SUGGESTED SOLUTION

FYJC

SUBJECT- STATISTICS

Test Code – FYJ 6017

BRANCH - () (Date :)

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ANSWER : 1

(A) Coefficient of correlation is a ratio of covariance and standard deviations.

Since, covariance and standard deviations are independent of units of measurement.

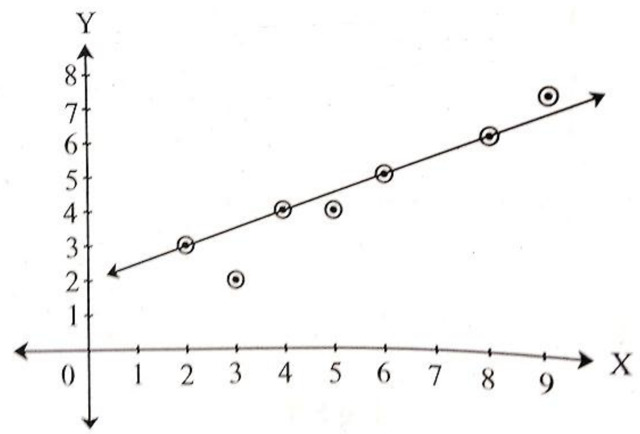
$\therefore$ coefficient of correlation is also independent of units of measurement.

$\therefore$ values of coefficient of correlation obtained by first and second investigators are same.

(02)

(B) Here we take capital on X – axis and profit on Y – axis and plot the points as below,

Scale : on X – axis 1 cm = 1 Cr
On Y – axis 1 cm = 1 L



(02)

(C) Given r = 0.48, Cov (X, Y) = 36

Since $\sigma_x^2 = 16$

$\therefore \sigma_x = 4$

Since, $r = \frac{Cov(X,Y)}{\sigma_x \sigma_y}$

$\therefore 0.48 = \frac{36}{4 \times \sigma_Y}$

$\therefore \sigma_Y = \frac{36}{0.48 \times 4} = \frac{9}{0.48}$

$= \frac{900}{48} = 18.75$

$\therefore$ Standard deviation of y is 18.75

(02)

ANSWER : 2

(A) Given, n = 50, σ_x =4.8, σ_y = 3.5, $\Sigma(x_i - \bar{x})\,(y_i - \bar{y})$ = 420

Cov (X, Y) = $\frac{1}{n}\sum(x_i - \bar{x})(y_i - \bar{y})$

= $\frac{1}{50} \times 420$

$\therefore$ Cov (X, Y) = 8.4

r = $\frac{\text{Cov}(X,Y)}{\sigma_X \sigma_Y} = \frac{8.4}{(4.8)(3.5)} = \frac{84 \times 10}{48 \times 35} = \frac{1}{2}$ = 0.5

(03)

(B) We are given that $\sum x_i = 140$, $\sum y_i = 150$, $\sum(x_i - 10)^2 = 180$, $\sum(y_i - 15)^2 = 500$, and $\sum(x_i - 10)\,(y_i - 15)$ = 60.

Let us define $u_i = x_i = 10$ and $v_i = y_i - 15$, then, we have,

$\sum u_i = \sum(x_i - 10) = \sum x_i - \sum 10 = \sum x_i - 10n$ = 140 – 10 × 10 = 40.

$\sum v_i = \sum(y_i - 15) = \sum y_i - \sum 15 = \sum y_i - 15n$ = 150 – 150 = 0.

$\sum u_i^2 = \sum(x_i - 10)^2$ = 180.

$\sum v_i^2 = \sum(y_i - 15)^2$ = 500.

$\sum u_i v_i = \sum(x_i - 10)(y_i - 15)$ = 60.

$\bar{u} = \frac{\sum u_i}{n} = \frac{40}{10}$ = 4. $\bar{v} = \frac{\sum v_i}{n} = \frac{0}{10}$ = 0.

$\sigma_u = \sqrt{\frac{1}{n}\sum_{i=1}^{n} u_i^2 - \bar{u}^2} = \sqrt{\frac{180}{10} - 4^2}$

$= \sqrt{18 - 16} = \sqrt{2}$

$\sigma_v = \sqrt{\frac{1}{n}\sum_{i=1}^{n} v_i^2 - \bar{v}^2} = \sqrt{\frac{500}{10} - 0^2}$

$= \sqrt{50 - 0} = \sqrt{50}$

$\therefore$ cov (u, v) $\frac{1}{n}\sum u_i v_i - \bar{u}\,\bar{v} = \frac{60}{10} - (4)(0)$ = 6

$r_{uv} = \frac{\text{cov}(u,v)}{\sigma_u \sigma_v} = \frac{6}{\sqrt{2}\sqrt{50}}$ = 0.6

But $r_{xy} = r_{uv}$ = 0.6.

(03)

ANSWER : 3

(A) Here, $r = 0.8$, $\Sigma x_i y_i = 60$, $\sigma_Y = 2.5$, $\sum x_i^2 = 90$

Here, x_i and y_i are the deviations from their respective means.

$\therefore$ If X_i, Y_i are elements of x and y series respectively, then

$X_i - \bar{x} = x_i$ and $Y_i - \bar{y} = y_i$

$\therefore \Sigma x_i y_i = \Sigma (X_i - \bar{x})(Y_i - \bar{y}) = 60, \sum x_i^2 = \sum (X_i - \bar{x})^2 = 90$

Now, $\sigma_x^2 = \frac{\sum (X_i - \bar{x})^2}{n}$

$\therefore \sigma_x^2 = \frac{90}{n}$

$\therefore \sigma_x = \sqrt{\frac{90}{n}}$

Also, $\text{Cov}(X, Y) = \frac{1}{n} \sum (X_i - \bar{x})(Y_i - \bar{y})$

$\therefore \text{Cov}(X, Y) = \frac{60}{n}$

$r = \frac{\text{Cov}(X,Y)}{\sigma_x \sigma_Y}$

$\therefore 0.8 = \frac{\frac{60}{n}}{\sqrt{\frac{90}{n}} \times 2.5}$

$\therefore 0.8 \times 2.5 \times \sqrt{\frac{90}{n}} = \frac{60}{n}$

$\therefore 2 \times \frac{\sqrt{90}}{\sqrt{n}} = \frac{60}{n}$

$\therefore \frac{n}{\sqrt{n}} = \frac{60}{2 \times \sqrt{90}}$

$\therefore \frac{\sqrt{n} \times \sqrt{n}}{\sqrt{n}} = \frac{30}{\sqrt{90}} = \frac{\sqrt{30} \times \sqrt{30}}{\sqrt{3}\,\sqrt{30}}$

$\therefore \sqrt{n} = \sqrt{10}$

$\therefore n = 10$

(04)

(B) (i) Let X = x_i, Y = y_i and missing observation be 'a'.

Given, $\bar{x}$ = 6, $\bar{y}$ = 8, n = 5

$\bar{y} = \frac{\sum y_i}{n}$

$\therefore$ 8 = $\frac{35+a}{5}$

$\therefore$ 40 = 35 + a

$\therefore$ a = 5

(ii) We construct the following table :

	x_i	y_i	x_i^2	y_i^2	x_iy_i
	6	9	36	81	54
	2	11	4	121	22
	10	a = 5	100	25	50
	4	8	16	64	32
	8	7	64	49	56
Total	**30**	**40**	**220**	**340**	**214**

From the table, we have

$\sum x_i$ = 30, $\sum y_i$ = 40, $\sum x_i^2$ = 220, $\sum y_i^2$ = 340, $\sum x_i y_i = 214$

Since, Cov (X, Y) = $\frac{1}{n}\sum x_i y_i - \bar{x}\ \bar{y}$

$\therefore$ Cov (X, Y) = $\frac{1}{5} \times 214 - 6\ \times 8$

= 42.8 – 48

= – 5.2

$\sigma_x^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$

= $\frac{220}{5} - (6)^2 - 44 - 36$

$\therefore\ \sigma_x^2 = 8$

$\therefore$ σ_X = $\sqrt{8} = 2\sqrt{2}$ = 2 (1.4142) = 2.83

σ_Y^2 = $\frac{\sum y_i^2}{n} - (\bar{y})^2$

= $\frac{340}{5} - (8)^2$ = 68 – 64

$\therefore$ σ_Y^2 = 4

$\therefore\ \sigma_Y = \sqrt{4} = 2$

Thus, the correlation coefficient between X and Y is

$$r = \frac{\text{Cov (X,Y)}}{\sigma_X \sigma_Y}$$

$$= \frac{-5.2}{2.83 \times 2}$$

$$= \frac{-2.6}{2.83}$$

$$= -0.92$$

(04)