FREQUENCY DISTRIBUTION OF Y

Y	6	7	8	9	TOTAL
F	5	5	7	3	20

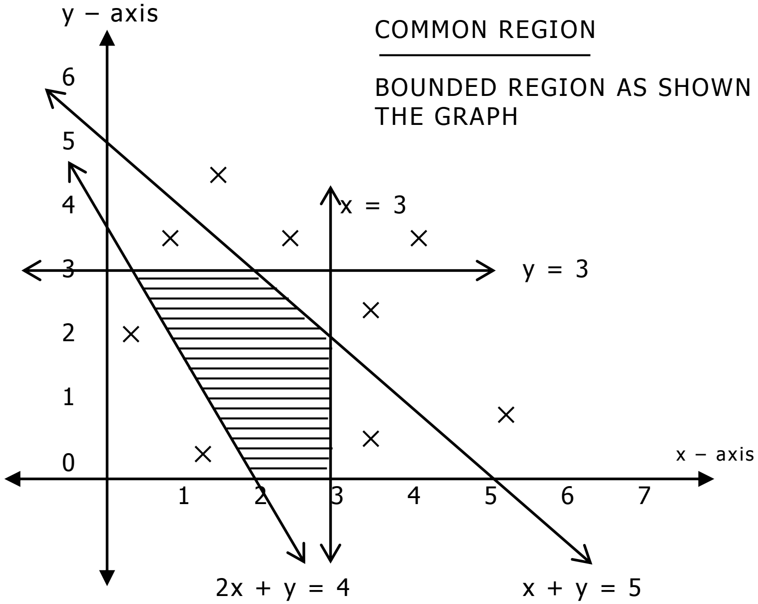
03. Find the feasible solution for the following system of linear inequations

$0 \le x \le 3$; $0 \le y \le 3$; $x + y \le 5$; $2x + y \ge 4$

SOLUTION

$x + y \le 5$	$x + y = 5$	Put (0,0) in $x + y \le 5$
	cuts x – axis at (5,0)	$0 \le 5$
	cuts y – axis at (0,5)	SS : ORIGIN SIDE
$2x + y \ge 4$	$2x + y = 4$	Put (0,0) in $2x + y \ge 4$
	cuts x – axis at (2,0)	$0 \ge 4$
	cuts y – axis at (0,4)	(NOT SATISIFIED)
		SS : NON ORIGIN SIDE
$x \le 3$	$x = 3$	Put (0,0) in $x \le 3$
	parallel to y – axis	$0 \le 3$
	passing through (3,0)	SS : ORIGIN SIDE
$y \le 3$	$y = 3$	Put (0,0) in $y \le 3$
	parallel to x – axis	$0 \le 3$
	passing through (0,3)	SS : ORIGIN SIDE
$x , y \ge 0$		SS : I QUADRANT

SCALE : 1 CM = 1 UNIT



COMMON REGION

BOUNDED REGION AS SHOWN IN THE GRAPH