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CLASSES

MATHEMATICS AND STATISTICS

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THE RANKERS FACTORY

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S.Y.J.C.
MATHEMATICS

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S.Y.J.C. – MATHEMATICS (PART 2)

INDEX

CHAPTER 1 - COMMISSION, BROKERAGE & DISCOUNT

1. Commission:

The charges paid to an agent for doing the work on behalf of principal is called commission.

Agent Commission = Sales X Rate of Commission

Seller Get = Sales – Agent Commission

2. Brokerage:

The broker is an agent who brings together the buyer and the seller for the purpose of purchase or sale.

This commission is called brokerage & is charged to both the parties.

3. Auctioneer:

An auctioneer is an agent who sells goods by auction. He sells goods to the highest bidder many a time name of principal is not disclosed.

4. Factor:

A factor is an agent who is given the possession of goods & enters a contract for Sale in his/her own name.

5. Del Credere agent

A Del Credere agent gives guarantee to his principal that the party to he/she sells the goods will pay the sale price of goods.

If the buyer is unable to pay after the transaction is completed, a Del Credere agent is liable for the payment.

Agent gets additional Commission other than the usual Commission for this. This commission is known as del credere Commission.

Del credere Commission = Sales X Rate of del credere

6. a. No. of Unit Sold = $\frac{\text{Sales}}{\text{Price per Unit}}$

b. r % Commission on Sales over Rs 10,000

It means r % comm. On Rs (X -10000)

↓

Total sales

E.g. suppose, Sales ⟶ Rs 28,000

5% Comm. On sales upto. Rs 10,000

7% comm. On sales over Rs 10,000

So, 5% comm. On Rs 10,000
7% comm. On Rs (28,000 – 10,000)
i.e. Rs 18,000

Comm. = $\left(\frac{5}{100} \times 10{,}000\right) + \left(\frac{7}{100} \times 18{,}000\right)$
= 500 + 1260
= Rs. 1760

7. List price, invoice price, Trade Disc. & Cash Discount Concept

a. List price/ marked price/catalogue price

b. Trade Disc.
- Always calculated on list price only
- Disc. Given on bulk Quantity

c. Invoice price
- Difference b/w list price & Trade disc

Invoice price = list Price – Trade discount

d. Cash Discount
- Always calculated on invoice price
- It is allowed in consideration of ready cash payment / on spot payment.

e. Net Price
- It is net selling price getting after allowing trade discount & cash discount.

Net Price = Invoice price – Cash discount

Banker's Discount

1. Present worth, sum due & True Discount Concept

a. Present worth (P.W) ⟶ Also Known as principal

b. True Discount (T.D)
- Difference b/w Sum due & Present worth
- Interest on present worth for the due period.

True discount = Sum due - Present Worth
= $\frac{\text{P.W} \times \text{n} \times \text{r}}{100}$

C. Sum due

Total amount due at the end of 'n' period

Sum due = Present worth + True discount

2. Discounting of bill Concept:

a. Drawer & Drawee:

- Person who draws the bill is called the drawer
- A person on when the bill is drawn is called Drawee.

b. Face Value

- Amount for which bill is drawn
- It is the sum due on present worth.

C. Date of bill – Date on which bill is drawn

d. Nominal due date

- Date on which the period of bill expires.

N.D.D = Date of bill Drawn + Period of Bill

e. Legal Due Date

- After bill expires drawee is allowed to pay 3 days later [i.e. 3 Grace Days]

L.D.D = N.D.D + 3 Grace days

Date of bill Drawn – 1st April, 2020
Period of Bill - 6 month

Date of Bill Drawn		01/04/2020
+ P.O.B	+	6 month
N.D.D	⟶	01/10/2020
	+	3 Grace days
L.D.D	⟶	04/10/2020

f. Banker's Discount

When a bill is discounted in a bank, the banker will deduct the amount from the face value of the bill at the given rate of interest for the period from the date of bill discounting to legal due date. This amount is known as Banker's Discount (B.D.)

The banker's discount is called commercial discount.

g. Cash value

Amount paid to the holder of the bill after deducting banker's discount is known as cash value

Cash value (CV) = F.V – B.D

h. **Banker's Gain**

True discount $\xrightarrow[\text{on}]{\text{Calculated}}$ Present worth

banker's discount $\xrightarrow[\text{on}]{\text{Calculated}}$ Face value/ sum due

It means Banker's Discount $>$ True Discount

- Difference b/w banker's discount & true discount is called Banker's gain.
- It is equal to interest on True Discount.

$$B.G. = BD - TD$$

OR

$$B.G. = \frac{TD \times n \times r}{100}$$

i. List of formula:

1. Banker's Discount (B.D) $= \frac{FV \times n \times r}{100}$

OR

$= FV - CV$

OR

$= BG + T.D$

here,

n - No of days from date of bill discounting to legal due date.

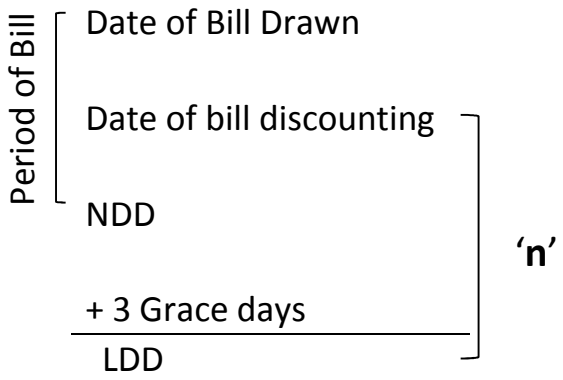
2. Cash Value (CV) = Face value – Banker's Discount

3. Banker's gain (BG) = BD -TD

OR

$= \frac{TD \times n \times r}{100}$

J. Sequence of Date's:



To find LDD ⟶ 1. NDD + 3 Grace days
OR
2. Date of bill disc. +n days

To find Date of bill discounting ⟶ **LDD** – '**n**' days

OBJECTIVES

I) Choose the correct alternative.

1. An agent who gives guarantee to his principal that the party will pay the sale price of goods is called
a. Auctioneer b. Del Credere Agent
c. Factor d. Broker

2. An agent who is given the possession of goods to be sold is known as
a. Factor b. Broker
c. Auctioneer d. Del Credere Agent

3. The date on which the period of the bill expires is called
a. Legal Due Date b. Grace Date
c. Nominal Due Date d. Date of Drawing

4. The payment date after adding 3 days of grace period is known as
a. The legal due date b. The nominal due date
c. Days of grace d. Date of drawing

5. The sum due is also called as
a. Face value b. Present value
c. Cash value d. True discount

6. P is the abbreviation of
a. Face value b. Present worth
c. Cash value d. True discount

7. Banker's gain is simple interest on
a. Banker's discount b. Face Value
c. Cash value d. True discount

8. The marked price is also called as
a. Cost price b. Selling price
c. List price d. Invoice price

9. When only one discount is given then
a. List price = Invoice price b. Invoice price = Net selling price
c. Invoice price = Cost price d. Cost price = Net selling price

10. The difference between face value and present worth is called
a. Banker's discount b. True discount
c. Banker's gain d. Cash value

II) Fill in the blanks.

1. A person who draws the bill is called_____
2. An_____is an agent who sells the goods by auction.
3. Trade discount is allowed on the_____ price.
4. The banker's discount is also called_____.
5. The banker's discount is always_____than the true discount.
6. The difference between the banker's discount and the true discount is called_____.
7. The date by which the buyer is legally allowed to pay the amount is known as_____.
8. A_____is an agent who brings together the buyer and the seller.
9. If buyer is allowed both trade and cash discounts, discount is first calculated on_____price.
10. _____= List price (catalogue Price) – Trade Discount.

III) State whether each of the following is True or False.

1. Broker is an agent who gives a guarantee to seller that the buyer will pay the sale price of goods.
2. Cash discount is allowed on list price.
3. Trade discount is allowed on catalogue price.
4. The buyer is legally allowed 6 days grace period.
5. The date on which the period of the bill expires is called the nominal due date.
6. The difference between the banker's discount and true discount is called sum due.
7. The banker's discount is always lower than the true discount.
8. The bankers discount is also called as commercial discount.
9. In general cash discount is more than trade discount.
10. A person can get both, trade discount and cash discount.

Answers:

I)
1. b. Del credere agent
2. a. factor
3. c. nominal due date
4. a. The legal due date
5. a. Face Value
6. b. Present worth
7. d. True discount
8. b. List Price
9. b. Invoice price = Net selling price
10. b. True discount

II)
1. Drawee
2. Auctioneer
3. Catalogue/list
4. Commercial Discount
5. higher
6. Bankers Gain
7. Legal due date
8. A broker
9. Trade, Catalogue / list
10. Invoice Price

III)
1. Flase
2. False
3. True
4. False
5. True
6. False
7. False
8. True

CHAPTER 2 - INSURANCE & ANNUITY

Introduction:

- The verb "to insure" means to arrange for compensation in the event of damage or total loss of property or injury or the death of someone, in exchange of regular payments to a company or to the state.
- Protection The word "insurance" means creation of some security or monetary against a possible damage or loss.
- An insurance policy is a legal document of the contract or agreement b/w the two paries, insurer (insurance company) & insured (person covered by insurance)

Types:

i) Life Insurance
ii) General Insurance

1. **Life Insurance**:
   - Person who wishes to be insured for life agree to pay the insurance company a certain amount of money called as premium.
   - In return, the insurance company agrees to pay a definite amount (called as policy value) in the event of death of insured person or maturity of the policy.

2. **General Insurance:**
   - Covers all types of insurance **except life insurance**
   - Allows a person to insure properties like building's, factories & godowns Containing goods against a possible loss (total or partial) due to fire, flood, etc.
   - Vehicles can be insured to cover risk of possible damage due to accidents.
   - All Contracts of general insurance are governed by the principle of indemnity
   - Principle of indemnity states that an insured may not be compensated by insurance company in an amount exceeding the insured's economic loss.

   As a result an insured person cannot make profit from an insurance company.

   A] Fire Insurance
   - Fire insurance is property insurance that covers damage & losses caused by fire to property like buildings, godowns containing goods, etc.
   - Period of fire insurance policy is **1 year**.

B] Accident Insurance

- Accident insurance allows insuring vehicles like cars, trucks, two wheelers etc. Against to a vehicle due to accidents.
- This policy also covers the liability of the insured person to third parties involved in the accident.
- Period of accident insurance policy is **1 year**.

C] Marine Insurance

- Marine insurance covers goods, freight cargo, etc. against loss or damage during transit by road, rail, sea or air.
- Shipments are protected from the time they leave the seller's warehouse till the time they reach the buyer's warehouse.

➢ **Terminology**:

1. Property Value:
Value of entire property

2. Policy Value:
Amount of property insured.

3. Premium:
Amount paid to the insurance company to insure the property.

Amount of policy = Policy Value X Rate of premium

4. Loss:
Value of damage.

- **Damage to the extent of 80% / reduced by 80%**
  Loss = 80% of property value
- **Reduced to 80%**
  Loss = 20% of property value

5. Claim:

- Amount paid by insurance company to insured person in the event of loss.

$$\text{Claim} = \frac{\text{Policy value}}{\text{Property value}} \times \text{Loss}$$

ANNUITY

- **Introduction:**
  An annuity is a series of payments at fixed intervals guaranteed for a fixed number of years or the lifetime of one or more individuals.

- **Terminology**:

A] Four parties of an Annuity:

1. Annuitant :
   A person who receives an annuity

2. Issuer :
   A Company (usually an insurance company) that issues an annuity.

3. Owner :
   An individual or an entity that buys an annuity from the issuer of the annuity & makes Contributions to the annuity.

4. Beneficiary:
   A person who receives a death benefit from an annuity at the death of the annuitant

B] **Two phases of an annuity**

1. Accumulation phase (OR Investment phase)
2. Distribution phase.

C] **Types of Annuities:** There are three types of annuities

1. Annuity Certain

An Annuity certain is an investment that provides a series of payments for a set period of time to a person or to person's beneficiary.

2. Contingent Annuity

- Contingent Annuity is a form of annuity contract that provides payments at the time when the named contingency occurs.
- Annuity paid till the happening of that event.

3. Perpetual Annuity or Perpetuity

- A perpetual annuity also called as perpetuity promises to pay a certain amount of money to its owner forever.
- Though a perpetuity may promise to pay you forever, its value isn't forever.

D] Classification of Annuities:

1. Immediate Annuity or Ordinary Annuity or Annuity Regular:
   - Annuity made at the end of each payment period.

2. Annuity Due:
   - Annuity made at the beginning of each payment period.
   - First payment is made as soon as the contract is finalized.

3. Deferred Annuity:
   - In this annuity contract the payment of annuity starts after a deferment period or at the attainment by the annuitant of a specified age.

4. Forborne annuity

   Annuity remains Unpaid for certain period of time.
   - Periodic Payment – Size of each Payment of an annuity.
   - Payment Period – Time b/w 2 successive payments
   - Present Value of an annuity

     The present value of an annuity is the current value of future payments from an annuity.

     The annuity's future cash flows are discounted at the discount rate.
   - Future value of an annuity

     Future value of an annuity represents the amount of money that will be accumulated by making consistent investments over a set period.

Annuity Immediate/ Ordinary Annuity/Annuity regular	**Annuity Due**
1. Future Value /Accumulated Value	
$A = \frac{C}{i}[(1+i)^n - 1]$	$A^1 = \frac{C}{i}[(1+i)^n - 1](\mathbf{1}+\mathbf{i})$
2. Present Value	
$P = \frac{C}{i}[1-(1+i)^{-n}]$	$P^1 = \frac{C}{i}[1-(1+i)^{-n}](\mathbf{1}+\mathbf{i})$
Relation b/w A & P	
$A = P(1+i)^n$ $\frac{1}{P} - \frac{1}{A} = \frac{i}{C}$	$A^1 = P^1(1+i)^n$ $\frac{1}{P^1} - \frac{1}{A^1} = \frac{i}{C(1+i)}$

Where

C - Amount of each annuity payment

i - Interest rate per compounding Period

n - No of times interest compounded in term

Interest compounded Monthly	Interest compounded Quarterly	Interest compounded half yearly	Interest compounded yearly
$i = \frac{r}{1200}$	$i = \frac{r}{400}$	$i = \frac{r}{200}$	$i = \frac{r}{100}$
$n = 12N$	$n = 4N$	$n = 2N$	$n = N$

Where

r – Rate of interest p.a.

N – No. of years.

Note:

1. If type of annuity not mentioned then always assume annuity immediate.
2. If type of interest is not mention then it is assumed that the interest is compounded per annum.

OBJECTIVES

I) choose the correct alternative.

1. "A contract that pledges payment of an agreed upon amount to the person (or his/ her nominee) on the happening of an event covered against" is technically known as
   a. Death coverage b. Savings for future
   c. Life insurance d. Provident fund

2. Insurance companies collect a fixed amount from their customers at a fixed interval of time. This amount is called
   a. EMI b. Installment
   c. Contribution d. Premium

3. Following are different types of insurance.
   I. Life insurance
   II. Health insurance
   III. Liability insurance
   (a) Only I (b) Only II (c) Only III (d) All the three

4. By taking insurance, an individual
   a. Reduces the risk of an accident
   b. Reduces the cost of an accident
   c. Transfers the risk to someone else.
   d. Converts the possibility of large loss to certainty of a small one.

5. You get payments of Rs.8,000 at the beginning of each year for five years at 6%, what is the value of this annuity?
   a. Rs 34,720 b. Rs 39,320 c. Rs 35,720 d. Rs. 40,000

6. In an ordinary annuity, payments or receipts occur at
   a. Beginning of each period b. End of each period
   c. Mid of each period d. Quarterly basis

7. Amount of money today which is equal to series of payments in future is called
   a. Normal value of annuity b. Sinking value of annuity
   c. Present value of annuity d. Future value of annuity

8. Rental payment for an apartment is an example of
   a. Annuity due b. Perpetuity
   c. Ordinary annuity d. Installment

9. is a series of constant cashflows over a limited period of time.
   a. Perpetuity b. Annuity
   c. Present value d. Future value

10. A retirement annuity is particularly attractive to someone who has
   a. A severe illness b. Risk of low longevity
   c. Large family d. Chance of high longevity

II) Fill in the blanks

1. An installment of money paid for insurance is called_____.
2. General insurance covers all risks except_____.
3. The value of insured property is called_____.
4. The proportion of property value to insured value is called_____.
5. The person who receives annuity is called_____.
6. The payment of each single annuity is called_____.
7. The intervening time between payment of two successive installments is called as_____.
8. An annuity where payments continue forever is called_____.
9. If payments of an annuity fall due at the beginning of every period, the series is called annuity_____.
10. If payments of an annuity fall due at the end of every period, the series is called annuity_____.

III) State whether each of the following is True or False.

1. General insurance covers life, fire, and theft.
2. The amount of claim cannot exceed the amount of loss.
3. Accident insurance has a period of five years.
4 Premium is the amount paid to the insurance company every month.
5. Payment of every annuity is called an installment.
6. Annuity certain begins on a fixed date and ends when an event happens.
7. Annuity contingent begins and ends on certain fixed dates.
8. The present value of an annuity is the sum of the present value of all installments.
9. The future value of an annuity is the accumulated values of all installments.
10. Sinking fund is set aside at the beginning of a business.

Answers:

I)
1. c
2. d
3. d
4. d
5. c
6. b
7. c
8. b
9. b
10. d

II)
1. Premium
2. Life
3. Property value
4. Policy value
5. Annuitant
6. Installment
7. Payment period
8. Perpetuity
9. Annuity due
10. Immediate annuity R.

III)
1. False
2. True
3. False
4. True
5. False
6. True
7. False
8. True
9. False
10. True

CHAPTER 3 - LINEAR REGRESSION

Basic Concepts

1. The technique used for predicting the value of one variable for a given value of the other variable is called regression.
2. Regression is a statistical tool for investigating the relationship between variables.
3. Value being predicted is called the response or dependent variable.
4. Value used for predicting the response or dependent variables are called predictors or independent variables.
5. A linear regression model consists of a linear equation with unknown coefficient (called as parameters of linear regression model.)
6. Correlation analysis is used for measuring the strength or degree of the relationship & between the independent & dependent variable.

Regression Coefficients of:

a) X on Y:

$$b_{xy} = \frac{\text{Cov.}(x,y)}{\text{var.}(y)}$$

$$= \frac{\text{Cov.}(x,y)}{(\sigma y)^2}$$

$$= \frac{\frac{\sum xy}{n} - (\bar{x}),(\bar{y})}{\frac{\sum y^2}{n} - (\bar{y})^2} \Rightarrow \frac{n.\sum xy - \sum x.\sum y}{n.\sum y^2 - (\sum y)^2}$$

$$= \frac{\frac{\sum(x-\bar{x}),(y-\bar{y})}{n}}{\frac{\sum(y-\bar{y})^2}{n}} \Rightarrow \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(y-\bar{y})^2}$$

$$= r.\frac{\sigma x}{\sigma y}$$

b) Y on X:

$$b_{yx} = \frac{\text{cov.}(x,y)}{\text{Var.}(x)}$$

$$= \frac{\text{cov.}(x,y)}{(\sigma x)^2}$$

$$= \frac{\frac{\sum xy}{n} - (\bar{x}),(\bar{y})}{\frac{(\sum x)^2}{n} - (x)^2} \Rightarrow \frac{n.\sum xy - (\sum x)(\sum y)}{n.\sum x^2 - (\sum x)^2}$$

$$= \frac{\frac{\sum(x-\bar{x}),(y-\bar{y})}{n}}{\frac{\sum(x-\bar{x})^2}{n}} \Rightarrow \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$$

$$= r.\frac{\sigma y}{\sigma x}$$

Where

- Covariance of X & Y:

$$\text{Cov.}(x,y) = \frac{\sum xy}{n} - (\bar{x}),(\bar{y})$$

$$= \frac{\sum(x-\bar{x}),(y-\bar{y})}{n}$$

- Variance of X

$$\text{Var}(x) = \frac{\sum x^2}{n} - (\bar{x})^2 \quad \text{Or} \quad \frac{\sum(x-\bar{x})^2}{n}$$

- Variance of Y

$$\text{Var}(y) = \frac{\sum y^2}{n} - (\bar{y})^2 \quad \text{Or} \quad \frac{\sum(y-\bar{y})^2}{n}$$

- Standard Deviation $(\text{SD}) = \sqrt{\text{Variance}}$

$$\sigma x = \sqrt{\text{Var}(x)}$$

$$\sigma y = \sqrt{\text{Var}(y)}$$

- Regression lines / Regression Equations

(a) x on y :

$x \to ?$ $\quad x \to$ Dependent variable
$y \to 10$ $\quad y \to$ Independent variable

x Dependend on y

$$(x - \bar{x}) = \text{b}xy\,(y - \bar{y})$$

(b) y on x :

$x \to 12$ $\quad x \to$ Independent variable
$y \to ?$ $\quad y \to$ Dependent variable

y Dependend on x

$$(y - \bar{y}) = \text{b}yx\,(x - \bar{x})$$

- Karl Person's Correlation coefficient/ product moment correlation coeff.

Denoted by "**r**"

$$-1 < \boldsymbol{r} < 1$$

$$r = \frac{Cov.(x,y)}{\sigma x,\sigma y} \quad \text{or} \quad \frac{Cov.(x,y)}{\sqrt{Var.(x)},\sqrt{Var.(y)}}$$

$$= \pm\sqrt{\text{b}xy.\,\text{by}x}$$

Note:
If bxy & by$x$ both are positive then r must be positive and vice-versa

- Identification of Regression lines

Given two regression lines without stating which is y on x & which is x on y.

i) Based on Assumption:

Follow the steps:

1. Assume one regression equation as Y on X & other as X on Y

2. Write Y on X equation as
   $y = Ax + B$
   Where $A \rightarrow byx$
   Write X on Y equation as
   $x = Ax + B$
   Where $A \rightarrow bxy$

3. Compute: r^2
   $r^2 = bxy, byx$

4. Check: $\boxed{0 \leq r^2 \leq 1}$

If satisfied then assumption are correct else reverse the assumption

ii) Slope method (Alternate Method)

Steps:

1. From 1st Eqn.
   $ax + by = c$
   Find: $\mathrm{M}_1 = \frac{-\text{Coeff of x}}{\text{Coeff of y}} = \frac{-\mathrm{a}}{\mathrm{b}}$

2. From 2nd Eqn.
   $dx + ey = f$
   Find: $\mathrm{M}_2 = \frac{-\text{Coeff of x}}{\text{Coeff of y}} = \frac{-\mathrm{d}}{\mathrm{e}}$

3. Compare
   $|\mathrm{M}_1| > |\mathrm{M}_2|$ or $|\mathrm{M}_1| < |\mathrm{M}_2|$

4. Allocate
   $byx = M_1 \text{ or } M_2$ (Whichever is less)
   $bxy = \frac{1}{\boldsymbol{M_1 \text{ or } M_2}}$ (Whichever is high)

5. Identify Reg. lines
   If byx get from M_1 then 1st Eqn. is y on x & if bxy get from M_2 then 2nd Eqn.is x on y and vice-versa.

- **Equation already mentioned that given eqn. is X on Y or Y on X**.

  From x on y Eqn.

  $\mathrm{b}xy = \frac{-\text{Coeff of y}}{\text{Coeff of } x}$

  From y on x Eqn.

  $\mathrm{b}yx = \frac{-\text{Coeff of } x}{\text{Coeff of } y}$

- To find $\bar{x}, \bar{y}$:
  - Line of regression have a point of intersection $(\bar{x}, \bar{y})$
  - Do Simultaneous Eqn. and find value of x & y.

  Value of $x \to \bar{x}$
  Value of $y \to \bar{y}$

- Shift of origin & change of scale
  - Regression coeff. are independent of change of origin but not of scale.
  - bxy & byx are not affected by change of origin but are affected by change of scale.
    This property is known as **Invariance property**
  - Invariant property states that bxy & byx are invariant under change of origin but are not invariant under change of scale.

    Let $\mathrm{U} = \frac{x-a}{c}$ & $\mathrm{V} = \frac{y-b}{d}$

    (i) $bxy = \frac{c}{d}$ buv $\qquad$ $byx = \frac{d}{c}$ bvu

    (ii) If c & d are of same sign
    Then, $r_{xy} = r_{uv}$

    If c & d are of opp. Sign
    Then, $r_{xy} - r_{uv}$

- Properties of Regression Coeff.
  1) Correlation Coeff. is symmetric
     i.e. $\mathrm{r}_{xy} = \mathrm{r}_{yx}$ but regression Coeff. are not symmetric $(\mathrm{b}xy \neq \mathrm{b}yx)$
  2) When $r = 0$, both regression coefficients are 0
  3) When $r = \pm 1$ then,
     a) Two regression lines become identical i.e. they coincide.
     b) $\mathrm{b}xy = \frac{1}{\mathrm{b}yx}$ & $\mathrm{b}yx = \frac{1}{\mathrm{b}xy}$
  4) If $\mathrm{b}yx > 1$ then $\mathrm{b}xy < 1$

5) $\left|\frac{bxy+byx}{2}\right| \geq |r|$

6) $r^2 = bxy.byx$

$r = \pm\sqrt{bxy.byx}$

Sign analogy of $bxy.byx$ & r

bxy	byx	r
+	+	+
-	-	-

7) **Case I**

bxy = + ve

byx = - ve

Case II

bxy = - ve

byx = + ve

In Both Case

↓

"r will be imaginary"

Data is In Consistent

OBJECTIVES

I) Choose the correct alternative.

1. Regression analysis is the theory of
   a) Estimation b) Prediction
   c) Both a and b d) Calculation

2. We can estimate the value of one variable with the help of other known variable only if they are
   a) Correlated b) Positively correlated
   c) Negatively correlated d) Uncorrelated

3. There are _____types of regression equations.
   a) 4 b) 2 c) 3 d) 1

4. In the regression equation of Y on X
   a) X is independent and Y is dependent.
   b) Y is independent and X is dependent.
   c) Both X and Y are independent.
   d) Both X and Y are dependent.

5. In the regression equation of X on Y
   a) X is independent and Y is dependent.
   b) Y is independent and X is dependent.
   c) Both X and Y are independent.
   d) Both X and Y are dependent.

6. b_{XY} is_____
   a) Regression coefficient of Y on X
   b) Regression coefficient of X on Y
   c) Correlation coefficient between X and Y
   d) Covariance between X and Y

7. b_{YX} is_____
   a) Regression coefficient of Y on X
   b) Regression coefficient of X on Y
   c) Correlation coefficient between X and Y
   d) Covariance between X and Y

8. 'r' is _____
   a) Regression coefficient of Y on X
   b) Regression coefficient of X on Y
   c) Correlation coefficient between X and Y
   d) Covariance between X and Y

9. $b_{XY}.\ b_{YX}$_____2
   a) $v(x)$ b) σ_x c) r^2 d)$(\sigma_y)^2$

10. If $b_{yx} > 1$ then b_{xy} is_____
    a) > 1 b) < 1 c) > 0 d) < 0

11. $|b_{xy} + b_{yx}| \geq$ _____
    a) $|r|$ b) $2|r|$ c) r d) 2r

12. b_{xy} and b_{yx} are_____
    a) Independent of change of origin and scale
    b) Independent of change of origin but not of scale
    c) Independent of change of scale but not of origin
    d. Affected by change of origin and scale

13. If u $= \frac{x-a}{c}$ and $v = \frac{y-b}{d}$ then $b_{yx} =$_____
    a) $\frac{d}{c}\ b_{vu}$ b) $\frac{c}{d}\ b_{vu}$
    c) $\frac{a}{b}\ b_{vu}$ d) $\frac{b}{a}\ b_{vu}$

14. If u $= \frac{x-a}{c}$ and $v = \frac{y-b}{d}$ then $b_{xy} =$_____
    a) $\frac{d}{c}\ b_{uv}$ b) $\frac{c}{d}\ b_{uv}$
    c) $\frac{a}{b}\ b_{uv}$ d) $\frac{b}{a}\ b_{uv}$

15. Corr $(x, x) =$_____
    a) 0 (b) 1 (c) −1 (d) can't be found

16. Corr (x, y) = ____
    a) corr(x, x) (b) corr(y, y)
    c) corr(y, x) (d) cov(y, x)

17. $Corr\left(\frac{x-a}{c}, \frac{y-b}{d}\right) = -corr(x, y)$if,
    a) c and d are opposite in sign
    b) c and d are same in sign
    c) a and b are opposite in sign
    d) a and b are same in sign

18. Regression equation of X on Y is_____
    a) $y - \bar{y} = b_{yx}(x - \bar{x})$
    b) $x - \bar{x} = b_{xy}(y - \bar{y})$
    c) $y - \bar{y} = b_{xy}(x - \bar{x})$
    d) $x - \bar{x} = b_{yx}(y - \bar{y})$

19. Regression equation of Y on X is_____
    a) $y - \bar{y} = b_{yx}(x - \bar{x})$
    b) $x - \bar{x} = b_{xy}(y - \bar{y})$
    c) $y - \bar{y} = b_{xy}(x - \bar{x})$
    d) $x - \bar{x} = b_{yx}(y - \bar{y})$

20. b_{yx} =_____
    a) $r\frac{\sigma_x}{\sigma_y}$ b) $r\frac{\sigma_y}{\sigma_x}$
    c) $\frac{1}{r}\frac{\sigma_y}{\sigma_y}$ d) $\frac{1}{r}\frac{\sigma_x}{\sigma_y}$

21. b_{xy} =_____
    a) $r\frac{\sigma_x}{\sigma_y}$ b) $r\frac{\sigma_y}{\sigma_x}$
    c) $\frac{1}{r}\frac{\sigma_y}{\sigma_y}$ d) $\frac{1}{r}\frac{\sigma_x}{\sigma_y}$

22. $Cov\ (x, y)$ =_____
    a) $\sum(x - \bar{x})(y - \bar{y})$
    b) $\frac{\sum(x-\bar{x})(y-\bar{y})}{n}$
    c) $\frac{\sum xy}{n} - \bar{x}\bar{y}$
    d) b and c both

23. If $b_{xy} < 0$ and $b_{yx} < 0$ then 'r' is_____
    a) > 0 b)< 0 c) > 1 d)not found

24. If equations of regression lines are $3x + 2y - 26 = 0$ and $6x + y - 31 = 0$ then means of x and y are
    a) (7,4) b) (4,7) c) (2,9) d) (−4,7)

II) Fill in the blanks

1. If $b_{xy} < 0$ and $b_{yx} < 0$ then 'r' is_____
2. Regression equation of Y on X is_____
3. Regression equation of X on Y is_____
4. There are_____ types of regression equations.
5. Corr $(x, -y)$ = _____
6. If u $= \frac{x-a}{c}$ and $v = \frac{y-b}{d}$ then b_{xy} =_____
7. If u $= \frac{x-a}{c}$ and $v = \frac{y-b}{d}$ then b_{yx} =_____
8. $|b_{xy} + b_{yx}| \geq$ _____
9. If $b_{yx} > 1$ then b_{xy} is_____
10. $b_{xy}.b_{yx}$ =_____

III) State whether each of the following is True or False.

1. Corr (X, X) =1
2. Regression equation of X on Y is
   $y - \bar{y} = b_{yx}(x - \bar{x})$
3. Regression equation of Y on X is
   $y - \bar{y} = b_{yx}(x - \bar{x})$
4. Corr (x, y) = Corr (y, x)
5. b_{xy} and b_{yx} are independent of change of origin and scale.
6. 'r' is regression coefficient of Y on X
7. b_{yx} is correlation coefficient between X and Y
8. If $u = x - a$ and $v = y - b$ then $b_{xy} = b_{uv}$
9. If $u = x - a$ and $v = y - b$ then $r_{xy} = r_{ur}$ = r
10. In the regression equation of Y on X, b_{yx} represents slope of the line.

Answers:

I)
1) c
2) a
3) b
4) a
5) b
6) b
7) a
8) c
9) c
10) b
11) b
12) b
13) a
14) b
15) b
16) c
17) a
18) b
19) a
20) b
21) a
22) d
23) b
24) b

II)
1) Negative
2) $y - \bar{y} = \mathrm{b}_{yx}(x - \bar{x})$
3) $x - \bar{x} = \mathrm{b}_{xy}(y - \bar{y})$
4) 2
5) -1
6) $\frac{c}{d}\, b_{uv}$
7) $\frac{d}{c}\, b_{uv}$
8) $2|r|$
9) < 1
10) r^2

III)

1) True	2) False	3) True	4) True	5) False
6) False	7) False	8) True	9) True	10) True

CHAPTER 4 - TIME SERIES

Introduction

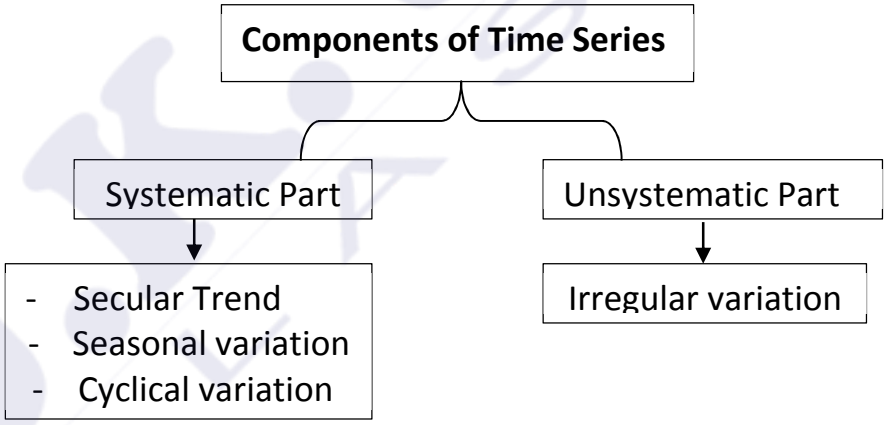
- Time series is a sequence of observations made on a variable at regular time intervals over a specified period of time.
- Time series Analysis help us in monitoring & forecasting data with help of appropriate statistical models.
- Analysis of time of time series data requires maintaining records of values of the variable overtime.
- Time series is statistical data that are arranged and presented in chronological order i.e. over a period of time.

Examples:

1. Monthly, Quarterly OR yearly production of an industrial product.
2. Monthly sales in a departmental store
3. Yearly GDP of a country
4. Daily closing price of a share at a stock exchange

Use of Time series Analysis:

- Used for studying past behaviour of a variable
- Used for forecasting future behaviour of a variable
- Used in evaluating the performance.
- Used in making a comparative study.



Note:

1. Every time series has some or all of these components.
2. Only the systematic components of a time series are useful in forecasting its future values.

A] Secular Trend or Simple Trend (T)

- It is the long term pattern of time series to move in upward or downward direction.

- Sector trend shows smooth & regular movement of time series.
- Does not include short term fluctuation but only consist of steady movement over a long period of time.
- General tendency of a variable to increase, decrease or remain constant in long term called trend of a variable

Eg.

1) Population of a country has increasing trend over a year.
2) Due to modern technology, agricultural & industrial production is increasing.

B] <u>Seasonal variation (S)</u>

- Over a span of one year, seasonal variation taken place due to rhythmic forces which operates in a regular & periodic manner.
  These forces have the same or almost similar pattern year after year.
- Seasonal variation could be seen & calculated if the data are recorded quarterly, monthly, weekly, Daily or hourly basis.
- Seasonal variation is measured with the help of seasonal indices, which are useful for short term forecasting.
- A bank manager can we such short term forecast in managing cash flow on diff. days of a week or month.
- In time series with only annual data no seasonal variation.

<u>Eg.</u>

1) Sale of cold drinks rise in summer & fall in winter
2) Sale of clothes might go up in festival like Diwali, Christmas, Navratri etc. as compared to other days.

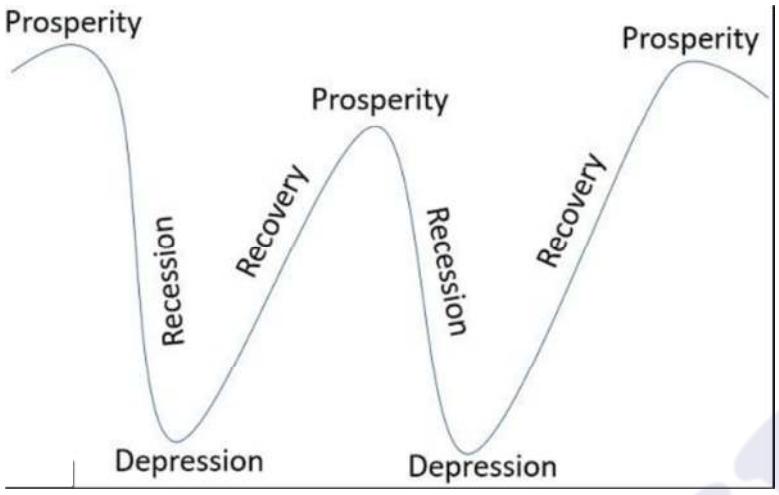
C] <u>Cyclical Variation</u>

- Cyclical variation is a long term oscillatory movement in values of a time series.
- Cyclical variation occurs over a long period.
- One complete round of Oscillation is called a cycle.
- Cyclical variation are also termed an business cycle or trade cycle

.

Typical business cycle consists of following 4 phases:

1) Prosperity
2) Recession
3) Recovery
4) Depression



D] **Irregular Variation**

Irregular Variation are unexpected variation in times series caused by unforeseen events include natural disasters like floods, or Famines, political events like strike, international events like wars or other conflicts.

- Irregular variation do not follow any patterns & it cannot be predicted in advance.
- Irregular variation are also known as unexplained variation or unaccounted variation.
- Mathematical models of Time series
  Based on **secular trend (T)**, **Seasonal variation (S)**, **Cyclical variation (C)** & **Irregular variation (I)**

Two standard Mathematical models of Time series

Additive model
$Y_t = T + S + C + I$

Multiplicative model
$Y_t = T .S. C. I$

Additive Model	**Multiplicative model**
1. Assumes that four component are independent	1. Does not assume independence of four components.
2. All four components must be measured in same unit of measurement	2. Trend (T) expressed as unit of measurement and others are expressed as percentage or relative values. Hence, are free from unit of measurement.
3. Magnitude of seasonal variation does no change as the series go up or down.	3. Magnitude of seasonal variation increases as the data value increases & decreases as data value decreases

- **Measurement of secular trend**

I) Method of free hand curve (graphical Method)

- Quit flexible to use but its risky
- Involves min. amt. of work.
- Helps to know trend not trend value.

II) <u>Method of moving Averages</u>.

- 3 yearly moving Avg.
- 4 yearly centered moving Avg.
- 5 Yearly moving Avg.

III) <u>Method of least square</u> :

Equation of Trend line are

$$Y_t = a^1 + b^1\,(U)$$

Where,

$$U = \frac{t - \text{middle } 't' \text{ value}}{h}$$

If n →odd

Or

$$= \frac{2(t - \text{mean of 2 middle } 't' \text{value})}{h}$$

If n →Even

t → time (yr)

<u>Two normal Equation are</u>

$$\sum Y_t = n.a^1 + b^1.\sum U \quad \rightarrow \quad 1$$

$$\sum UY_t = a^1.\sum U + b^1.\sum U^2 \quad \rightarrow \quad 2$$

OBJECTIVES

I) Choose the correct alternative.

1. Which of the following can't be a component of a time series?
(a)Seasonality (b) Cyclical
(c) Trend (d) Mean

2. The first step in time series analysis is to
(a) Perform regression calculations
(b) Calculate a moving average
(c) Plot the data on a graph
(d) Identify seasonal variation

3. Time-series analysis is based on the assumption that
(a) Random error terms are normally distributed.
(b) The variable to be forecast and other independent variables are correlated.
(c) Past patterns in the variable to be forecast will continue unchanged into the future.
(d) The data do not exhibit a trend.

4. Moving averages are useful in identifying
(a) Seasonal component
(b) Irregular component
(c) Trend component
(d) Cyclical component

5. We can use regression line for past data to forecast future data. We then use the line which
(a) Minimizes the sum of squared deviations of past data from the line
(b) Minimizes the sum of deviations of past data from the line.
(c) Maximizes the sum of squared deviations of past data from the line
(d) Maximizes the sum of deviations of past data from the line.

6. Which of the following is a major problem for forecasting, especially when using the method of least squares?
(a) The past cannot be known
(b) The future is not entirely certain
(c) The future exactly follows the patterns of the past
(d) The future may not follow the patterns of the past

7. An overall upward or downward pattern in an annual time series would be contained in which component of the times series
(a) Trend (b) Cyclical (c) Irregular (d) Seasonal

8. The following trend line equation was developed for annual sales from 1984 to 1990 with 1984 as base or zero year.
   Y1 = 500 + 60X (in 1000 Rs). The estimated sales for 1984 (in 1000 Rs) is:
   (a) Rs 500 (b) Rs 560 (c) Rs 1,040 (d) Rs 1,100

9. What is a disadvantage of the graphical method of determining a trend line?
   (a) Provides quick approximations
   (b) Is subject to human error
   (c) Provides accurate forecasts
   (d) Is too difficult to calculate

10. Which component of time series refers to erratic time series movements that follow no recognizable or regular pattern.
    (a) Trend (b) Seasonal (c) Cyclical (d) Irregular

II) Fill in the blanks

1. _____components of time series is indicated by a smooth line.
2. component of time series is indicated by periodic variation year after year.
3. _____component of time series is indicated by a long wave spanning two or more years.
4. _____component of time series is indicated by up and down movements without any pattern.
5. Addictive models of time series_____ independence of its components.
6. Multiplicative models of time series_____ independence of its components.
7. The simplest method of measuring trend of time series is_____
8. The method of measuring trend of time series using only averages is_____.
9. The complicated but efficient method of measuring trend of time series is_____.
10. The graph of time series clearly shows of_____ it is monotone.

III) State whether each of the following is True or False.

1. The secular trend component of time series represents irregular variations.
2. Seasonal variation can be observed over several years.
3. Cyclical variation can occur several times in a year.
4. Irregular variation is not a random component of time series.
5. Additive model of time series does not require the assumption of independence of its components.

6. Multiplicative model of time series does not require the assumption of independence of its components.
7. Graphical method of finding trend is very complicated and involves several calculations.
8. Moving average method of finding trend is very complicated and involves several calculations.
9. Least squares method of finding trend is very simple and does not involve any calculations.
10. All the three methods of measuring trend will always give the same results.

Answers:

I)
1. (d)
2. (c)
3. (c)
4. (c)
5. (a)
6. (d)
7. (a)
8. (a)
9. (b)
10. (a)

II)
1. Trend
2. Seasonal
3. Cyclical
4. Irregular
5. Assume
6. Does not assume
7. Graphical
8. Moving average
9. Least square
10. Trend

III)
1. False
2. True
3. False
4. False
5. False
6. True
7. False
8. False
9. False
10. False

CHAPTER 5 - INDEX NUMBER

- **Basic concept**
  1. Index Numbers are special kind of average, expressed in ratio, calculated as percentage & used as numbers.
  2. Index number is a number which is used as a tool for comparing prices & quantities of a particular commodity or a group of commodities in a particular time period with respect to other time period or periods.
  3. Index Numbers indicate relative change in price or quantity or value expressed in percentage.
  4. Index Numbers are always unit free.
  5. The year in which comparison is made is called the "Current Year" & the year with respect to which the comparison is made is the "Base year".

- **Types**:
  1) Price Index:
  When the comparison is made in respect of prices is called price index numbers.
  2) **Quantity Index:**
  When the comparison is made in respect of Quantities it is called Quantity Index numbers.
  3) **Value Index:**
  When comparison is made in respect of values (value = Price X Qty.) it is called value Index Number.

- **Terminology OR Notation**
  1) P_o = Price of Commodity in Base year
  2) P_1 = Price of Commodity in Current year
  3) q_o = Quantity of commodity consumed in Base year
  4) q_1 = Quantity of commodity consumed in Current year
  5) V_o = Value spent on a commodity during the base year

  $$V_o = P_o q_o$$

  6) V_1 = Value spent on a commodity during the Current year

  $$V_1 = P_1 q_1$$

  7) i = Price relative

  $$V_1 = \frac{P_1}{p_o} \times 100$$

  8) P_{01} = Price Index number of current year with respect to base year.
  9) Q_{01} = Quantity Index number of current year with respect to base year.
  10) V_{01} = Value index number of current year with respect to base year.

- **Construction of Index Numbers**

Index Number are constructed by the following two methods.

i) Simple Aggregate method:

Price Index No. (P_{01}) $= \frac{\sum P_1}{\sum p_o} \times 100$

Quantity Index No. (Q_{01}) $= \frac{\sum q_1}{\sum q_o} \times 100$

Value Index No. (V_{01}) $= \frac{\sum P_1 q_1}{\sum p_o q_o} \times 100$

ii) Weighted Aggregate Method:

$P_{01} = \frac{\sum P_1 W}{\sum p_o W} \times 100$

In most of cases, quantities are taken as weights.

1) **Laspeyear's price Index No.**

$P_{01}(L) = \frac{\sum P_1 q_0}{\sum p_o q_o} \times 100$

Here, Base year Quantities are taken as weights.

2) **Passche's Price Index No.**

$P_{01}(P) = \frac{\sum P_1 q_1}{\sum p_o q_1} \times 100$

Here, Current year Quantities are taken as weights.

3) **Dorbish – Bowley's price Index No.**

$P_{01}(D - B) = \frac{\frac{\sum P_1 q_0}{\sum p_o q_o} + \frac{\sum P_1 q_1}{\sum p_o q_1}}{2} \times 100$

OR

$= \frac{P_{01}(L) + P_{01}(P)}{2}$

A.M. of Laspeyre's & Paasche's Index No.

4) **Fisher's price Index No.**

$P_{01}(F) = \sqrt{\frac{\sum P_1 q_0}{\sum p_o q_o} + \frac{\sum P_1 q_1}{\sum p_o q_1}} \times 100$

OR

$= \sqrt{P_{01}(L) \times P_{01}(P)}$

GM of Laspeyre's & Paasche's Index No.

5) **Marshall – Edgeworth price Index No.**

$$P_{01}(ME) = \frac{\sum P_1 q_0}{\sum p_0 q_0} + \frac{\sum P_1 q_1}{\sum p_0 q_1} \times 100$$

OR

$$= \frac{\sum P_1 (q_0 + q_1)}{\sum p_0 (q_0 + q_1)} \times 100$$

Here, sum of base Year & Current year quantities are taken as weight.

6) **Walsch's price Index No.**

$$P_{01}(W) = \frac{\sum P_1 \sqrt{q_0 q_1}}{\sum p_0 \sqrt{q_0 q_1}} \times 100$$

Here, GM of Base year & Current year quantities are taken as weight

Cost of living index Number:

- Also known as consumer price index Number.
- An index number of the cost of buying goods & services in day-to-day life for a specific consumer class.
- Cost of living Index Number is used in calculating purchasing power of money. Also used in determining Real wages

Method

1. Family budget method

$$CLI = \frac{\sum IW}{\sum W}$$

Whether I = Price relative

$$= \frac{P_1}{P_0} \times 100$$

W = Exp. Incurred against each commodity in the base years.

$= P_0 q_0$

2. Aggregates Expenditure methods:

$$CLT = \frac{\sum P_1 q_0}{\sum p_0 q_0} \times 100$$

In this method, quantities consumed in base year taken as weights.

- **Real Income:**

$$\text{Real Income} = \frac{\text{Income}}{\text{CLI}} \times 100$$

To maintain the same standard of living in the subsequent years his real income in all those subsequent years must be same as his base year income.

OBJECTIVES

I) choose the correct alternative.

1. Price Index Number by Simple Aggregate Method is given by

(a) $\sum \frac{p_1}{p0} \times 100$ (b) $\sum \frac{p_0}{p1} \times 100$

(c) $\frac{\sum p_1}{\sum p_0} \times 100$ (d) $\frac{\sum p_0}{\sum p_1} \times 100$

2. Quantity Index Number by Simple Aggregate Method is given by

(a) $\sum \frac{q_1}{q_0} \times 100$ (b) $\sum \frac{q_0}{q1} \times 100$

(c) $\frac{\sum q_1}{\sum q_0} \times 100$ (d) $\frac{\sum q_0}{\sum q_1} \times 100$

3. Value Index Number by Simple Aggregate Method is given by

(a) $\sum \frac{p_1 q_0}{p0 q_1} \times 100$ (b) $\sum \frac{p_0 q_1}{p_0 q_0} \times 100$

(c) $\frac{\sum p_1 q_1}{\sum p_1 q_0} \times 100$ (d) $\frac{\sum p_1 q_1}{\sum p_0 q_0} \times 100$

4. Price Index Number by Weighted Aggregate Method is given by

(a) $\sum \frac{p_1 w}{p_0 w} \times 100$ (b) $\sum \frac{p_0 w}{p_1 w} \times 100$

(c) $\frac{\sum p_1 w}{\sum p_0 w} \times 100$ (d) $\frac{\sum p_0 w}{\sum p_1 w} \times 100$

5. Quantity Index Number by Weighted Aggregate Method is given by

(a) $\sum \frac{q_1 w}{q_0 w} \times 100$ (b) $\sum \frac{q_0 w}{q_1 w} \times 100$

(c) $\frac{\sum q_1 w}{\sum q_0 w} \times 100$ (d) $\frac{\sum q_0}{\sum p_1} \times 100$

6. Value Index Number by Weighted Aggregate Method is given by

(a) $\sum \frac{p_1 q_0 w}{p_0 q_0 w} \times 100$ (b) $\sum \frac{p_0 q_1 w}{p_0 q_0 w} \times 100$

(c) $\frac{\sum p_1 q_1 w}{\sum p_0 q_1 w} \times 100$ (d) $\frac{\sum p_1 q_1 w}{\sum q_0 q_0 w} \times 100$

7. Laspeyre's Price Index Number is given by

(a) $\frac{\sum p_0 q_0}{\sum p_1 q_0} \times 100$ (b) $\frac{\sum p_0 q_1}{\sum p_1 q_1} \times 100$

(c) $\frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$ (d) $\frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$

8. Paasche's Price Index Number is given by

(a) $\frac{\sum p_0q_0}{\sum p_1q_0} \times 100$ (b) $\frac{\sum p_0q_1}{\sum p_1q_1} \times 100$

(c) $\frac{\sum p_1q_0}{\sum p_0q_0} \times 100$ (d) $\frac{\sum p_1q_1}{\sum p_0q_1} \times 100$

9. Dorbish-Bowley's Price Index Number is given by

(a) $\frac{\frac{\sum p_1q_0}{\sum p_0q_1} + \frac{\sum p_0q_1}{\sum p_1q_0}}{2} \times 100$ (b) $\frac{\frac{\sum p_1q_1}{\sum p_0q_0} + \frac{\sum p_0q_0}{\sum p_1q_1}}{2} \times 100$

(c) $\frac{\frac{\sum p_1q_0}{\sum p_0q_0} + \frac{\sum p_1q_1}{\sum p_0q_1}}{2} \times 100$ (d) $\frac{\frac{\sum p_0q_0}{\sum p_1q_0} + \frac{\sum p_0q_1}{\sum p_1q_1}}{2} \times 100$

10) Fisher's Price Number is given by

(a) $\sqrt{\frac{\sum p_1q_0}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_0q_1}} \times 100$

(b) $\sqrt{\frac{\sum p_0q_0}{\sum p_1q_0} \times \frac{\sum p_0q_1}{\sum p_1q_1}} \times 100$

(c) $\sqrt{\frac{\sum p_0q_1}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_1q_0}} \times 100$

(d) $\sqrt{\frac{\sum p_1q_0}{\sum p_1q_1} \times \frac{\sum p_0q_0}{\sum p_0q_1}} \times 100$

11) Marshall-Edgeworth's Price Index Number is given by

(a) $\frac{\sum p_1(q_0+q_1)}{\sum p_0(q_0+q_1)} \times 100$

(b) $\frac{\sum p_0(q_0+q_1)}{\sum p_1(q_0+q_1)} \times 100$

(c) $\frac{\sum q_1(p_0+p_1)}{\sum q_0(p_0+p_1)} \times 100$

(d) $\frac{\sum q_1(p_0+p_1)}{\sum q_0(p_0+p_1)} \times 100$

12) Walsh's Price Index Number is given by

(a) $\frac{\sum p_1\sqrt{q_0q_1}}{\sum p\sqrt{q_0q_1}} \times 100$

(b) $\frac{\sum p_0\sqrt{q_0q_1}}{\sum p_1\sqrt{q_0q_1}} \times 100$

(c) $\frac{\sum q_1\sqrt{p_0p_1}}{\sum q_0\sqrt{p_0p_1}} \times 100$

(d) $\frac{\sum q_0\sqrt{p_0p_1}}{\sum q_1\sqrt{p_0p_1}} \times 100$

13) The Cost of Living Index Number using Aggregate Expenditure Method is given by

(a) $\frac{\sum p_1q_0}{\sum p_0q_0} \times 100$

(b) $\sum \frac{p_1q_1}{p_0q_1} \times 100$

(c) $\frac{\sum p_1q_1}{\sum p_0q_1} \times 100$

(d) $\sum \frac{p_1q_0}{p_0q_0} \times 100$

14) The Cost of Living Index Number using Weighted Relative Method is given by

(a) $\frac{\sum IW}{\sum W}$

(b) $\sum \frac{W}{IW}$

(c) $\frac{\sum W}{\sum IW}$

(d) $\sum \frac{IW}{W}$

II) Fill in the blanks

1. Price Index Number by Simple Aggregate Method is given by_____.
2. Quantity Index Number by Simple Aggregate Method is given by____.
3. Value Index Number by Simple Aggregate Method is given by_____.
4. Price Index Number by Weighted Aggregate Method is given by_____.
5. Quantity Index Number by Weighted Aggregate Method is given by____.
6. Value Index Number by Weighted Aggregate Method is given by_____.

7. Laspeyre's Price Index Number is given by_____
8. Paasche's Price Index Number is given by_____.
9. Dorbish-Bowley's Price Index Number is given by_____.
10. Fisher's Price Index Number is given by_____.
11. Marshall-Edgeworth's Price Index Number is given by_____.
12. Walsh's Price Index Number is given by_____.

III) State whether each of the following is True or False.

1. $\frac{\sum p_1}{\sum p_0} \times 100$ is the Price Index Number by Simple Aggregate Method.

2. $\frac{\sum q_0}{\sum q_1} \times 100$ is the Quantity Index Number by Simple Aggregate Method.

3. $\sum \frac{p_0 q_0}{p_1 q_1} \times 100$ is Value Index Number by Simple Aggregate Method.

4. $\sum \frac{p_1 q_0}{p_0 q_0} \times 100$ is Paasche's Price Index Number.

5. $\frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$ is Laspeyre's Price Index Number.

6. $\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$ is Dorbish-Bowley's Price Index Number.

7. $\frac{1}{2}\left[\sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0}} + \frac{\sqrt{p_1 q_1}}{\sqrt{p_0 q_1}}\right] \times 100$ is Fisher's Price Index Number.

8. $\frac{\sum p_0 (q_0 + q_1)}{\sum p_1 (q_0 + q_1)} \times 100$ is Marshall- Edgeworth's Price Index Number.

9. $\frac{\sum p_0 \sqrt{q_0 q_1}}{\sum p_1 \sqrt{q_0 q_1}} \times 100$ is Walsh's Price Index Number.

10. $\sqrt{\frac{p_1 q_0}{\sum p_0 q_0}} \times \sqrt{\frac{\sum p_1 q_1}{\sum p_0 q_1}} \times 100$ is Fisher's Price Index Number.

Answers:

I)
1. (c)
2. (c)
3. (d)
4. (c)
5. (c)
6. (d)
7. (c)
8. (d)
9. (c)
10. (a)
11. (a)
12. (a)
13. (a)
14. (a) .

II)
1. $\frac{\sum p_1}{\sum p_0} \times 100$
2. $\frac{\sum q_1}{\sum q_0} \times 100$
3. $\frac{\sum p_1 q_1}{\sum q_1 q_0} \times 100$
4. $\frac{\sum p_1 w}{\sum p_0 w} \times 100$
5. $\frac{\sum q_1 w}{\sum q_0 w} \times 100$
6. $\frac{\sum p_1 q_1 w}{\sum q_0 q_0 w} \times 100$
7. $\frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$
8. $\frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$
9. $\frac{1}{2}\left[\sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0}} + \frac{\sqrt{p_1 q_1}}{\sqrt{p_0 q_1}}\right] \times 100$
10. $\sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}}$

11. $\frac{\sum_1(q_0+q_1)}{\sum p_0(q_0+q_1)} \times 100$

12. $\frac{\sum p_1\sqrt{q_0q_1}}{\sum p_0\sqrt{p_0q_1}} \times 100$

III) 1. True
2. False
3. False
4. False
5. False
6. False
7. I?
8. False
9. False
10. False

CHAPTER 6 - LINEAR PROGRAMMING

- **Introduction :**

  Linear Programming is a mathematical technique designed to help for planning & design making.

  Linear programming problems are also known as optimization problems. Mathematical programming involves optimization of a certain function, called objective function, subject to given conditions or restrictions known as constraints.

  L.P.P. may be defined as the problem of maximizing or minimizing a linear function subject to linear constraints.

- **Terminology :**

  1) Decision variables:
     Variables involved in L.P.P. are called Decision variables.

  2) Objective function:
     Linear function of decision variables which is to be optimized i.e. either maximized or minimized is called objective function.

  3) Constraints:
     Condition under which the objective function is to be optimized are called constraints.
     These are in the form of equations or inequations

  4) Non-negativity constraints:
     Values of the variables under consideration may be positive or Zero due to imposed conditions.

  5) Solution:
     Set of values of variable which satisfies all the constraints of the LPP.

  6) Feasible solution:
     Solution which satisfy all constraints.

  7) Feasible region:
     Common region determined by all the constraints & non-negativity restrictions of the linear programming problem.
     Boundaries of the region may or may not be include in the feasible region.

OBJECTIVES

I) choose the correct alternative.

1. The value of objective function is maximize under linear constraints.
   a) at the centre of feasible region
   b) at (0, 0)
   c) at any vertex of feasible region.
   d) The vertex which is at maximum distance from (0, 0).

2. Which of the following is correct?
   a) Every LPP has on optional solution
   b) Every LPP has unique optional solution.
   c) If LPP has two optional solution the it has infinitely many solutions.
   d) The set of all feasible solutions of LPP may not be a convex set.

3. Objective function of LPP is
   a) a constraint
   b) a function to be maximized or minimized
   c) a relation between the decision variables
   d) a feasible region.

4. The maximum value of $z = 5x + 3y$. subject to the constraints 3x $+ 5y = 15$; $5x + 2y \leq 10, x, y \geq 0$ is.
   a) 235 b) 235/9
   c) 235/19 d) 235/3

5. The maximum value of $z = 10x + 6y$, subjected to the constraints $3x + y \leq 12, 2x + 5y \leq 34, x \geq 0, y \geq 0$ is.
   a) 56 b) 65
   c) 55 d) 66

6. The point at which the maximum value of $z = x + y$ subject to the constraints $x + 2y \leq 70, 2x + y \leq 15, x \geq 0, y \geq 0$ is
   a) (36, 25) b) (20, 35)
   c) (35, 20) d) (40, 15)

7. Of all the points of the feasible region the optimal value of z is obtained at a point
   a) inside the feasible region.
   b) at the boundary of the feasible region.
   c) at vertex of feasible region.
   d) on x - axis.

8. Feasible region; the set of points which satify.
   a) The objective function.
   b) All of the given function.
   c) Some of the given constraints
   d) Only non-negative constrains

9. Solution of LPP to minimize $z = 2x + 3y$ st. $x \geq 0, y \geq 0, 1 \leq x + 2y \leq 10$ is
   a) $x = 0, y = 1/2$ b) $x = 1/2, y = 0$
   c) $x = 1, y = -2$ d) $x = y = 1/2$.

10. The corner points of the feasible region given by the inequations $x + y \leq 4, 2x + y \leq 7, x \geq 0, y \geq 0$, are
   a) (0, 0), (4, 0), (3, 1), (0, 4).
   b) (0, 0), (7/2, 0), (3, 1), (0, 4).
   c) (0, 0), (7/2, 0), (3, 1), (5, 7).
   d) (6, 0), (4, 0), (3, 1), (0, 7).

11. The corner points of the feasible region are (0, 0), (2, 0), (12/7, 3/7) and (0,1) then the point of maximum $z = 6.5x + y = 13$
   a) (0, 0) b) (2, 0)
   c) (11/7, 3/7) d) (0, 1)

12. If the corner points of the feasible region are (0, 0), (3, 0), (2, 1) and (0, 7/3) the maximum value of $z = 4x + 5y$ is .
   a) 12 b) 13
   c) 35/2 d) 0

13. If the corner points of the feasible region are (0, 10), (2, 2), and (4, 0) then the point of minimum $z = 3x + 2y\ is$.
   a) (2, 2) b) (0, 10)
   c) (4, 0) d) (2, 4)

14. The half plane represented by $3x + 2y \leq 0$ constraints the point.
   a) (1, 5/2) b) (2,1)
   c) (0, 0) d) (5, 1)

15. The half plane represented by $4x + 3y \geq 14$ contains the point
   a) (0, 0) b) (2, 2)
   c) (3, 4) d) (1, 1)

II) Fill in the blanks

1) Graphical solution set of the in equations $x \geq 0, y \geq 0$ is in_____quadrant

2) The region represented by the in equations $x \geq 0, y \geq 0$ lines in_____ quadrants

3) The optimal value of the objective function is attained at the____points of feasible region.

4) The region represented by the inequality $y \leq 0$ lies in____ quadrants

5) The constraint that a factory has to employ more women (y) than men (x) is given by_____.

6) A garage employs eight men to work in its showroom and repair shop. The constants that there must be not least 3 men in showroom and repair shop. The constrains that there must be at least 3 men in showroom and at least 2 men in repair shop are_____and _____respectively

7) A train carries at least twice as many first class passengers (y) as second class passengers (x) The constraint is given by____

8) A dish washing machine holds up to 40 pieces of large crockery (x) This constraint is given by_____

III) State whether each of the following is True or False.

1) The region represented by the inequalities $x \geq 0, y \geq 0$ lies in first quadrant.
2) The region represented by the in qualities $x \leq 0, y \leq 0$ lies in first quadrant.
3) The optimum value of the objective function of LPP occurs at the center of the feasible region.
4) Graphical solution set of $x \leq 0, y \geq 0$ in xy system lies in second quadrant.
5) Saina wants to invest at most Rs. 24000 in bonds and fixed deposits. Mathematically this constraints is written as $x + y \leq 24000$ where x is investment in bond and y is in fixed deposits.
6) The point (1, 2) is not a vertex of the feasible region bounded by $2x + 3y \leq 6, 5x + 3y \leq 15, x \geq 0, y \geq 0$.

7)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
a	c	b	c	a	d	c	b	a	b	b	b	a	c	c

The feasible solution of LPP belongs to only quadrant I The Feasible region of graph $x + y \leq 1 \; and \; 2x + 2y \geq 6$ exists.

Answers:

I)

II) 1) I
2) III
3) vertex
4) III and IV
5) $y > x$
6) $x \geq 3, y \geq 2,$
7) $x \geq 2y$
8) $x \leq 40$

III) 1) True
2) False
3) False
4) True
5) True
6) True
7) True

CHAPTER 8 - ASSIGNMENT, PROBLEM & SEQUENCING

Introduction to Assignment Problem

1. An assignment problem can be represented, by **n*x*n matrix** which constitutes **n**! possible ways of making assignments
2. Assignment problem is a special case of **Linear programming problem**.
3. Assignment problem is a special type of problem which deals with allocation of various resources to various activities on one to one basis.
   It is done in such a way that the total cost or time involved in the process is minimum or total profit is maximum
4. **Hungarian method** is an optimization algorithm that solves an Assignment problem

Steps

- For minimization problem
  1. **Row Reduction**
     Reduce each row by subtracting minimum value from each row.
  2. **Column Reduction**
     Reduce each column by subtracting minimum value from each Column.
  3. **Assign Zero**
     After applying first 2 steps you will get minimum 1 Zero (0) in each Row & Column.
     - Assign Single Zero ($\boxed{0}$) in each row & Cancel Zero [$\otimes$] from respective column And vice-versa.
  4. After applying step 3 you will get assign Zero ($\boxed{0}$) in each row & column.
     And if not, then apply rule of Ticked column & Unticked Row
  5. Do Allocation & Find minimum values.

- Rule of Ticked Column & Unticked Row
  1. Ticked ($\surd_1$) the Row where there is **No assign Zero ($\boxed{0}$)**

     ↓ In that row
     Check cancels Zero[$\otimes$]

     Ticked ($\surd_2$) the Column where there is **Cancelled Zero** [$\otimes$]

     ↓ In that Column
     Check the assign zero ($\boxed{0}$)

     Ticked ($\surd_3$) the Row where there is **assign Zero ($\boxed{0}$)**

2. Draw minimum line on Ticked column & Unticked Row

3. Reduce all the uncovered values by subtracting minimum value & Add the Same at the intersection.

4. You will get revised matrix
   Now do Re-assigning & Re-allocation process.

- **Unbalanced matrix (Dummy Method)**

  Here, | No of rows ≠ No. of. Columns |

  Take dummy Row or dummy column whichever is less and all values are Zero. (0).
  Now, follow steps of minimisation or maximisation depends upon question.

- **Restricted Assignment Problem (Dash Method)**
  - Assign very high value " ∞" to the prohibited cell

    So that | ∞ - Any No. = ∞ |

  - It means that particular cell/job cannot be assigned
  - Do steps of minimization problem.

- **Maximization Problem**
  - Find maximum value from overall
  - Subtract all Elements in the matrix from the maximum value
  - Now apply Rules of Minimization but at last find Maximum value. Instead of Minimum value.

SEQUENCING

Introduction:

Such problem are called sequencing problems where one has to sequence the order in which 'n' jobs are to be allotted to 'm' machines so that the total time required to complete all the jobs is minimized.

- Terminology
  1. **Total elapsed Time: (T):**
     - Time taken to complete all jobs
     - Time b/w the beginning of the first job at the first machine till the completion of last job on the last machine.
     - Since we begin with '0' time total elapsed time is last figure in worktable.
  2. **Idle time for machines:**
     Idle time is the time when the machine is available but waiting for a job to be processed.
  3. **General Sequence** $\longrightarrow (n!)^M$
     n $\longrightarrow$ Jobs
     M $\longrightarrow$ dift. Machines

     Optimal Sequence $\longrightarrow$ Sequence out of General sequence which minimizes the total elapsed time.

- **Types of sequencing problem**
  1. Sequencing 'n' jobs on 2 machines
  2. Sequencing 'n' jobs on 3 machines

- **Sequencing 'n' jobs on 3 machines**

  Reduce the problem to 2 machines & find the required sequence in the same way as we did in 2 machine problem. Then worktable by taking 3 machine data, to find total elapsed time & idle time of all 3 machine.

  Condition
  - Min. $(M_1) \geq$ Max. (M_2)
    OR
  - Min. $(M_3) \geq$ Max. (M_2)

  Convert into 2 machines
  - If Conditions are satisfied then **convert 3 machines into 2 fictitious machine say G & H**
    $G = M_1 + M_2$
    $H = M_2 + M_3$
    Now find optimal sequence and the Do worktable to find elapsed time & idle time for 3 machines

    Note 1: Conversion of 3 machines to 2 machines is only for finding sequence

    Note 2: Worktable prepared for 3 machines only.

OBJECTIVES

I) Choose the correct alternative.

1. In sequencing, an optimal path is one that minimizes_____
   (a) Elapsed time (b) Idle time
   (c) Both (a) and (b) (d) Ready time

2. If job A to D have processing times as 5, 6, 8, 4 on first machine and 4, 7, 9, 10 on second machine then the optimal sequence is :
   (a) CDAB (b) DBCA
   (c) BCDA (d) ABCD

3. The objective of sequencing problem is
   (a) to find the order in which jobs are to be made
   (b) to find the time required for the completing all the job on hand
   (c) to find the sequence in which jobs on hand are to be processed to minimize the total time required for processing the jobs
   (d) to maximize the cost

4. If there are n jobs and m machines, then there will be..... sequences of doing the jobs.
   (a) mn (b) m(n!) (c) n^m (d) $(n!)^m$

5. The Assignment Problem is solved by
   (a) Simplex method,
   (b) Hungarian method
   (c) Vector method,
   (d) Graphical method,

6. In solving 2 machine and n jobs sequencing problem, the following assumption is wrong
   (a) No passing is allowed
   (b) Processing times are known
   (c) Handling time is negligible
   (d) The time of passing depends on the order of machining

7. To use the Hungarian method, a profit maximization assignment problem requires
   (a) Converting all profits to opportunity losses
   (b) A dummy person or job
   (c) Matrix expansion
   (d) Finding the maximum number of lines to cover all the zeros in the reduced matrix

8. Using Hungarian method the optimal assignment obtained for the following assignment problem to minimize the total cost is:

Agent	Job			
	A	B	C	D
1	10	12	15	25
2	14	11	19	32
3	18	21	23	29
4	15	20	26	28

(a) 1 — C, 2 — B , 3 — D, 4 — A
(b) 1 — B, 2 — C, 3 — A, 4 — D
(c) 1 — A, 2 — B, 3 — C, 4 — D
(d) 1 — D, 2 — A, 3 — B, 4 — C

9. The assignment problem is said to be unbalance if
(a) Number of rows is greater than number of columns
(b) Number of rows is lesser than number of columns
(c) Number of rows is equal to number of columns
(d) Both (a) and (b)

10. The assignment problem is said to be balanced if
(a) Number of rows is greater than number of columns
(b) Number of rows is lesser than number of columns
(c) Number of rows is equal to number of columns
(d) If the entry of row is zero

11. The assignment problem is said to be balanced if it is a
(a) Square matrix (b) Rectangular matrix
(c) Unit matrix (d) Triangular matrix

12. In an assignment problem if number of rows is greater than number of columns then
(a) Dummy column is added (b) Dummy row is added
(c) Row with cost 1 is added (d) Column with cost 1 is added

13. In a 3 machine and 5 jobs problem, the least of processing times on machine A, B and C are 5, 1. and 3 hours and the highest processing times are 9, 5, and 7 respectively, then it can be converted to a 2 machine problem if order of the machines is:
(a) B-A-C, (b) A-B-C (c) C - B - A (d) Any order

14. The objective of an assignment problem is to assign
(a) Number of jobs to equal number of persons at maximum cost
(b) Number of jobs to equal number of persons at minimum cost
(c) Only the maximize cost (d) Only to minimize cost

II) Fill in the blanks

1. An assignment problem is said to be unbalanced when .....
2. When the number of rows is equal to the number of columns then the problem is said to be assignment problem.
3. For solving an assignment problem the matrix should be a matrix.
4. If the given matrix is not a ..... matrix, the assignment problem is called an unbalanced problem.
5. A dummy row(s) or column(s) with the cost elements as ..... the matrix of an unbalanced assignment problem as a square matrix.
6. The time interval between starting the first job and completing the last. job including the idle time (if any) in a particular order by the given set of machines is called .....
7. The time for which a machine j does not have a job to process to the start of job i is called .....
8. Maximization assignment problem is transformed to minimization problem by subtracting each entry in the table from the._____value in the table.
9. When an assignment problem has more than one solution, then it is..... optimal solution.
10. The time required for printing of four books A, B, C and D is 5, 8, 10 and 7 hours. While its data entry requires 7, 4, 3 and 6 hrs respectively. The sequence that minimizes total elapsed time is.....

III) State whether each of the following is True or False.

1. One machine - one job is not an assumption in solving sequencing problems.
2. If there are two least processing times for machine A and machine B, priority is given for the processing time which has lowest time of the adjacent machine.
3. To convert the assignment problem into a maximization problem, the smallest element in the matrix is deducted from all other elements.
4. The Hungarian method operates on the principle of matrix reduction, whereby the cost table is reduced to a set of opportunity costs.
5. In a sequencing problem, the processing times are dependent of order of processing the jobs on machines.

6. Optimal assignments are made in the Hungarian method to cells in the reduced matrix that contain a zero.
7. Using the Hungarian method, the optimal solution to an assignment problem is found when the minimum number of lines required to cover the zero cells in the reduced matrix equals the no of persons.
8. In an assignment problem, if number of column is greater than number of rows, then a dummy column is added,
9. The purpose of dummy row or column in an assignment problem is to obtain balance between total number of activities and total number of resources.
10. One of the assumptions made while sequencing n jobs on 2 machines is : two jobs must be loaded at a time on any machine.

Answers:

I)

1	2	3	4	5	6	7	8	9	10	11	12	13	14
c	b	c	d	b	d	a	a	d	c	a	a	b	b

II)
1. Number of rows is not equal to the number of columns.
2. Balanced
3. Square
4. Square
5. Zero
6. Total elapsed time
7. Idle time
8. Maximum
9. Multiple
10. A-D-B-C

III)
1. False
2. True
3. False
4. True
5. False
6. True
7. True
8. False
9. True
10. False

CHAPTER 8 - RANDOM VARIABLE & PROBABILITY DISTRIBUTION

- **Basic Concepts**

1) Random Variable
   - A Random variable is a real-valued function defined on sample space of a random experiment.
   - Domain of random variable is the sample space of a random experiment, while its co-domain is the real line.
   - Abbreviation r.v. used for random variable.

2) Types of Random variable:

A. Discrete Random Variable
   - Possible values from a countable set, which may be finite or infinite.
   - Non - negative integers
   - Value of discrete r.v. are obtained by counting.

   Examples
   a. No. of children in a family
   b. No. of cars sold by dealer, etc.

B. Continuous Random Variable
   - Possible values form an interval of real numbers.
   - A Continuous r.v. has uncontably infinite possible values and these values form an interval of real numbers.
   - Values of continuous r.v. is obtained by measurement.

   Examples
   a. Height of trees in forest,
   b. Weights of students in a class etc.

3) Coin concept

1C	2C	3C			
H	HH	HHH	HHT	HTH	HTT
T	HT	THH	THT	TTH	TTT
	TH				
	TT				

4) Dice Concept

1D → 1, 2, 3, 4, 5, 6,

2D →
(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4,) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6, 6)

5) Probability – distribution of a discrete random variable

Possible values of X be denoted by x_1, x_2 ……..,and the corresponding probabilities be denoted by P_1, P_2 , …….

Then the set of ordered pairs { (x_1, P_1) (x_2, P_2)……} is called the probability distribution of the random variable X.

6) Probability mass function (PMF)

f is said to be PMF

If,

i) $0 \leq P(x = x) \leq 1$ for $\forall\, x$

ii) $\sum P(x = x) = 1$

7) Cumulative Distribution function (cdf)

Cumulative distribution function (cdf) of a discrete random variable X is denoted by F.

$$F(X) = P(X \leq x)$$

8) Expected value of discrete random variable

- Sum of product of values of x & their respective probabilities
- Also known as mean or Average

$$E(X) = \sum X.P(X)$$

9) Variance & standard deviation of discrete random variable

Variance (x) = $\sum x^2.p(x) - [\sum x.p(x)]^2$

Or

$$= E(x)^2 - [E(x)]^2$$

$$SD(\sigma) = \sqrt{Var(x)}$$

10) Probability Density function (Pdf)

f is Pdf if:

- $f(x) \geq 0$ for all $x \in (a, b)$
- $\int_a^b f(x)dx = 1$

11) Cumulative Distribution function of continuous r.v. [Cdf of x]

$F(x) = \int_a^x f(x)dx$ for all $x \geq a$

12) Expected value, variance & SD of Continuous r.v.

$$\text{Expected value } [\mathrm{E}(x)] = \int_{-\infty}^{\infty} x.f(x)\ \mathrm{d}x$$

$$\text{Variance of } x\ [\mathrm{V}(x)] = \int_{-\infty}^{\infty} x^2.f(x)\ \mathrm{d}x - \left[\int_{-\infty}^{\infty} x.f(x)dx\right]^2$$

$$\mathrm{SD} = \sqrt{\text{Variance}}$$

13) Bernoulli trial

- Having only 2 mutually exclusive outcomes. i.e. success & failure.
- A sequence of dichotomous experiments is called a sequence of Bernoulli trials, **if it satisfies the following conditions**
  - Trials are independent
  - Probability of success remains the same in all trials.

14) Binomial Distribution

→ Based on Bernoulli trial

2 Outcome → Success, Failure

→ X~B (n,p)

X follows Binomial Distribution having 2 parameter n & P.

→ $P(x) = n_{c_x}, p^x, q^{n-x}$

Where,

n → no. of trials

P → Probability of Success

q → Probability of Failure

$$p = 1 - q$$

$$p + q = 1$$

$x \rightarrow$ No. of successes.
$C \rightarrow$ Combination.

$$n_{c_r} = \frac{n!}{r!(n-r)!}$$

$n \rightarrow$ No. of observation
$r \rightarrow$ No. of Objects to be selected.

$n_{c_n=1}$	$n_{c_0=1}$	$n_{c_1=n}$

- In case of coin

  p $\rightarrow$ prob. Of getting head = $^1/_2$

  q $\rightarrow$ prob. Of getting tail = $^1/_2$

- In Binomial Distribution

  Mean = n.p | SD= $\sqrt{npq}$
  Variance = npq

  Mean > variance

15) Poisson Distribution

- It is a discrete probability distribution

  X ~ P (M)

  X follows Poisson distribution having only 1 parameter i.e.'M'

  $$P(x) = \frac{e^{-m}.(m)^x}{x!}$$

  Where,
  M = n.p
  = Average

- Mean = M

  Variance = M

  Mean = Variance

OBJECTIVES

I) 1. **choose the correct alternative.**

F(x) is c.d.f. of discrete r.v. X whose p.m.f. is given by $P(x) = k\binom{4}{x}$, for x = 0, 1, 2, 3, 4 & $P(x) = 0$ otherwise then F(5) = ...

(a) $\frac{1}{16}$ (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ (d) 1

2. F(x) is c.d.f. of discrete r.v. X whose distribution is

X_i	-2	-1	0	1	2
P_i	0.2	0.3	0.15	0.25	0.1

then F(- 3) = ...

(a) 0 (b) 1 (c) 0.2 (d) 0.15

3. x : is number obtained on upper most free when a fair die ..... thrown then E(x) = ...

(a) 3.0 (b) 3.5 (c) 4.0 (d) 4.5

4. If p.m.f. of r. v. X is given below.

x	0	1	2
P(x)	q^2	2pq	p^2

then Var(x) = ...

(a) p^2 (b) q^2 (c) pq (d) 2pq

5. The expected value of the sum of two numbers obtained when two fair dice are rolled is .....

(a) 5 (b) 6 (c) 7 (d) 8

6. Given p.d.f. of a continuous r.v. X as

$f(x) = \frac{x^2}{3}$ for $-1 < x < 2$

= 0 otherwise then

$F(x) =$

(a) $\frac{1}{9}$ (b) $\frac{2}{9}$ (c) $\frac{3}{9}$ (d) $\frac{4}{9}$

7. X is r.v. with p.d.f $(x) = \frac{k}{\sqrt{x}}$, $0 < x < 4$
$= 0$ otherwise
Then E $(x) =$_____
(a) $\frac{1}{3}$ (b)$\frac{4}{3}$ (c)$\frac{2}{3}$ (d)1

8. If X ~B $(20, \frac{1}{10})$ then E $(x) =$_____
(a) 2 (b) 5 (c) 4 (d) 3

9. $If\ E\ (x)\ =\ m$ and Var (x) = m then X follows.....
(a) Binomial distribution (b) Possion distribution
(c) Normal distribution (d) none of the above

10. If E $(x)\ >\ Var\ (x)$ then X follows .....
(a) Binomial distribution (b) Possion distribution
(c) Normal distribution (d) none of the above

II) Fill in the blanks

1. The values of discrete r.v. are generally obtained by .....
2. The values of continuous r.v. are generally obtained by .....
3. If X is discrete random variable takes the
Values$x_1, x_2, x_3, \ldots\ldots\ldots x_n$then $\sum_{i=1}^{n} p(x_i) = \ldots\ldots\ldots\ldots$

4. If F(x) is distribution function of discrete r.v.x with p.m.f.p$(x) = \frac{x-1}{3}$ for $x = 1,2,3$ & p$(x) = 0$ otherwise then F(4) = ...

5. If F(x) is distribution function of discrete r.v.X with p.m.f.$p(x) = k\binom{4}{x}$ for $x = 0, 1,2,3,4$ and $(p(x) = 0$ otherwise then F(-1) = ...

6. (x) is considered to be..... of the probability distribution of x.

7. If x is continuous r.v. and $F(x_i)\ =\ P(X\ \leq\ x_i)$
$\int_{-\infty}^{x_i} f(x)\,dx\ then\ F(x) is\ called \ldots\ldots\ldots\ldots\ldots\ldots$

8. In Binomial distribution probability of success_____from trial to trial.

9. In Binomial distribution if n is very large and probability success of p is very small such that np = m (constant) then ..... distribution is applied

III) State whether each of the following is True or False.

1. If $P(X = x) = k\binom{4}{x}$ for $x = 0,1,2,3,4$, then $F(5) = \frac{1}{4}$ when $F(x)$ is c. d. f.

2.

X	-2	-1	0	1	2
$P(X = x)$	0.2	0.3	0.15	0.25	0.1

If $F(x)$ is c.d.f. of discrete r.v. X then F (-3) = 0.

3. X is the number obtained on upper most face when a die is thrown then $E(x) = 3.5$.

4. If p.m.f. of discrete r.v. X is

X	0	1	2
$P(X = x)$	a^2	$2pq$	p^2

then $E(x) = 2p$.

5. The p.m.f. of a r.v. X is
$P(x) = \frac{2x}{n(n+1)}, x = 1, 2 \ldots\ldots\ldots\ldots n$
= 0 otherwise,
Then
$E(x) = \frac{2n+1}{3}$

6. If $f(x) = kx(1-x)$ for $0 < x < 1$
= 0 Otherwise when
K = 12

7. If X ∞ B $(n_1 p)$ and $n = 6$ & $P(x = 4)$ $= p(x = 2)$ then $P = \frac{1}{2}$

8. If r.v. X assumes values 1, 2, 3,_____n with equal probabilities then $E(x) = \frac{n+1}{2}$

9. If r.v. X assumes the values 1,2,3,_____, 9 with equal probabilities, $E(x) = 5$.

Answers:

I)

1	2	3	4	5	6	7	8	9	10
d	a	b	d	c	b	b	a	b	a

II)
1. Counting
2. Measurement
3. 1
4. 1
5. 0
6. Centre of gravity
7. Distribution function
8. Remains constant / independent
9. Poisson

III)
1. False
2. True
3. True
4. True
5. True
6. False
7. True
8. True
9. True

HSC RESULT 19-20

Name	Percentage
Khushbu Mali	95.54
Priyanka Udeshi	94.92
Smruti Suresh Jagdale	94.92
Nidhi Dhanani	94.77
Ishika Pravin Sanghavi	94.62
Vansh Vora	94.46
Aishwarya Vijay Badhe	94.46
Khushi Vipul Darji	94.46
Kushal Thakkar	94.46
Sampreeth Jayantha Poojary	94.31
Janahvi Bharat Dayare	94.31
Kriti Khatri	94.31
Sindhu Umesh Gawde	94.31
Dhruvi Sanghvi	94.31
Gautami Taggerse	94.15
Sudhanshu Singh	94.15
Komal Jitesh Gandhi	94.15
Vedika Mediboina	94.15
Sharvari Dilip Sawant	94.15
Rashi Sanjay Jain	94.15
Saniya Kulkarni	94.15
Rochelle Menezes	94.15
Aditi Mogaveera	94.00
Arundatii Singh	94.00
Yukta Sukerkar	94.00
Megha J Hinduja	94.00
Shreya Harlalka	93.85
Mansi Kadian	93.85
Sakshi Shankar Sudrik	93.85
Ridhi Ajit Rikame	93.85
Rutika Vartak	93.69
Vaedik Khatod	93.69
Bhavya Bhandari	93.69
Vidisha Shetty	93.69
Parth Dubey	93.54
Rohan Subramanian	93.54
Kervi Singhvi	93.54
Diya Khaturia	93.54
Hetal Poonamchand Hingad	93.54
Anushka M Dalvi	93.54
Jay Singh	93.54
Saras Sali	93.54
Yashasvi Maheshwari	93.38
Livya Noronha	93.38
Ishita Kute	93.38
Khushi Agrawal	93.38
Khushboo Shah	93.38
Khushee Shah	93.23
Deep Jayesh Gada	93.23
Siddharth Manoj Sethia	93.23
Aditya Kanal	93.23
Kosha Shah	93.23
Roshni Keshav Iddya	93.23
Neha Motwani	93.23
Parth Agarwal	93.23

Name	Percentage
Netri Shah	93.23
Sakshi Navin Shetty	93.23
Parth Patki	93.23
Pratham Shah	93.08
Prishita Shah	93.08
Prachi Parkar	93.08
Pratishta Pravin Shetty	93.08
Pallavi Jha	93.08
Nameera Ahmed	93.08
Shreya Bharat Jain	93.08
Yashvi	93.08
Sakshi Kothari	92.92
Khushi Nayan Makadia	92.92
Kashti Mehta	92.92
Kevin Patel	92.77
Priyanshi Mihir Shah	92.77
Prabhankit Shinde	92.77
Krupa Bidye	92.77
Prasham Gandhi	92.77
Nisha Surendra Rai	92.77
Devank S. Mayekar	92.77
Abhishek Dhuri	92.77
Shravani Wabekar	92.77
Shreya Niranjan Bhorawat	92.77
Tithi Parmar	92.62
Kamlesh Suthar	92.62
Akshat Choudhary	92.62
Khushal Parihar	92.62
Devanshi Kapadia	92.62
Amey Mhaskar	92.62
Keya Trivedi	92.62
Neer Shah	92.62
Yashvi Shah	92.62
Soham Angre	92.62
Ayush Ajay Sawant	92.62
Ankita Kewalramani	92.62
Deepam	92.62
Prasanna	92.62
Prasanna Suresh	92.62
Hrishita Raghu Poojari	92.46
Devdas Ranjeet Patole	92.46
Anannya Mhatre	92.46
Sanskruti Shashikant Phavade	92.46
Sanskar Maheshwari	92.46
Neeti Vakharia	92.46
Payas Mehta	92.46
Shobhit Maliwal	92.46
Leesha Gupta	92.46
Nikunj Jain	92.46
Siddhant Hemant Avhad	92.46
Khushi Maheshwari	92.46
Chaitra Billava	92.31
Hitakshi Mehta	92.31
Smit Manish Fofaria	92.31
Khushi Varaiya	92.31

Name	Percentage
Riya Mahyavanshi	92.31
Ayush Agrawal	92.31
Gautam Bhavesh Shah	92.31
Bhumit Mehta	92.31
Saakshi Deepak Karia	92.31
Palak Jaitly	92.31
Prerna Rajen Vora	92.31
Manasvi Patankar	92.31
Hetavi Shah	92.15
Bansi Madlani	92.15
Deeksha Kapoor	92.15
Yash Nautiyal	92.15
Shruti Jain	92.15
Mahek Payak	92.15
Raksha Shekhar Shetty	92.15
Dev Shah	92.15
Aditi Ashok Shetty	92.15
Athira Vaipur	92.15
Jahnavi	92.15
Manas Shetty	92.00
Neeraj Shah	92.00
Yash Shah	92.00
Yash Divyank Dhah	92.00
Aditya Kandari	92.00
Isha Chotai	92.00
Breanna Fernandes	92.00
Kashish Bhargava	92.00
Krishna Bharat Bhanushali	92.00
Keni Mehta	92.00
Khushi Kanodia	91.85
Shreya Tatke	91.85
Pratyush Deepak Rajgor	91.85
Bhaktee Shah	91.85
Bhargavraju Veerla	91.85
Krupa Rakesh Gajre	91.85
Swizal Gomes	91.85
Heli Sanjay Dhruv	91.85
Parth Upadhyay	91.85
Vinit	91.85
Cheryl Andrade	91.85
Yash Thakare	91.69
Vitrag Singhi	91.69
Radhika Dabholkar	91.69
Aastha Hari Chand	91.69
Dhruvi Desai	91.69
Rohit Baviskar	91.69
Bhinde Parth Mahendra	91.69
Parth Mahendra bhinde	91.69
Tanish Agarwal	91.69
Mokshitha Sherty	91.69
Sanchit Jain	91.69
Samiksha Bhatt	91.69
Sejal Phapale	91.69
Isha Bathia	91.69
Radhika Garg	91.69

HSC RESULT 19-20

Name	Percentage
Om Kedia	91.69
Esha Trisom Sonkusale	91.69
Samarth	91.54
Viraj Mehta	91.54
Mangesh Gadewar	91.54
Murtaza Saria	91.54
Disha Mody	91.54
Samma Naresh Kewlani	91.54
Ayush Panchamiya	91.54
Priya Rao	91.54
Kiara Xavier	91.54
Hansika Gupte	91.54
Deval Mehta	91.38
Nagesh Banne	91.38
Ojas	91.38
Tanaya	91.38
Jhanvi	91.38
Aparna Ramanathan	91.38
Mahek Shah	91.38
Niel Patade	91.38
Harshi Kothari	91.38
Aryan Karnawat	91.38
Ananya Akerkar	91.38
Aabeid Shaikh	91.38
Shubham Modi	91.38
Isha Shah	91.23
Neeraj Kishore Udasi	91.23
Honey Waghela	91.23
Vanshita Devadiga	91.23
Kavish Garg	91.23
Mit Shah	91.23
Ayushi Dhruva	91.23
Cheril Nitin Shah	91.23
Mohak Savla	91.23
Bhakti Deshmukh	91.23
Kaivan Dhruval Doshi	91.23
Shweta Lackdivey	91.23
Shreyas Badiger	91.08
Sneha Chavan	91.08
Sathvika Shetty	91.08
Ankita Joshi	91.08
Mansi Lad	91.08
Nitansh Shah	91.08
Shree Joshi	91.08
Zubiya Ansari	91.08
Mitali Shetty	91.08
Ashmita Devadiga	91.08
Vidhi Shah	91.08
Diya Chheda	91.08
Dimple Dangi	91.08
Chandan Tiwari	90.92
Disha N Shah	90.92
Gauri Ojha	90.92
Tanish Dhami	90.92
Arishit Shetty	90.92

Name	Percentage
Swastik	90.92
Shayan Sadik Desai	90.92
Khushi Rakesh Chordia	90.92
Krish Parmar	90.92
Vidhi Singh	90.92
Saloni	90.92
Shreya Reddy	90.92
Diya Dedhia	90.92
Shambhavi Pai	90.92
Vrunda Atul Mehta	90.77
Parikshit Vanjara	90.77
Khushi Soni	90.77
Esha Hingarh	90.77
Merill D'souza	90.77
Riya Patel	90.77
Poojan Sanghavi	90.77
Maurya Borse	90.77
Ashi Devang Dhruva	90.77
Heena	90.77
Khush Agarwal	90.77
Siddhi Panchal	90.77
Siddhi	90.77
Tania	90.77
Srivatsa Patil	90.77
Rahul Medda	90.77
Nishi Jagdish Punmiya	90.77
Tanushree Yadav	90.77
Vedant Keluskar	90.77
Nishtha jain	90.62
Krish Shah	90.62
Amisha Mehta	90.62
Fenil Soneji	90.62
Richa Pravin Naik	90.62
Jhanvi Joshi	90.62
Smriti Jain	90.62
Priya Mangesh Jagtap	90.62
Sakshi Kalpesh Shah	90.62
Rajlaxmi Magadum	90.62
Devanshi Vira	90.62
Kashissh Singhania	90.62
Disha Shah	90.62
Vidhi	90.62
Kaushik K Bhartiya	90.62
Krina Satra	90.62
Vedant Shriyan	90.46
Krish Jain	90.46
Lokesh M Jain	90.46
Sanskar Agarwal	90.46
Narayani Gaur	90.46
Jahnvi Shah	90.46
Shreeya Deorukhkar	90.46
Aryaa Punyarthi	90.46
Sneha Ashok Shinde	90.46
Sneha Shinde	90.46
Yuvraj Abhaykumar Gandhi	90.46

Name	Percentage
Tanvi Rasal	90.46
Hatim Sonkachwala	90.46
Meet Paresh Kanakia	90.46
Meet Kanakia	90.46
Jaineel Dalal	90.46
Disha Biyani	90.46
Vishakha Ranga	90.31
Devesh Dilip Pimpale	90.31
Khushi Vinod Bhanushali	90.31
Vini Desai	90.31
Pauravi Nitin Baikar	90.31
Sharvari Deshpande	90.31
Nisha Rajesh Rao	90.31
Vanshita Vora	90.31
Sayed Mohammed Junaid	90.31
Anish	90.31
Shubham Vora	90.31
Harshit Kedia	90.31
Kirti Balu Hase	90.31
Deepal Vikas Gohel	90.31
Deepal Gohel	90.31
Gautam Kothari	90.15
Preksha Patel	90.15
Pratik Dattu Koakte	90.15
Sanika Shivaji Varal	90.15
Gandhali Sumukh Desai	90.15
Surabhi Sonar	90.15
Jainam Swayam Shah	90.15
Siddhi Tiwari	90.15
Het Fariya	90.15
Nemin Doshi	90.15
Jeni Shah	90.15
Hayyan Badamia	90.15
Arushi Keniya	90.15
Roshan Jain	90.00
Shruti Shetty	90.00
Ramnek Chhipa	90.00
Riya Shirvaikar	90.00
Ayush Barbhaya	90.00
Pranav	90.00
Riddhi	90.00
Sakshi Raut	90.00
Manjiri Parab	90.00
Pal Shah	90.00
Yash Ganesh Khanolkar	90.00
Prasesh Mehta	90.00
Disha Bucha	90.00
Tanish Dharmendra Parmar	90.00
Palak Jain	90.00