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MATHEMATICS AND STATISTICS

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(PART 1)

Head Office

Shraddha, 4th Floor, Old Nagardas Road,
Near Chinai College, Andheri (E), Mumbai - 400 069.

022 - 2683 66 66

S.Y.J.C. – MATHEMATICS (PART 1)

INDEX

CH 1 - MATHEMATICAL LOGIC

STATEMENT

A statement is declarative sentence which is either TRUE or FALSE but not both simultaneously

EXAMPLE 1

i. the sun rises in the East

ii. The square of a real number is negative

iii. Sum of two odd numbers is odd

iv. Sum of opposite angles in a cyclic rectangle is 180o

truth values of statements (i) and (iv) is T and that of (ii) and (iii) is F

EXAMPLE 2

NOTE :

Sentences like exclamatory , interrogative , imperative are not considered as statements

i. May God bless you !
….EXCLAMATORY

ii. Why are you so unhappy ?
….INTERROGATIVE

iii Remember me when we are parted
….IMPERATIVE

iv Don't ever touch my phone
….IMPERATIVE

v. I hate you !
….EXCLAMATORY

vi. Where do you want to go today ?
….INTERROGATIVE

OPEN SENTENCE

An open sentence is a sentence whose truth can vary according to some condition which are not stated in the sentence

EXAMPLE

i. $x + 4 = 8$

truth of the above sentence depends on the value of x which is not given . Hence it can go true or false

ii. Chinese food is very tasty

truth value of the above sentence varies as degree of taste varies from individual to individual

OPEN SENTENCE are not considered as STATEMENTS in LOGIC

State which of the following sentences are statements . Justify your answer if it's a statement . Write down its truth value

01. The sum of interior angles of a triangles is 180^o

It's a statement , TRUTH VALUE : T

02. You are amazing !

It's not a statement . Its an exclamatory sentence

03. Please grant me a loan

It's not a statement . Its an imperative sentence

04. He is an actor

It's not a statement . Its an open sentence

Pronoun he is not known , therefore we can state the truth value of the given sentence

05. Did you eat lunch yet ?

It's not a statement . Its an interrogative sentence

06. $x + 5 < 14$

It's not a statement . Its an open sentence

Since value of x is not known , we can state the truth value of the above sentence

LOGICAL CONNECTIVES

NOT : THE NEGATION (UNARY CONNECTIVE)

If p is a statement then Negation of p i.e NOT of p is denoted by $\sim p$

Example : $p \equiv$ 2 is a prime number $\quad p \equiv 2 + 2 = 4$

$\sim p \equiv$ 2 is not a prime number $\quad \sim p \equiv 2 + 2 \neq 4$

1

Truth Table :

p	~p
T	F
F	T

OR : DISJUNCTION

The disjunction of two statements p and q is denoted by p v q

There exists two types of 'OR' one is exclusive and other is inclusive

2

Example 1 : a. throwing a coin will get a head OR a tail

b. the amount should be paid by cheque or by demand draft

In the above examples 'OR' is used in the the sense that only one of the two possibilities exists but not both . Hence it is called EXCLUSIVE

Example 2 : a. Graduate or employee persons are eligible to apply for this post

b. the child must be accompanied by father or mother

in the above statements 'OR' is used in the sense that first or second or BOTH possibilities exists . Hence it is called INCLUSIVE

IN mathematics 'OR' is used in the INCLUSIVE sense

Hence p v q means p or q or both p and q

TRUTH TABLE

p	q	p v q
T	T	T
T	F	T
F	T	T
F	F	F

AND : CONJUNCTION

3

The conjunction of two statements p and q is denoted by $p \wedge q$

Example : $p \equiv$ Ali is handsome ; $q \equiv$ Ali is intelligent

$p \wedge q \equiv$ Ali is handsome and intelligent

For $(p \wedge q)$ to be true , both p and q must be true

TRUTH TABLE

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Note : 'and' can be replaced by

- As well as ..... : He is good in studies as well as sports
- neither… nor : Ali is neither handsome nor intelligent
- But ……. : it is a cloudy day but the climate is pleasant
- Though … : though he failed the exam , he was happy
- Still …. : he is fat still he managed to catch the train
- While ….. : he was playing cricket while I was playing music

IF …THEN…. : CONDITIONAL

4

Symbol : $\rightarrow$ / $\Rightarrow$

Example : $p \equiv$ Rhombus ; $q \equiv$ Parallelogram

$p \rightarrow q \equiv$ if Rhombus then Parallelogram

note : If p happens then q needs to happen , else the statement will go false

As in the above example , If Quadrilateral is a rhombus then it needs to be a parallelogram . We cannot have Rhombus without the parallelogram i.e

We cannot have P WITHOUT Q , if P happens and Q not it will be false

CHECK IN THE TRUTH TABLE : $T \rightarrow F \equiv F$

TRUTH TABLE

p	q	$p \to q$	
T	T	T	
T	F	F	here p happens without q hence false
F	T	T	
F	F	T	

Different forms of writing if ….. then

a) if p then q : if rhombus then parallelogram

b) p implies q : Rhombus implies parallelogram

c) p only if q : Rhombus only if parallelogram

d) q is necessary for p : Parallelogram is necessary for Rhombus

e) p is sufficient for q : Rhombus is sufficient condition for Parallelogram

IF AND ONLY IF : BI-CONDITIONAL

5

Symbol : $\leftrightarrow$ / $\Leftrightarrow$

Example : p $\equiv$ An angle is right angle ; q $\equiv$ It is of measure 90^o

$p \leftrightarrow q$ $\equiv$ An angle is right angle IFF It is of measure 90^o

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$\equiv$ If angle is right angle then it is of measure 90^o

AND

If it is of measure 90^o then it is right angle

NOTE : from the example cited above , it is clear that either both will happen OR both will not happen . Hence the statement is false when one happens without the other

TRUTH TABLE

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Write the following statements in SYMBOLIC FORM

Q1

01. An angle is right angle AND its measure is 90^o

P ≡ An angle is right angle

Q ≡ its measure is 90^o

SYMBOLIC FORM : $P \wedge Q$

02. Rohit is smart OR he is healthy

P ≡ Rohit is smart

Q ≡ Rohit is healthy

SYMBOLIC FORM : $P \vee Q$

03. India is a democratic country OR China is NOT a communist country

P ≡ India is a democratic country

Q ≡ China is a communist country

SYMBOLIC FORM : $P \vee \sim Q$

04. Mango is a fruit BUT potato is a vegetable

P ≡ Mango is a fruit

Q ≡ Potato is a vegetable

SYMBOLIC FORM : $P \wedge Q$

05. Even though it is cloudy , it is still raining

P ≡ It is cloudy

Q ≡ It is still raining

SYMBOLIC FORM : $P \wedge Q$

06. Price increases if and only if demand falls

P ≡ price increases

Q ≡ demand falls

SYMBOLIC FORM : $P \leftrightarrow Q$

07. If Rome is in Italy then Paris is NOT in France

P ≡ Rome is in Italy

Q ≡ Paris is in France

SYMBOLIC FORM : $P \rightarrow \sim Q$

08. the drug is effective though it has side effects

P ≡ Drug is effective

Q ≡ Drug has side effects

SYMBOLIC FORM : $P \wedge Q$

09 it is not true that Ram is tall and handsome

P ≡ Ram is tall

Q ≡ Ram is handsome

SYMBOLIC FORM : $\sim(P \wedge Q)$

10. It is not true that intelligent persons are neither polite nor helpful

NOTE

Intelligent persons are neither polite nor helpful can be read as

Intelligent persons are not polite and not helpful

P ≡ Intelligent persons are polite

Q ≡ Intelligent persons are helpful

SYMBOLIC FORM : $\sim(\sim P \wedge \sim Q)$

Q1

If p and q are true and r and s are false , find the truth values of each of the following compound statements

01.

$\sim [(\sim p \wedge s) \wedge (\sim q \wedge r)]$

$\equiv \sim [(\sim T \wedge F) \wedge (\sim T \wedge F)]$

$\equiv \sim [(F \wedge F) \wedge (F \wedge F)]$

$\equiv \sim (F \wedge F)$

$\equiv \sim F$

$\equiv T$

02.

$(p \leftrightarrow q) \wedge (r \leftrightarrow \sim s)$

$\equiv (T \leftrightarrow T) \wedge (F \leftrightarrow \sim F)$

$\equiv (T \leftrightarrow T) \wedge (F \leftrightarrow T)$

$\equiv T \wedge F$

$\equiv F$

Q4

Express the following statements in the symbolic form and write their truth values

1) It is not true that √2 is a rational number

P ≡ √2 is a rational number (F)

It is not true that √2 is a rational number

$\equiv \sim P$

$\equiv \sim F$

$\equiv T$

2) 16 is an even number and 8 is a perfect square

P ≡ 16 is an even number (T)

Q ≡ 8 is a perfect square (F)

16 is an even number and 8 is a perfect square

$\equiv P \wedge Q$

$\equiv T \vee F$

$\equiv F$

3) 4 is an odd number iff 3 is not a prime factor of 6

P ≡ 4 is an odd number (F)

Q ≡ 3 is a prime factor of 6 (T)

4 is an odd number iff 3 is not a prime factor of 6

$\equiv P \leftrightarrow \sim Q$

$\equiv F \leftrightarrow \sim T$

$\equiv F \leftrightarrow F$

$\equiv T$

4) If $9 > 1$ then $x^2 - 2x + 1 = 0$ for $x = 1$

P ≡ $9 > 1$ (T)

Q ≡ $x^2 - 2x + 1 = 0$ for $x = 1$ (T)

If $9 > 1$ then $x^2 - 2x + 1 = 0$ for $x = 1$

$\equiv P \rightarrow Q$

$\equiv T \rightarrow T$

$\equiv T$

QUANTIFIERS AND QUANTIFIED STATEMENTS

$\forall\ x \in R\ ,\ x^2 \geq 0$

the symbol $\forall$ stands for

' FOR ALL VAUES OF'

$\forall$ is known as UNIVERSAL QUANTIFIER

$\exists\ x \in N$, such that $x + 4 = 7$

the symbol $\exists$ stands for

'THERE EXIST AT LEAST ONE'

$\exists$ is known as EXISTENTIAL QUANTIFIER

AN OPEN SENTENCE WITH A QUANTIFIER BECOMES A STATEMENT AND IS CALLED A QUANTIFIED STATEMENT

Q1

USE QUANTIFIERS TO CONVERT EACH OF THE FOLLOWING OPEN SENTENCES DEFINED ON N , INTO A TRUE STATEMENT

01. $2n - 1 = 5$

$\exists\ n \in N$, such that $2n - 1 = 5$

$2n - 1 = 5$

$n = 3 \in N$ satisfies the above statement and hence TRUE

02. $x^3 < 64$

$\exists\ x \in N$, such that $x^3 < 64$

$x = 1 , 2 , 3 \in N$ satisfies the above statement and hence TRUE

03. $3x - 4 < 9$

$\exists\ x \in N$, such that $3x - 4 < 9$

$3x - 4 < 9$

$3x < 13$

$x = 1 , 2 , 3 , 4 \in N$ satisfies the above statement and hence TRUE

04. $n^2 \geq 1$

$\forall\ n \in N\ ,\ n^2 \geq 1$

all $n \in N$ satisfies the above condition , hence TRUE

NOTE : if at all you happen to use $\exists$ in the above sum , the sentence will become a statement , but FALSE

Q2

If A = {1 , 3 , 5 , 7 } , determine the truth value of each of the following statements

01. $\exists\ x \in A$, such that $x^2 < 1$

No element in A satisfies $x^2 < 1$

Hence

TRUTH VALUE is F

02. $\forall\ x \in A\ ,\ x + 3 < 9$

$x + 3 < 9$

$x < 6$

$x = 7 \in A$ does not satisfy the given condition

Hence

TRUTH VALUE is F

Q1 CONSTRUCT TRUTH TABLE FOR THE GIVEN STATEMENTS

EXERCISE 1.6

01. $(\sim p \vee q) \wedge (\sim p \vee \sim q)$

p	q	~ p	~ q	~p ∨ q	~p ∨ ~q	(~p ∨ q) ∧ (~p ∧ ~q)
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

02. $(\sim q \wedge p) \wedge (p \wedge \sim p)$

p	q	~ p	~ q	~q ∧ p	p ∧ ~p	(~q ∧ p) ∧ (p ∧ ~p)
T	T	F	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	F	F	F
F	F	T	T	F	F	F

Since all the truth values in the last column are 'F' , statement is CONTRADICTION

03. $\sim(p \vee q) \rightarrow \sim(p \wedge q)$

p	q	p ∨ q	~(p ∨ q)	(p ∧ q)	~(p ∧ q)	~(p ∨ q) → ~(p ∧ q)
T	T	T	F	T	F	T
T	F	T	F	F	T	T
F	T	T	F	F	T	T
F	F	F	T	F	T	T

Since all the truth values in the last column are 'T' , statement is TAUTOLOGY

04. $(\sim p \vee \sim q) \leftrightarrow \sim(p \wedge q)$

p	q	~ p	~ q	~p ∨ ~q	p ∧ q	~(p ∧ q)	(~p ∨ ~q) ↔ ~(p ∧ q)
T	T	F	F	F	T	F	T
T	F	F	T	T	F	T	T
F	T	T	F	T	F	T	T
F	F	T	T	T	F	T	T

Since all the truth values in the last column are 'T' , statement is TAUTOLOGY

Q2

Prove that the following statements are LOGICALLY EQUIVALENT

01. $\sim (p \vee q) \equiv \sim p \wedge \sim q$

p	q	~ p	~ q	p ∨ q	COL A: ~ (p ∨ q)	COL B: ~p ∧ ~q
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

Since truth values in col A and col B are identical $\sim (p \vee q) \equiv \sim p \wedge \sim q$

02. $(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

p	q	r	q ∨ r	p ∨ q	COL A: (p ∨ q) → r	p → r	q → r	COL B: (p →r) ∧ (q →r)
T	T	T	T	T	T	T	T	T
T	T	F	T	T	F	F	F	F
T	F	T	T	T	T	T	T	T
T	F	F	F	T	F	F	T	F
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	T	F	F
F	F	T	T	F	T	T	T	T
F	F	F	F	F	T	T	T	T

Since truth values in col A and col B are identical $(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

DUAL STATEMENT

To write dual of any statement , we need to replace $\wedge$ by $\vee$, $\vee$ by $\wedge$, t by c and c by t where t denotes TAUTOLOGY and c denotes CONTRADICTION

DUALITY THEOREM

CONSIDER

$\sim(P \wedge Q) \equiv \sim P \vee \sim Q$

IF WE WRITE THE DUAL OF THE ABOVE

$\sim(P \vee Q) \equiv \sim P \wedge \sim Q$ THIS IS TRUE

NOW THAT'S WHAT DUALITY THEOREM STATES ,

FOR ANY GIVEN PAIR OF LOGICALLY EQUIVALENT STATEMENTS IF WE WRITE THE DUAL ,

THE NEW PAIR OF STATEMENTS FORMED ARE ALSO LOGICALLY EQUIVALENT

Q1

Write the duals of the following statements

01 $\sim(p \wedge q) \vee (\sim q \wedge \sim p)$

DUAL

$\sim(p \vee q) \wedge (\sim q \vee \sim p)$

02. $(\sim p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$

DUAL

$(\sim p \vee q) \wedge (p \vee \sim q) \wedge (\sim p \vee \sim q)$

Q2

State the dual of each of the following statements by applying the principle of duality

01. $p \vee (q \vee r) \equiv (p \vee q) \vee r$

DUAL

$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$

02. $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

DUAL

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

03. $\sim(p \wedge q) \equiv \sim p \vee \sim q$

DUAL

$\sim(p \vee q) \equiv \sim p \wedge \sim q$

Q3

Write the duals of the following statements

01. All natural numbers are integers OR rational numbers

DUAL

All natural numbers are integers AND rational numbers

02. Some roses are red AND all lilies are white

DUAL

Some roses are red OR all lilies are white

03. 13 is a prime number OR India is a democratic country

DUAL

13 is a prime number AND India is a democratic country

NEGATION OF A COMPOUND STATEMENT

$\sim(\sim P) \equiv P$

$\sim(P \vee Q) \equiv \sim P \wedge \sim Q$

$\sim(P \wedge Q) \equiv \sim P \vee \sim Q$

…… DE MORGAN'S LAWS

$\sim(P \rightarrow Q) \equiv P \wedge \sim Q$

NOTE :

IN $\sim(P \rightarrow Q)$, WE ARE TRYING TO WRITE WHEN IS IMPLICATION FALSE . ITS WHEN 'P IS HAPPENING AND Q IS NOT HAPPENING'

HENCE THE ANSWER WAS '$P \wedge \sim Q$'

$\sim(P \leftrightarrow Q) \equiv (P \wedge \sim Q) \vee (Q \wedge \sim P)$

NOTE :

IN $\sim(P \leftrightarrow Q)$, WE ARE TRYING TO WRITE WHEN IS DOUBLE IMPLICATION FALSE .

NOW DOUBLE IMPLICATION IS TRUE WHEN EITHER BOTH ARE HAPPENING OR BOTH ARE NOT HAPPENING

HENCE ITS FALSE WHEN 'P IS HAPPENING AND Q IS NOT HAPPENING OR Q IS HAPPENING AND P IS NOT HAPPENING'

HENCE THE ANSWER WAS

$(P \wedge \sim Q) \vee (Q \wedge \sim P)$

Q1

Using rules of negations , write negation of the following statements

01.

Ramesh is intelligent and he is hard working

$\equiv P \wedge Q$

NEGATION

$\sim(P \wedge Q)$

$\equiv \sim P \vee \sim Q$

≡ Ramesh is NOT intelligent OR Ramesh is NOT is hard working

02.

Kanchanganga is in India and Everest is in Nepal

$\equiv P \wedge Q$

NEGATION

$\sim(P \wedge Q)$

$\equiv \sim P \vee \sim Q$

≡ Kanchanganga is NOT in India OR Everest is NOT in Nepal

03.

Parth plays cricket and chess

$\equiv P \wedge Q$

NEGATION

$\sim(P \wedge Q)$

$\equiv \sim P \vee \sim Q$

≡ Parth does NOT play cricket OR Parth does NOT play chess

04.

If every planet moves around the Sun then every moon of the planet moves around the Sun

$\equiv$ $P \rightarrow Q$

NEGATION

$\sim (P \rightarrow Q)$

$\equiv$ $P \wedge \sim Q$

$\equiv$ Every planet moves around the Sun BUT every moon of the planet does NOT move around the Sun

NOTE : In place of AND , we have used BUT in the above answer

05.

An angle is right angle if and only if it is of measure 90^{o}

$\equiv$ $P \leftrightarrow Q$

NEGATION

$\sim (P \leftrightarrow Q)$

$\equiv$ $(P \wedge \sim Q) \vee (Q \wedge \sim P)$

$\equiv$ an angle is right angle AND it is NOT of measure 90^{o}

OR

Angle is of measure 90^{o} AND it is NOT a right angle

Q2

Using rules of Negation , write the negation of the following

01. $(\sim p \wedge r) \vee (p \vee \sim r)$

$\sim [(\sim p \wedge r) \vee (p \vee \sim r)]$

$\equiv \sim (\sim p \wedge r) \wedge \sim (p \vee \sim r)$

$\equiv (\sim (\sim p) \vee \sim r) \wedge (\sim p \wedge \sim (\sim r))$

$\equiv (p \vee \sim r) \wedge (\sim p \wedge \sim r)$

02. $\sim (p \vee q) \rightarrow r$

$\sim [\sim (p \vee q) \rightarrow r]$

$\equiv \sim (p \vee q) \wedge \sim r$

.... Using $\sim (P \rightarrow Q) \equiv P \wedge \sim Q$

$\equiv (\sim p \wedge \sim q) \wedge \sim r$

.... Using De Morgan's Law

03. $(p \rightarrow q) \wedge r$

$\sim [(p \rightarrow q) \wedge r]$

$\equiv \sim (p \rightarrow q) \vee \sim r$

...... Using De Morgan's Law

$\equiv (p \wedge \sim q) \vee \sim r$

.... Using $\sim (P \rightarrow Q) \equiv P \wedge \sim Q$

04. $(p \leftrightarrow q) \vee (\sim q \rightarrow \sim r)$

$\sim [(p \leftrightarrow q) \vee (\sim q \rightarrow \sim r)]$

$\equiv \sim (p \leftrightarrow q) \wedge \sim (\sim q \rightarrow \sim r)$

...... Using De Morgan's Law

$\equiv ((p \wedge \sim q) \vee (q \wedge \sim p)) \wedge ((\sim q) \wedge \sim (\sim r))$

Using

$\sim (P \rightarrow Q) \equiv P \wedge \sim Q$,

$\sim (P \leftrightarrow Q) \equiv (P \wedge \sim Q) \vee (Q \wedge \sim P)$

$\equiv (p \wedge \sim q) \vee (q \wedge \sim p) \wedge (\sim q \wedge r)$

NEGATIONS OF QUANTIFIED STATEMENTS

CHANGE

EVERY THING TO SOMETHING

SOMETHING TO NOTHING

NOTHING TO SOMETHING

NOTE 1 :

While Changing SOMETHING to NOTHING , it gives a feeling of HA TO NA and hence negation is done

Example :

~ (Some bosses are good)

≡ No boss is good

Similarly while changing NOTHING TO SOMETHING , it gives a feeling of NA TO HA and hence negation is done

Example :

~ (No dog is intelligent)

≡ Some dogs are intelligent

NOTE 2 :

However while changing EVERYTHING TO SOMETHING , negation needs to be done by put a NOT in the statement

Example :

~ (Every student is hardworking)

≡ Some students are NOT hardworking

Q1

Write the negation of the following

01.

All girls are sincere

NEGATION

Some girls are NOT sincere

02.

Some bureaucrats are efficient

NEGATION

NO bureaucrat is efficient

03.

No man is animal

NEGATION

Some men are animals

04.

All integers are rational numbers and all rational numbers are real

$\equiv P \wedge Q$

NEGATION

$\sim (P \wedge Q)$

$\equiv \sim P \vee \sim Q$

≡ SOME integers are not rational numbers OR SOME rational numbers are not real

Q2

NOTE 1

$\forall$ n , condition …..

READ AS :

all n satisfies the given condition

NEGATION

$\exists$ n , such that condition is not satisfied

READ AS :

There exist at least one n , such that the condition is NOT satisfied

NOTE 2

$\exists$ n , such that condition is satisfied

READ AS :

There exist at least one n , such that the condition is satisfied

NEGATION

$\forall$ n , condition is NOT satisfied

READ AS :

all n DO NOT satisfy the given condition

Write the negation of the following

01.

$\forall$ $n \in N$, $n + 1 > 0$

NEGATION

$\exists$ $n \in N$, such that $n + 1 \leq 0$

02.

$\exists$ $n \in N$, $n^2 + 2$ is odd number

NEGATION

$\forall$ $n \in N$, $n^2 + 2$ is NOT odd number

CONVERSE – INVERSE – CONTRAPOSITIVE

FOR A GIVEN 'IF ….. THEN ……' STATEMENT

GIVEN $P \to Q$

CONVERSE : $Q \to P$

CONTRAPOSITVE : $\sim Q \to \sim P$

INVERSE : $\sim P \to \sim Q$

Write Converse - Contrapositive – Invesre of the given statement

01.

if a man is bachelor then he is happy

$\equiv P \to Q$

CONVERSE : $Q \to P$

If man is happy then he is a bachelor

CONTRAPOSITIVE : $\sim Q \to \sim P$

If man is NOT happy then he is NOT a bachelor

INVERSE : $\sim P \to \sim Q$

If man is NOT a bachelor then he is NOT happy

02.

if I do not work hard , then I do not prosper

$\equiv P \to Q$

CONVERSE : $Q \to P$

If I do not prosper , then I do not work hard

CONTRAPOSITIVE : $\sim Q \to \sim P$

If I prosper then I work hard

INVERSE : $\sim P \to \sim Q$

If I work hard then I prosper

ALGEBRA OF STATEMENTS

1. IDEMPOTENT LAW

$p \vee p \equiv p$

$p \wedge p \equiv p$

2. COMMUTATIVE LAW

$p \vee q \equiv q \vee p$

$p \wedge q \equiv q \wedge p$

3. ASSOCIATIVE LAW

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

4. DISTRIBUTIVE LAW

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

5. DE MORGAN'S LAW

$\sim (p \vee q) \equiv \sim p \wedge \sim q$

$\sim (p \wedge q) \equiv \sim p \vee \sim q$.

6. COMPLEMENT LAWS

$p \vee \sim p \equiv t$

$p \wedge \sim p \equiv c$

7. IDENTITY LAWS

$p \vee c \equiv p$	$p \vee t \equiv t$
$p \wedge c \equiv c$	$p \wedge t \equiv p$

EXPLAINATIONS

COMPLEMENT LAWS

$p \vee \sim p \equiv t$

In the above statement , one is true and other is false . In disjunction ($\vee$) , such a statement where one is happening and other is not happening is always TRUE and hence the statement is a TAUTOLOGY

$p \wedge \sim p \equiv c$

In the above statement , one is true and other is false . In conjunction ($\wedge$), such a statement where one is happening and other is not happening is always FALSE and hence the statement is a CONTRADICTION

IDENTITY LAWS

In Disjunction ($\vee$) of two statements , atleast one needs to be true , for the statement to be true

$p \vee c \equiv p$

In the above statement , the second statement is a contradiction i.e false and hence the outcome of the statement now depends on P . If P is true , the statement goes true , If P is false , the statement goes false . Hence 'p'

$p \vee t \equiv t$

In the above statement , the second statement is tautology and hence the statement on the whole is TRUE

In conjunction ($\wedge$) of two statements , both need to be true , for the statement to be true

$p \wedge c \equiv c$

In the above case , second statement is false and hence the entire statement is a contradiction

$p \wedge t \equiv p$

In the above statement , the second part of the statement is a tautology i.e true and hence the outcome of the statement now depends on P . If P is true , the statement goes true , If P is false , the statement goes false . Hence .....

Using ALGEBRA OF STATEMENTS / WITHOUT TRUTH TABLES prove :

1. $\sim (p \vee q) \vee (\sim p \wedge q) \equiv \sim p$

Solution $\sim (p \vee q) \vee (\sim p \wedge q)$

$\equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q)$ .... DE MORGAN'S LAW

$\equiv \sim p \wedge (\sim q \vee q)$ .... DISTRIBUTIVE LAW

$\equiv \sim p \wedge t$ .... COMPLEMENT LAW

$\equiv \sim p$ .... IDENTITY LAW

2. $p \wedge [(\sim p \vee q) \vee \sim q] \equiv p$

Solution $p \wedge [(\sim p \vee q) \vee \sim q]$

$\equiv p \wedge [\sim p \vee (q \vee \sim q]$ .... ASSOCIATIVE LAW

$\equiv p \wedge (\sim p \vee t)$ .... COMPLEMENT LAW

$\equiv p \wedge t$ .... IDENTITY LAW

$\equiv p$ .... IDENTITY LAW

3. $(p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q) \equiv p \vee \sim q$

Solution $[(p \wedge q) \vee (p \wedge \sim q)] \vee (\sim p \wedge \sim q)$

$\equiv [p \wedge (q \vee \sim q)] \vee (\sim p \wedge \sim q)$ .... DISTRIBUTIVE LAW

$\equiv (p \wedge t) \vee (\sim p \wedge \sim q)$ .... COMPLEMENT LAW

$\equiv p \vee (\sim p \wedge \sim q)$ .... IDENTITY LAW

$\equiv (p \vee \sim p) \wedge (p \vee \sim q)$ .... DISTRIBUTIVE LAW

$\equiv t \wedge (p \vee \sim q)$ .... COMPLEMENT LAW

$\equiv p \vee \sim q$ .... IDENTITY LAW

4. $[(p \vee q) \wedge \sim p] \to q$ is a tautology

Solution $[(p \vee q) \wedge \sim p] \to q$

$\equiv [(p \wedge \sim p) \vee (q \wedge \sim p)] \to q$ ……… DISTRIBUTIVE LAW

$\equiv [c \vee (q \wedge \sim p)] \to q$ ……… COMPLEMENT LAW

$\equiv (q \wedge \sim p) \to q$ ……… IDENTITY LAW

$\equiv \sim (q \wedge \sim p) \vee q$ ……… $P \to Q \equiv \sim P \vee Q$

$\equiv (\sim q \vee p) \vee q$ ……… DE MORGAN'S LAW

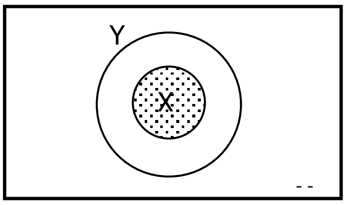
$\equiv q \vee (\sim q \vee p)$ ……… COMMUTATIVE LAW

$\equiv (q \vee \sim q) \vee p$ ……… ASSOCIATIVE LAW

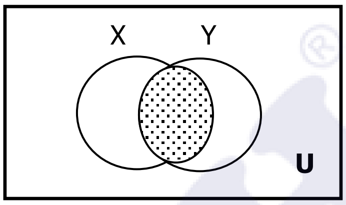
$\equiv t \vee p$ ……… COMPLEMENT LAW

$\equiv t$ ……… IDENTITY LAW

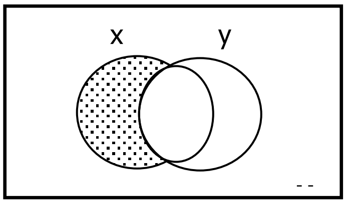
TYPES OF VENN DIAGRAMS



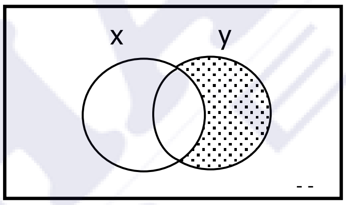
ALL X ARE Y , X→Y



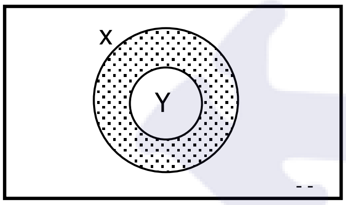
SOME X ARE Y



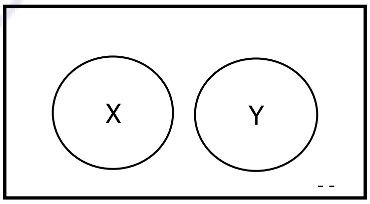
SOME X ARE NOT Y



SOME Y ARE NOT X



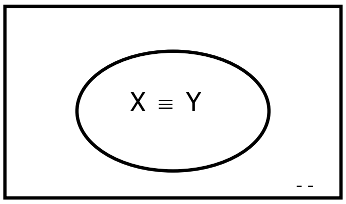
SOME X ARE NOT Y



NO X IS Y

NOTE :

BEFORE SUBMITTING THE ABOVE VENN DIAGRAM , STUDENT MUST ASSURE THAT THE SILENT PART OF THE ABOVE STATEMENT DID MEAN OR SAY 'BUT ALL Y ARE X'



ALL X ARE Y AND ALL Y ARE X , X ↔ Y

Q1

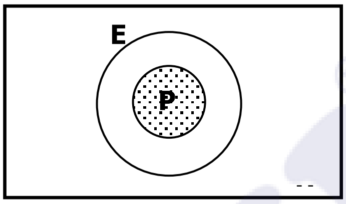
Express the truth of each of the following statement by VENN DIAGRAM

01. all professors are educated

P ≡ set of all professors

E ≡ set of all educated people

U ≡ set of all human beings



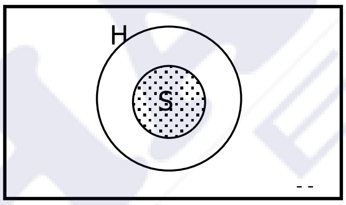
02. Sunday implies holiday

(It means : - All Sundays are Holidays)

S ≡ set of all Sundays

H ≡ set of all Holidays

U ≡ set of all days in a year



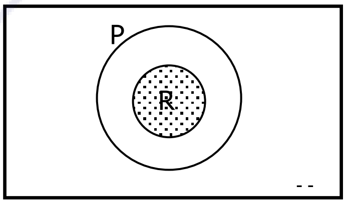
03. If a quadrilateral is a rhombus then its is a parallelogram

(It means : - All rhombus are parallelograms)

R ≡ set of all rhombus

P ≡ set of all parallelograms

U ≡ set of all quadrilaterals



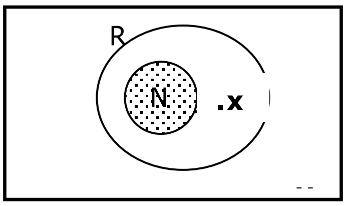
04. All natural numbers are real numbers and x is not a natural number

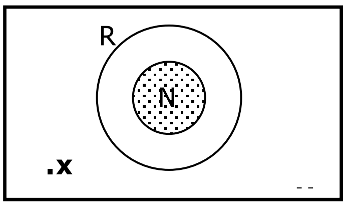
(It means : - All rhombus are parallelograms)

N ≡ set of all natural numbers

R ≡ set of all real numbers

U ≡ set of all numbers (complex)



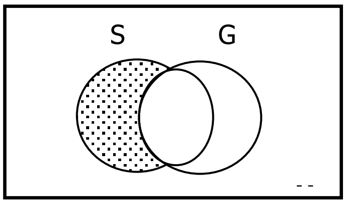


05. Many servants are not graduates

S ≡ set of all servants

G ≡ set of all graduates

U ≡ set of all human beings

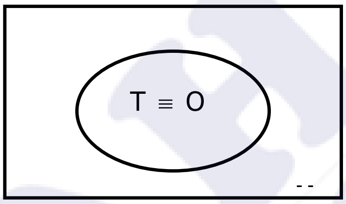


06. All teachers are scholars and scholars are teachers

T ≡ set of all teachers

O ≡ set of all scholars

U ≡ set of all human beings

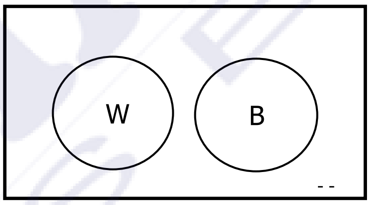


07. No wicketkeeper is bowler in a cricket team

W ≡ set of all wicket keepers

B ≡ set of all bowlers

U ≡ set of all players in a cricket team



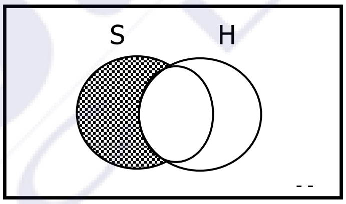
Q2 Using Venn Diagram , examine the LOGICAL EQUIVALENCE of the following statements

01. a. There are students who are not scholars

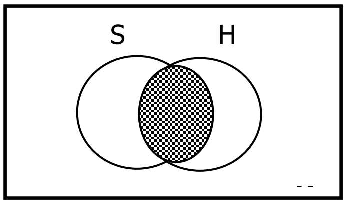
b. There are scholars who are students

c. There are persons who are scholars and students

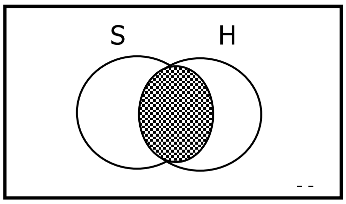
S ≡ set of all students ; H ≡ set of all scholars ; U ≡ set of all human beings



Statement (a)



Statement (b)



Statement (c)

CONCLUSION

Statement (b) & (c)

are LOGICALLY EQUIVALENT

Q3

Express the truth of each of the following statement by VENN DIAGRAM

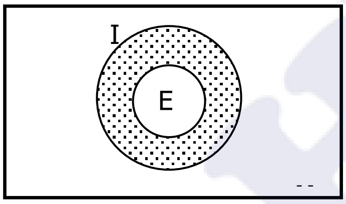
01. Some Isosceles triangles are not equilateral triangles

(Some Isosceles triangles are not equilateral triangles but all equilateral triangles are isosceles)

E ≡ set of all equilateral triangles

I ≡ set of all isosceles triangles

U ≡ set of all triangles



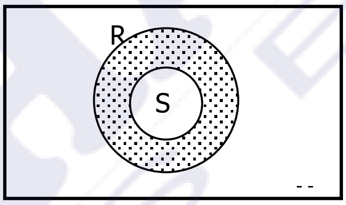
02. Some rectangles are not squares

(Some rectangles are not squares BUT all squares are rectangles)

R ≡ set of all rectangles

S ≡ set of all squares

U ≡ set of all quadrilaterals



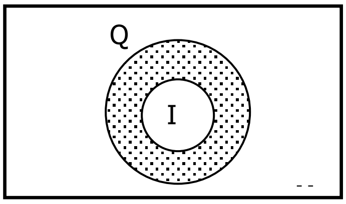
03. Some rational numbers are not integers

(Some rational numbers are not integers but all integers are rational)

Q ≡ set of all rational numbers

I ≡ set of all integers

U ≡ set of all real numbers



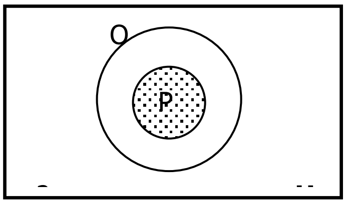
04. If n is a prime number and n ≠ 2 , then it is odd

(all prime numbers except 2 are odd)

P ≡ set of all prime numbers n , n≠2

O ≡ set of all odd numbers

U ≡ set of all real numbers



PAPER - I CHAPTER - 2

MATRICES

CLASSIFICATION OF MATRICES

MATRICES IN GENERAL

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2 \times 3}$$

A → NAME OF THE MATRIX

2 → NO. OF ROWS

3 → NO. OF COLUMNS

ROW MATRIX

$A = \begin{bmatrix} 3 & 4 & 5 \end{bmatrix}$

CONTAINS SINGLE ROW

COLUMN MATRIX

$B = \begin{bmatrix} 2 \\ 7 \\ 3 \end{bmatrix}$

CONTAINS SINGLE COLUMN

NULL MATRIX

$O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

RECTANGULAR MATRIX

$C = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 4 & 6 \end{bmatrix}$

NO. OF ROWS ≠ NO. OF COL'S

SQUARE MATRIX

$X = \begin{bmatrix} 1 & 1 & 3 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{bmatrix}$

NO OF ROW = NO. OF COL'S

SQUARE MATRIX → DIAGONAL MATRIX → SCALAR MATRIX → IDENTITY MATRIX

SQUARE MATRIX → SYMMETRIC MATRIX, SKEW SYMMETRIC MATRIX

DIAGONAL MATRIX

$E = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

ALL NON DIAGONAL ELEMENTS = 0

SCALAR MATRIX

$D = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

ALL DIAGONAL ELEMENTS ARE EQUAL

IDENTITY MATRIX

$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

ALL DIAGONAL ELEMENTS ARE EQUAL TO '1'

SYMMETRIC MATRIX

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$

$a_{ij} = a_{ji}, \ \forall\ i,j$

SKEW SYMMETRIC MATRIX

$A = \begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & -4 \\ -3 & 4 & 0 \end{bmatrix}$

$a_{ij} = -a_{ji}, \ \forall\ i,j$

OPERATIONS ON MATRICES

EQUALITY OF MATRICES

> MATRCIES SHOULD BE OF SAME ORDER

> ALL THE CORRESPONDING ELEMENTS MUST BE EQUAL

> EXAMPLE

$$\begin{bmatrix} a & b \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix}$$

THEN a = 4 , b = 5

ADD / SUBTRACT BETWEEN MATRICES

> MATRICES MUST BE OF SAME ORDER

> ADD/SUB THE CORRESPONING ELEMENTS

> EXAMPLE

$$\begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 5 & 7 \end{bmatrix}$$

SCALAR MULTIPLICATION

> MULTIPLY MATRIX WITH A SCALAR

> MULTIPLY ALL THE ELEMENTS IN THE MATRIX BY THAT SCALAR

> EXAMPLE

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$$

THEN $2A = \begin{bmatrix} 4 & 6 \\ 2 & 2 \end{bmatrix}$

TRANSPOSE OF A MATRIX

> CHANGE ROW TO COLUMNS

> EXAMPLE

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

TRANSPOSE OF A

$$A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

DETERMINANT OF A MATRIX

> MATRIX HAS TO BE A SQUARE MATRIX

> EXAMPLE $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

DET. OF A = $|A|$ = 4 – 6 = – 2

> IF |A| = 0 THEN MATRIX A IS CALLED A SINGUALR MATRIX ELSE IT IS CALLED NON – SINGULAR MATRIX

PROPERTIES ON TRANSPOSE OF A MATRIX , SYMMTERIC & SKEW SYMMETRIC MATRIX

> $(A^T)^T = A$

> $(A+B)^T = A^T + B^T$

> $(AB)T = B^T.A^T$

> If A IS A SYMMETRIC MATRIX , THEN $A^T = A$

> If A IS SKEW SYMMETRIC MATRIX , THEN $A^T = -A$

> FOR ANY SQUARE MATRIX A ,
$A + A^T$ = SYMMTRIC MATRIX
$A - A^T$ = SKEW SYMMTRIC MATRIX

> ANY SQUARE MATRIX A CAN BE EXPRESSED AS SUM OF SYMMETRIC & SKEW SYMMETRIC MATRIX

$$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$$

MULTIPLICATION BETWEEN MATRICES

FOR AB TO EXISTS : **NO OF COLUMNS OF 'A' = NO. OF ROWS OF 'B'**

EXAMPLE1

$$A = \begin{matrix} R1 \\ R2 \end{matrix}\begin{bmatrix} 2 & 1 & 3 \\ 3 & 0 & 1 \end{bmatrix} \quad B = \begin{matrix} C1 & C2 \\ \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 2 \end{bmatrix} \end{matrix} \quad AB = \begin{bmatrix} R1C1 & R1C2 \\ R2C1 & R2C2 \end{bmatrix} = \begin{bmatrix} 2(1) + 1(2) + 3(3) & 2(4) + 1(5) + 3(2) \\ 3(1) + 0(2) + 1(3) & 3(4) + 0(5) + 1(2) \end{bmatrix} = \begin{bmatrix} 13 & 19 \\ 6 & 14 \end{bmatrix}$$

EXAMPLE 2

LET US TRY 'BA'

$$BA = \begin{matrix} R1 \\ R2 \\ R3 \end{matrix}\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 2 \end{bmatrix}\begin{matrix} C1 & C2 & C3 \\ \begin{bmatrix} 2 & 1 & 3 \\ 3 & 0 & 1 \end{bmatrix} \end{matrix} = \begin{bmatrix} R1C1 & R1C2 & R1C3 \\ R2C1 & R2C2 & R2C3 \\ R3C1 & R3C2 & R3C3 \end{bmatrix} = \begin{bmatrix} 1(2) + 4(3) & 1(1) + 4(0) & 1(3) + 4(1) \\ 2(2) + 5(3) & 2(1) + 5(0) & 2(3) + 5(1) \\ 3(2) + 2(3) & 3(1) + 2(0) & 3(3) + 2(1) \end{bmatrix} = \begin{bmatrix} 14 & 4 & 7 \\ 19 & 2 & 11 \\ 12 & 3 & 11 \end{bmatrix}$$

NOTE : $AB \neq BA$. HENCE MATRIX MULTIPLICATION IS NON COMMUTATIVE

SUMS ON ADDITION – SUBTRACTION BETWEEN MATRICES , SINGULAR MATRIX , SYMMETRIC & SKEW SYMMETRIC MATRIX

01.

Construct a matrix $A = [a_{ij}]_{3\times 2}$ whose element a_{ij} is given by $a_{ij} = i - 3j$

$a_{11} = 1 - 3(1) = -2$ $\quad a_{12} = 1 - 3(2) = -5$

$a_{21} = 2 - 3(1) = -1$ $\quad a_{22} = 2 - 3(2) = -4$

$a_{31} = 3 - 3(1) = 0$ $\quad a_{32} = 3 - 3(2) = -3$

$$A = \begin{pmatrix} -2 & -5 \\ -1 & -4 \\ 0 & -3 \end{pmatrix}$$

02. Find K if the folloiwng matrices are singular

$$A = \begin{pmatrix} 4 & 3 & 1 \\ 7 & k & 1 \\ 10 & 9 & 1 \end{pmatrix}$$

Since A is singular matrix ,

$|A| = 0$

$4(k - 9) - 3(7 - 10) + 1(63 - 10k) = 0$

$4k - 36 + 9 + 63 - 10k = 0$

$36 - 6k = 0$

$k = 6$

03. $A = \begin{pmatrix} 1 & -2 \\ 3 & -5 \\ -6 & 0 \end{pmatrix}$ $B = \begin{pmatrix} -1 & -2 \\ 4 & 2 \\ 1 & 5 \end{pmatrix}$ & $C = \begin{pmatrix} 2 & 4 \\ -1 & -4 \\ -3 & 6 \end{pmatrix}$ find matrix X such that $3A - 4B + 5X = C$

$3A - 4B + 5X = C$

$5X = C - 3A + 4B$

$$5X = \begin{pmatrix} 2 & 4 \\ -1 & -4 \\ -3 & 6 \end{pmatrix} - 3\begin{pmatrix} 1 & -2 \\ 3 & -5 \\ -6 & 0 \end{pmatrix} + 4\begin{pmatrix} -1 & -2 \\ 4 & 2 \\ 1 & 5 \end{pmatrix}$$

$$5X = \begin{pmatrix} 2 & 4 \\ -1 & -4 \\ -3 & 6 \end{pmatrix} - \begin{pmatrix} 3 & -6 \\ 9 & -15 \\ -18 & 0 \end{pmatrix} + \begin{pmatrix} -4 & -8 \\ 16 & 8 \\ 4 & 20 \end{pmatrix}$$

$$5X = \begin{pmatrix} 2 - 3 - 4 & 4 + 6 - 8 \\ -1 - 9 + 16 & -4 + 15 + 8 \\ -3 + 18 + 4 & 6 - 0 + 20 \end{pmatrix}$$

$$X = \frac{1}{5}\begin{pmatrix} -5 & 2 \\ 6 & 19 \\ 19 & 26 \end{pmatrix}$$

04. find a , b , c if $\begin{bmatrix} 1 & 3/5 & a \\ b & -5 & -7 \\ -4 & c & 0 \end{bmatrix}$ is a symmetric matrix

in a symmetric matrix $a_{ij} = a_{ji}$ for all i and j , $\therefore$ $b = 3/5$, $a = -4$, $c = -7$

05. find x , y , z if $\begin{bmatrix} 0 & -5i & x \\ y & 0 & z \\ 3/2 & -\sqrt{2} & 0 \end{bmatrix}$ is a skew symmetric matrix

in a skew symmetric matrix $a_{ij} = -a_{ji}$ for all i and j

$\therefore$ $y = -(-5i) = 5i$, $x = -3/2$, $z = -(-\sqrt{2}) = \sqrt{2}$

SUMS ON MULTIPLICATION BETWEEN MATRICES

01. $A = \begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 1 \\ 2 & -1 \end{bmatrix}$ Verify : A(B + C) = AB + AC

LHS

$$A(B + C) = \begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix} \left\{ \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 2 & -1 \end{bmatrix} \right\}$$

$$= \begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 12 - 10 & 8 + 6 \\ 6 + 15 & 4 - 9 \end{bmatrix} = \begin{bmatrix} 2 & 14 \\ 21 & -5 \end{bmatrix}$$

RHS

$$AB = \begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} -4 - 6 & 4 + 4 \\ -2 + 9 & 2 - 6 \end{bmatrix} = \begin{bmatrix} -10 & 8 \\ 7 & -4 \end{bmatrix}$$

$$AC = \begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 16 - 4 & 4 + 2 \\ 8 + 6 & 2 - 3 \end{bmatrix} = \begin{bmatrix} 12 & 6 \\ 14 & -1 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} -10 & 8 \\ 7 & -4 \end{bmatrix} + \begin{bmatrix} 12 & 6 \\ 14 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 14 \\ 21 & -5 \end{bmatrix}$$

PROVED : A(B + C) = AB + AC

02. if $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$; show that $A^2 - 4A$ is a scalar matrix

$$A^2 = A.A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

$$A^2 - 4A = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

= scalar matrix .... By definition

03. Find k if $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $A^2 = kA - 2I$

$$A^2 = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

GIVEN : $A^2 = kA - 2I$

$$\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

By Equality of two matrices ,

$1 = 3k - 2 \quad \therefore k = 1$

$4 = 4k \quad \therefore k = 1$

$-2 = -2k \quad \therefore k = 1$

$-4 = -2k - 2 \quad \therefore k = 1$

04. Find x , y , z if $\left\{3\begin{bmatrix}2 & 0\\0 & 2\\2 & 2\end{bmatrix} - 4\begin{bmatrix}1 & 1\\-1 & 2\\3 & 1\end{bmatrix}\right\}\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$

$$\left\{3\begin{bmatrix}2 & 0\\0 & 2\\2 & 2\end{bmatrix} - 4\begin{bmatrix}1 & 1\\-1 & 2\\3 & 1\end{bmatrix}\right\}\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$$

$$\left\{\begin{bmatrix}6 & 0\\0 & 6\\6 & 6\end{bmatrix} - \begin{bmatrix}4 & 4\\-4 & 8\\12 & 4\end{bmatrix}\right\}\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$$

$$\begin{bmatrix}2 & -4\\4 & -2\\-6 & 2\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$$

$$\begin{bmatrix}2-8\\4-4\\-6+4\end{bmatrix} = \begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$$

$$\begin{bmatrix}-6\\0\\-2\end{bmatrix} = \begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$$

By equality of matrices

$x - 3 = -6$

$x = -3$

$y - 1 = 0$

$y = 1$

$2z = -2$

$z = -1$

05. $A = \begin{bmatrix}1 & 2\\-1 & -2\end{bmatrix}$; $B = \begin{bmatrix}2 & a\\-1 & b\end{bmatrix}$, if $(A+B)^2 = A^2 + B^2$, find A and B

GIVEN $(A + B)^2 = A^2 + B^2$

$(A + B)(A + B) = A^2 + B^2$

$A^2 + AB + BA + B^2 = A^2 + B^2$

$AB + BA = 0$

$$AB = \begin{bmatrix}1 & 2\\-1 & -2\end{bmatrix}\begin{bmatrix}2 & a\\-1 & b\end{bmatrix} = \begin{bmatrix}2-2 & a+2b\\-2+2 & -a-2b\end{bmatrix} = \begin{bmatrix}0 & a+2b\\0 & -a-2b\end{bmatrix}$$

$$BA = \begin{bmatrix}2 & a\\-1 & b\end{bmatrix}\begin{bmatrix}1 & 2\\-1 & -2\end{bmatrix} = \begin{bmatrix}2-a & 4-2a\\-1-b & -2-2b\end{bmatrix}$$

$AB + BA = 0$

$$\begin{bmatrix}0 & a+2b\\0 & -a-2b\end{bmatrix} + \begin{bmatrix}2-a & 4-2a\\-1-b & -2-2b\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

$$\begin{bmatrix} 2-a & -a+2b+4 \\ -1-b & -a-4b-2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

By equality of matrices , $2 - a = 0 \quad \therefore\ a = 2$, $-1 - b = 0 \quad \therefore\ b = -1$

NOTE	Student must substitute $a = 2$, $b = -1$ in other two equations to check			
	Substituting $a = 2$, $b = -1$	in	$-a + 2b + 4 = 0$	
			$-2 - 2 + 4 = 0$	satisfied
		in	$-a - 4b - 2 = 0$	
			$-2 + 4 - 2 = 0$	satisfied

SUMS ON PROPERTIES OF TRANSPOSE – SYMMETRIC & SKEW SYMMETRIC MATRIX

01. if $A = \begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 4 & -1 \\ -3 & 3 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ -2 & 3 \end{bmatrix}$, then show that

$(A + B)^T = A^T + B^T$

LHS : $(A + B) = \begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 4 & -1 \\ -3 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 9 & -5 \\ -9 & 4 \end{bmatrix}$

$(A + B)^T = \begin{bmatrix} 4 & 9 & -9 \\ -2 & -5 & 4 \end{bmatrix}$

RHS : $A^T + B^T = \begin{bmatrix} 2 & 5 & -6 \\ -3 & -4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 4 & -3 \\ 1 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 9 & -9 \\ -2 & -5 & 4 \end{bmatrix}$

02. Prove that $A + A^T$ is a symmetric and $A - A^T$ is a skew symmetric matrix , where

$A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ -2 & -3 & 2 \end{bmatrix}$

$A + A^T = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ -2 & -3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 & -2 \\ 2 & 2 & -3 \\ 4 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 5 & 2 \\ 5 & 4 & -2 \\ 2 & -2 & 4 \end{bmatrix}$ = symmetric by definition

$A - A^T = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ -2 & -3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 & -2 \\ 2 & 2 & -3 \\ 4 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 6 \\ 1 & 0 & 4 \\ -6 & -4 & 0 \end{bmatrix}$ = skew symmetric by definition

03. Express the following matrix as a sum of a symmetric and a skew symmetric matrix

$$A = \begin{bmatrix} 4 & -2 \\ 3 & -5 \end{bmatrix}$$

$$A + A^T = \begin{bmatrix} 4 & -2 \\ 3 & -5 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ -2 & -5 \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ 1 & -10 \end{bmatrix}$$ = symmetric matrix , by definition

$$A - A^T = \begin{bmatrix} 4 & -2 \\ 3 & -5 \end{bmatrix} - \begin{bmatrix} 4 & 3 \\ -2 & -5 \end{bmatrix} = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$$ = skew symmetric matrix by definition

NOW , $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$

$$A = \frac{1}{2}\begin{bmatrix} 8 & 1 \\ 1 & -10 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$$

04. $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \\ 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 3 & -4 \\ 2 & -1 & 1 \end{bmatrix}$ VERIFY $(AB)^T = B^T.A^T$

LHS

$$AB = \begin{bmatrix} 2 & -1 \\ 3 & -2 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 0 & 3 & -4 \\ 2 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 0-2 & 6+1 & -8-1 \\ 0-4 & 9+2 & -12-2 \\ 0+2 & 12-1 & -16+1 \end{bmatrix} = \begin{bmatrix} -2 & 7 & -9 \\ -4 & 11 & -14 \\ 2 & 11 & -15 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} -2 & -4 & 2 \\ 7 & 11 & 11 \\ -9 & -14 & -15 \end{bmatrix}$$

RHS

$$B^T.A^T = \begin{bmatrix} 0 & 2 \\ 3 & -1 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 \\ -1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 0-2 & 0-4 & 0+2 \\ 6+1 & 9+2 & 12-1 \\ -8-1 & -12-2 & -16+1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -4 & 2 \\ 7 & 11 & 11 \\ -9 & -14 & -15 \end{bmatrix}$$

LHS = RHS

INVERSE OF A MATRIX USING ERO /ECO

WORKING

STEP 1 : $|A| \neq 0$, A^{-1} EXISTS

STEP 2 : START $A.A^{-1} = I$

STEP 3 : APPLY ROW OPERTAIONS ON 'A' TO CONVERT IT TO 'I' . SIMULATENOUSLY APPLY SAME SET OF ROW OPERATIONS ON 'I' ON THE OTHER SIDE

STEP 4 : REACH TO A STAGE OF : $IA^{-1} = B$ (SAY)

$A^{-1} = B$

NOTE –

1. AX = B , THE TWO SIDES WILL KEEP CONFIRMING WITH EACH OTHER ONLY WHEN ERO'S ARE APPLIED SIMULATENAOUSLY ON A AND B . SO IF ASKED TO FIND X , USE ERO TO CHANGE $A \rightarrow I$

2. XA = B , THE TWO SIDES WILL KEEP CONFIRMING WITH EACH OTHER ONLY WHEN ECO'S ARE APPLIED SIMULATENAOUSLY ON A AND B . SO IF ASKED TO FIND X , USE ECO TO CHANGE $A \rightarrow I$

SYSTEM/SEQUENCE TO BE FOLLOWED WHILE CHANGING 'A' TO 'I'

FOR 2 X 2 MATRIX

USING ERO

$A \longrightarrow I$

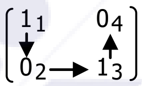
$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

STEP 1 : CHANGE 'a_{11}' TO '1'

STEP 2 : MOVE TO 'a_{21}' . MAKE IT '0'

STEP 3 : MOVE TO 'a_{22}' , MAKE IT '1'

STEP 4 : MOVE TO 'a_{12}' , MAKE IT '0'



USING ECO

$A \longrightarrow I$

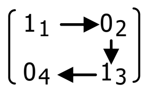
$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

STEP 1 : CHANGE 'a_{11}' TO '1'

STEP 2 : MOVE TO 'a_{12}' . MAKE IT '0'

STEP 3 : MOVE TO 'a_{22}' , MAKE IT '1'

STEP 4 : MOVE TO 'a_{21}' , MAKE IT '0

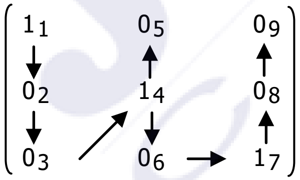


FOR 3 X 3 MATRIX

USING ERO

$A \longrightarrow I$

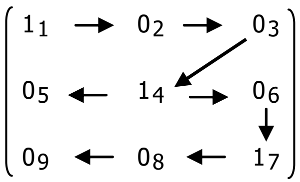
$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$



USING ECO

$A \longrightarrow I$

$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$



01. FIND A^{-1} USING ERO

$A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

$|A| = 6 - 5 = 1 \neq 0$

Hence A^{-1} exists

$AA^{-1} = I$

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$R_1 - R_2$

$$\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$R_2 - R_1$

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

$R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

$$I.\ A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

02. FIND A^{-1} USING ERO

$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$

$|A| = 1(7 - 20) - 2(7 - 10) + 3(4 - 2)$

$= 1(-13) - 2(-3) + 3(2)$

$= -13 + 6 + 6$

$= -1$

$\neq 0$ Hence A^{-1} exists

$AA^{-1} = I$

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 - R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \\ 2 & 4 & 7 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 - 2R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$(-1)R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -1 & 2 & 0 \\ 1 & -1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$R_2 + 2R_3$

$$\begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -1 & 2 & 0 \\ -3 & -1 & 2 \\ -2 & 0 & 1 \end{bmatrix}$$

$R_1 - 7R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 13 & 2 & -7 \\ -3 & -1 & 2 \\ -2 & 0 & 1 \end{bmatrix}$$

$$I.\ A^{-1} = \begin{bmatrix} 13 & 2 & -7 \\ -3 & -1 & 2 \\ -2 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 13 & 2 & -7 \\ -3 & -1 & 2 \\ -2 & 0 & 1 \end{bmatrix}$$

03. FIND A^{-1} USING ERO

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

$$|A| = 2(3-0) - 0(15-0) - 1(5-0)$$
$$= 6 - 5$$
$$= 1$$
$$\neq 0 \quad \text{Hence } A^{-1} \text{ exists}$$

$$AA^{-1} = I$$

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$3R_1$

$$\begin{bmatrix} 6 & 0 & -3 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_1 - R_2$

$$\begin{bmatrix} 1 & -1 & -3 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 - 5R_1$

$$\begin{bmatrix} 1 & -1 & -3 \\ 0 & 6 & 15 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 0 \\ -15 & 6 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 - 5R_3$

$$\begin{bmatrix} 1 & -1 & -3 \\ 0 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 0 \\ -15 & 6 & -5 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 - R_2$

$$\begin{bmatrix} 1 & -1 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 0 \\ -15 & 6 & -5 \\ 15 & -6 & 6 \end{bmatrix}$$

$R_1 + R_2$

$$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} -12 & 5 & -5 \\ -15 & 6 & -5 \\ 15 & -6 & 6 \end{bmatrix}$$

$R_2 / 3$

$$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -12 & 5 & -5 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$R_1 + 3R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$I.\,A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

04. FIND A^{-1} USING ECO

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

$|A| = 2(3-0) - 0(15-0) - 1(5-0)$
$= 6 - 5$
$= 1$
$\neq 0$ Hence A^{-1} exists

$A^{-1}.A = I$

$$A^{-1}\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_1 + C_3$

$$A^{-1}\begin{bmatrix} 1 & 0 & -1 \\ 5 & 1 & 0 \\ 3 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$C_3 + C_1$

$$A^{-1}\begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 5 \\ 3 & 1 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

$C_1 - 5C_2$

$$A^{-1}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 5 \\ -2 & 1 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ -5 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

$C_3 - 5C_2$

$$A^{-1}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ -5 & 1 & -5 \\ 1 & 0 & 2 \end{bmatrix}$$

$C_2 - C_3$

$$A^{-1}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -5 & 6 & -5 \\ 1 & -2 & 2 \end{bmatrix}$$

$C_1 + 2C_3$

$$A^{-1}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$A^{-1}.I = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

INVERSE OF A MATRIX USING ADJOINT METHOD

GIVEN

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

FIND A^{-1} USING ADJOINT METHOD

STEP 1 : FIND $|A|$, $|A| \neq 0$, A^{-1} EXISTS

STEP 2 : COFACTOR'S

$$A_{11} = (-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

STEP 3 : COFACTOR MATRIX = $\begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}$

STEP 4 : ADJ A = TRANSPOSE OF THE COFACTOR MATRIX

$$= \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$$

STEP 5 : $A^{-1} = \dfrac{1}{|A|}$ (ADJ A)

Find the inverse of the following matrices by the adjoint method

01.

$A = \begin{bmatrix} 3 & -1 \\ 2 & -1 \end{bmatrix}$ $|A| = -3 + 2 = -1 \neq 0$

A^{-1} exist

COFACTORS

$A_{11} = (-)^{1+1}(-1) = -1$

$A_{12} = (-)^{1+2}(2) = -2$

$A_{21} = (-)^{2+1}(-1) = 1$

$A_{22} = (-)^{2+2}(3) = 3$

COFACTOR MATRIX $= \begin{bmatrix} -1 & -2 \\ 1 & 3 \end{bmatrix}$

Adj A = transpose of cofactor matrix

$= \begin{bmatrix} -1 & 1 \\ -2 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{ adj } A$

$= -1 \begin{bmatrix} -1 & 1 \\ -2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix}$

02.

$A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 3 & 5 \\ -2 & 0 & -1 \end{bmatrix}$

$|A| = 1(-3 - 0) + 1(2 + 10) + 2(0+6)$

$= -3 + 12 + 12 = 21$

COFACTORS

$A_{11} = (-)^{1+1}\begin{vmatrix} 3 & 5 \\ 0 & -1 \end{vmatrix} = +(-3-0) = -3$

$A_{12} = (-)^{1+2}\begin{vmatrix} -2 & 5 \\ -2 & -1 \end{vmatrix} = -(2+10) = -12$

$A_{13} = (-)^{1+3}\begin{vmatrix} -2 & 3 \\ -2 & 0 \end{vmatrix} = +(-0+6) = 6$

$A_{21} = (-)^{2+1}\begin{vmatrix} -1 & 2 \\ 0 & -1 \end{vmatrix} = -(1-0) = -1$

$A_{22} = (-)^{2+2}\begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix} = +(-1+4) = 3$

$A_{23} = (-)^{2+3}\begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = -(0-2) = 2$

$A_{31} = (-)^{3+1}\begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix} = +(-5-6) = -11$

$A_{32} = (-)^{3+2}\begin{vmatrix} 1 & 2 \\ -2 & 5 \end{vmatrix} = -(5+4) = -9$

$A_{33} = (-)^{3+3}\begin{vmatrix} 1 & -1 \\ -2 & 3 \end{vmatrix} = +(3-2) = 1$

COFACTOR MATRIX

$= \begin{bmatrix} -3 & -12 & 6 \\ -1 & 3 & 2 \\ -11 & -9 & 1 \end{bmatrix}$

Adj A = transpose of cofactor matrix

$= \begin{bmatrix} -3 & -1 & -11 \\ -12 & 3 & -9 \\ 6 & 2 & 1 \end{bmatrix}$

$A-1 = \frac{1}{|A|} (\text{ADJ } A)$

$= \frac{1}{21} \begin{bmatrix} -3 & -1 & -11 \\ -12 & 3 & -9 \\ 6 & 2 & 1 \end{bmatrix}$

SOLVE SYSTEM OF LINEAR EQUATIONS (REDUCTION METHOD)

GIVEN SET OF EQUATIONS

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

SOLVE FOR x , y & z

STEP 1 : WRITE THE ABOVE SET OF EQUATIONS IN THE MATRIX FORM

AX = B

$$\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$$

STEP 2 : APPLY 'ERO' SIMULTANEOUSLY ON A & B TO REACH TO

$$\begin{pmatrix} 1 & -- & -- \\ 0 & \boxed{\begin{matrix} -- & -- \\ -- & -- \end{matrix}} \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} \quad \text{(SAY)}$$

STEP 3 : USING 'ERO' CONVERT ANY ONE OF THE 4 ELEMENTS ENCLOSED IN A BOX ABOVE TO '0'

STEP 4 : HAVING DONE THAT MULTIPLY THE MATRICES AND USING EQUALITY OF MATRICES COME BACK TO EQUATION FORM TO SOLVE FOR x , y & z

SOLVE SYSTEM OF LINEAR EQUATIONS (REDUCTION METHOD)

GIVEN SET OF EQUATIONS

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

SOLVE FOR x , y & z

STEP 1 : WRITE THE ABOVE SET OF EQUATIONS IN THE MATRIX FORM

$$\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$$

AX = B

$X = A^{-1}B$

STEP 2 : FIND $|A|$, $|A| \neq 0$, A^{-1} EXISTS

STEP 3 : FIND A^{-1} USING ERO OR ADJOINT METHOD

STEP 4 : GO BACK TO STEP 1 TO FIND X

SOLVE BY METHOD OF INVERSION

$2x - y + z = 1$

$x + 2y + 3z = 8$

$3x + y - 4z = 1$

$$\begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 8 \\ 1 \end{pmatrix}$$

$AX = B$

$X = A^{-1}B$

$$A = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & -4 \end{pmatrix}$$

$|A| = 2(-8-3) + 1(-4-9) + 1(1-6)$

$= -22 - 13 - 5$

$= -40 \neq 0$, A^{-1} exists

COFACTORS

$A_{11} = (-)^{1+1}\begin{vmatrix} 2 & 3 \\ 1 & -4 \end{vmatrix} = +(-8-3) = -11$

$A_{12} = (-)^{1+2}\begin{vmatrix} 1 & 3 \\ 3 & -4 \end{vmatrix} = -(-4-9) = 13$

$A_{13} = (-)^{1+3}\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = +(1-6) = -5$

$A_{21} = (-)^{2+1}\begin{vmatrix} -1 & 1 \\ 1 & -4 \end{vmatrix} = -(4-1) = -3$

$A_{22} = (-)^{2+2}\begin{vmatrix} 2 & 1 \\ 3 & -4 \end{vmatrix} = +(-8-3) = -11$

$A_{23} = (-)^{2+3}\begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} = -(2+3) = -5$

$A_{31} = (-)^{3+1}\begin{vmatrix} -1 & 1 \\ 2 & 3 \end{vmatrix} = +(-3-2) = -5$

$A_{32} = (-)^{3+2}\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = -(6-1) = -5$

$A_{33} = (-)^{3+3}\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = +(4+1) = 5$

COFACTOR MATRIX

$$= \begin{pmatrix} -11 & 13 & -5 \\ -3 & -11 & -5 \\ -5 & -5 & 5 \end{pmatrix}$$

Adj A = transpose of cofactor matrix

$$= \begin{pmatrix} -11 & -3 & -5 \\ 13 & -11 & -5 \\ -5 & -5 & 5 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} . \text{adj } A = \frac{-1}{40}\begin{pmatrix} -11 & -3 & -5 \\ 13 & -11 & -5 \\ -5 & -5 & 5 \end{pmatrix}$$

$$= \frac{1}{40}\begin{pmatrix} 11 & 3 & 5 \\ -13 & 11 & 5 \\ 5 & 5 & -5 \end{pmatrix}$$

$X = A^{-1}.B$

$$= \frac{1}{40}\begin{pmatrix} 11 & 3 & 5 \\ -13 & 11 & 5 \\ 5 & 5 & -5 \end{pmatrix}\begin{pmatrix} 1 \\ 8 \\ 1 \end{pmatrix}$$

$$= \frac{1}{40}\begin{pmatrix} 11 + 24 + 5 \\ -13 + 88 + 5 \\ 5 + 40 - 5 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{40}\begin{pmatrix} 40 \\ 80 \\ 40 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

x = 1 , y = 2 , z = 1 . SS {1 , 2 , 1 }

SOLVE BY METHOD OF REDUCTION

$x + 2y + z = 8$

$2x + 3y - z = 11$

$3x - y - 2z = 5$

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 3 & -1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \\ 5 \end{bmatrix}$$

$R_2 - 2R_1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -3 \\ 3 & -1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \\ 5 \end{bmatrix}$$

$R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -3 \\ 0 & -7 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \\ -19 \end{bmatrix}$$

$R_2 \times (-1)$, $R_3 \times (-1)$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & 7 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \\ 19 \end{bmatrix}$$

$R_3 - 7R_2$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & -16 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \\ -16 \end{bmatrix}$$

$$\begin{bmatrix} x + 2y + z \\ y + 3z \\ -16z \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \\ -16 \end{bmatrix}$$

By equality of matrices

$-16z = -16$ $\quad\therefore z = 1$

$y + 3z = 5$

$y + 3 = 5$ $\quad\therefore y = 2$

$x + 2y + z = 8$

$x + 4 + 1 = 8$ $\quad\therefore x = 3$

SS: $\{3, 2, 1\}$

PAPER - I CHAPTER - 3

DIFFERENTIATION

DERIVATIVES OF STANDARD FUNCTIONS

$$\frac{d}{dx} K = 0$$

$$\frac{d}{dx} x = 1$$

$$\frac{d}{dx} x^n = nx^{n-1}$$

$$\frac{d}{dx} x^2 = 2x$$

$$\frac{d}{dx} x^3 = 3x^2$$

$$\frac{d}{dx} x^4 = 4x^3$$

$$\frac{d}{dx} 5x^3 - 4x^2 - 8x = 15x^2 - 8x - 8$$

$$\frac{d}{dx} 3x^4 - 7x^3 + 5x = 12x^3 - 21x^2 + 5$$

$$\frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx} \frac{1}{x} = \frac{-1}{x^2}$$

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} a^x = a^x.\log a$$

$$\frac{d}{dx} 3^x = 3^x.\log 3$$

$$\frac{d}{dx} 4^x = 4^x.\log 4$$

$$\frac{d}{dx} 5^x = 5^x.\log 5$$

$$\frac{d}{dx} \log x = \frac{1}{x}$$

Ex 3.1 – DERIVATIVES OF COMPOSITE FUNCTIONS

NOTE : if $y = f(u)$ where $u = g(x)$ then

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

01.

$y = (4x^3 + 3x^2 - 2x)^6$ $\quad$ u^6

$$\frac{dy}{dx} = 6(4x^3+3x^2-2x)^5 \frac{d}{dx}(4x^3+3x^2-2x)$$

$$= 6(4x^3+3x^2-2x)^5 .(12x^2 + 6x - 2)$$

02.

$y = \sqrt[5]{(3x^2 + 8x + 5)^4}$

$y = (3x^2 + 8x + 5)^{4/5}$ $\quad$ $u^{4/5}$

$$\frac{dy}{dx} = \frac{4}{5}(3x^2 + 8x + 5)^{4/5-1} \frac{d}{dx}(3x^2 + 8x + 5)$$

$$= \frac{4}{5}(3x^2 + 8x + 5)^{-1/5}(6x + 8)$$

03.

$y = \sqrt{2x^2 + 3x - 4}$ $\quad$ $\sqrt{u}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{2x^2 + 3x - 4}} \frac{d}{dx}(2x^2 + 3x - 4)$$

$$= \frac{4x + 3}{2\sqrt{2x^2 + 3x - 4}}$$

04.

$y = e^{5x^2-2x+4}$ $\quad$ e^u

$$\frac{dy}{dx} = e^{5x^2-2x+4} \frac{d}{dx} 5x^2-2x+4$$

$$= (10x-2).e^{5x^2-2x+4}$$

05.

$y = 5^{x + \log x}$ $\quad$ 5^u

$$\frac{dy}{dx} = 5^{x+\log x} \log 5 \frac{d}{dx} x+\log x$$

$$= 5^{x+\log x} \log 5 . \left(1+ \frac{1}{x}\right)$$

06.

$y = \log (2x^2+3x-4)$ $\quad$ $\log u$

$$\frac{dy}{dx} = \frac{1}{2x^2+3x-4} \frac{d}{dx}(2x^2+3x-4)$$

$$= \frac{4x + 3}{2x^2+3x-4}$$

Ex 3.2 – DERIVATIVES OF INVERSE FUNCTIONS

NOTE 1

$y = f(x)$

then

$\frac{dy}{dx}$ is called as 'RATE OF CHANGE OF Y' OR 'MARGINAL Y'

NOTE 2

$y = f(x)$

but the question asked is

FIND 'RATE OF CHANGE OF X' OR 'MARGINAL X'

i.e dx/dy

then student is suppose to first find dy/dx since he will be comfortable doing that as $y = f(x)$.

Having done that

$$\frac{dx}{dy} = \frac{1}{dy/dx}$$

NOTE 3 : PRODUCT RULE $y = u.v$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

NOTE 4 : QUOTIENT RULE $y = \frac{u}{v}$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Find the rate of change of demand (x) of a commodity with respect to its price (y)

01. if $y = 12 + 10x + 25x^2$

$y = 12 + 10x + 25x^2$

$$\frac{dy}{dx} = 10 + 50x$$

Hence rate of change of demand wrt price

$$= \frac{dx}{dy} = \frac{1}{{}^{dy}/_{dx}} = \frac{1}{10+50x}$$

(By Derivative of the Inverse function)

02. if $y = 25x + \log(1+x^2)$

$y = 25x + \log(1+x^2)$

$$\frac{dy}{dx} = 25 + \frac{1}{1+x^2}\frac{d}{dx}1+x^2 = 25 + \frac{2x}{1+x^2}$$

$$= \frac{25 + 25x^2 + 2x}{1 + x^2}$$

Hence rate of change of demand wrt price

$$= \frac{dx}{dy} = \frac{1}{{}^{dy}/_{dx}} = \frac{1 + x^2}{25x^2 + 2x + 25}$$

(By Derivative of the Inverse function)

Find the MARGINAL DEMAND where x is the demand & y is the price

NOTE :

Marginal demand means marginal x and hence student is asked to find dx/dy

01. if $y = \frac{5x + 7}{2x - 13}$

$$\frac{dy}{dx} = \frac{(2x-13)\,{}^{d}/_{dx}(5x+7) - (5x+7)\,{}^{d}/_{dx}(2x-13)}{(2x-13)^2}$$

$$= \frac{(2x-13).5 - (5x+7).2}{(2x-13)^2}$$

$$= \frac{10x - 65 - 10x - 14}{(2x-13)^2} = \frac{-79}{(2x-13)^2}$$

Hence marginal demand i.e. rate of change of demand wrt price

$$= \frac{dx}{dy} = \frac{1}{{}^{dy}/_{dx}} = -\frac{(2x-13)^2}{79}$$

(By Derivative of the Inverse function)

Ex 3.3 – LOGARITHMIC DIFFERENTIATION

WHEN TO GO FOR LOGARITHMIC DIFFERENTIATION

CASE 1

$$y = \frac{f_1(x)^{n_1} . f_2(x)^{n_2}}{g_1(x)^{n_3} . g_2(x)^{n_4}}$$

Taking log on both sides

$$\log y = n_1.\log f_1(x) + n_2 \log f_2(x) - n_3.\log g_1(x) - n_4.\log g_2(x)$$

Differentiating wrt x

$$\frac{1}{y}\frac{dy}{dx} = n_1 \frac{1}{f_1(x)} \frac{d}{dx} f_1(x) + n_2 \frac{1}{f_2(x)} \frac{d}{dx} f_2(x) - n_3 \frac{1}{g_1(x)} \frac{d}{dx} g_1(x) - n_4 \frac{1}{g_2(x)} \frac{d}{dx} g_2(x)$$

$$\frac{dy}{dx} = y\left(n_1 \frac{f_1'(x)}{f_1(x)} + n_2 \frac{f_2'(x)}{f_2(x)} - n_3 \frac{g_1'(x)}{g_1(x)} - n_4 \frac{g_2'(x)}{g_2(x)}\right)$$

01. $y = \sqrt{\dfrac{(3x-4)^3}{(x+1)^4 . (x+2)}}$

Taking log on both sides

$$\text{Log } y = \frac{1}{2}\left(3 \log (3x-4) - 4 \log(x+1) - \log(x+2)\right)$$

Differentiating wrt x

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left(3 \frac{1}{3x-4} \frac{d}{dx}(3x-4) - 4 \frac{1}{x+1} \frac{d}{dx}(x+1) - \frac{1}{x+2} \frac{d}{dx}(x+2)\right)$$

$$\frac{dy}{dx} = \frac{y}{2}\left(3 \frac{1}{3x-4} 3 - 4 \frac{1}{x+1} - \frac{1}{x+2}\right)$$

$$\frac{dy}{dx} = \frac{y}{2}\left(\frac{9}{3x-4} - \frac{4}{x+1} - \frac{1}{x+2}\right)$$

$$\frac{dy}{dx} = \frac{1}{2}\sqrt{\frac{(3x-4)^3}{(x+1)^4.(x+2)}}\left(\frac{9}{3x-4} - \frac{4}{x+1} - \frac{1}{x+2}\right)$$

CASE 2

Consider $y = f(x)^{g(x)}$

Taking log on both sides

$\log y = g(x) \cdot \log f(x)$

Differentiating both sides wrt x

$$\frac{1}{y}\frac{dy}{dx} = g(x)\frac{d}{dx}\log f(x) + \log f(x)\frac{d}{dx}g(x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{g(x)}{f(x)} \cdot f'(x) + \log f(x) \cdot g'(x)$$

$$\frac{dy}{dx} = y\left(\frac{g(x)}{f(x)} \cdot f'(x) + \log f(x)\, g'(x)\right)$$

$$\frac{dy}{dx} = f(x)^{g(x)}\left(\frac{g(x)}{f(x)} \cdot f'(x) + \log f(x).g'(x)\right)$$

01.

$y = (2x + 5)^x$

taking log on both sides

$\log y = x.\log(2x+5)$

Differentiating wrt x

$$\frac{1}{y}\frac{dy}{dx} = x.\frac{d}{dx}\log(2x+5) + \log(2x+5)\frac{d}{dx}x$$

$$\frac{dy}{dx} = y\left[x\,\frac{1}{2x+5}\,\frac{d}{dx}(2x+5) + \log(2x+5)\right]$$

$$\frac{dy}{dx} = (2x+5)^x\left[\frac{2x}{2x+5} + \log(2x+5)\right]$$

02.

$y = (2x^2+1)^{3x+4}$

$\log y = (3x+4).\log(2x^2+1)$

Differentiating wrt x

$$\frac{1}{y}\frac{dy}{dx} = (3x+4)\frac{d}{dx}\log(2x^2+1) + \log(2x^2+1)\frac{d(3x+4)}{dx}$$

$$\frac{dy}{dx} = y\left[(3x+4)\frac{1}{2x^2+1}\frac{d}{dx}(2x^2+1) + \log(2x^2+1) \cdot 3\right]$$

$$\frac{dy}{dx} = (2x^2+1)^{3x+4}\left[\frac{4x.(3x+4) + 3.\log(2x^2+1)}{2x^2+1}\right]$$

WORD OF CAUTION

log (ab) = log a + log b

log (a+b) ≠ log a + log b

In sums where

$y = u + v$,

u ,v are of the form $f(x)^{g(x)}$,

students do tend to take log on both sides

THEY DO THIS

log y = log (u + v)

bcoz u and v are of the form $f(x)^{g(x)}$

next step they move log inside

log y = log u + log v

MAJBOORI MEIN BHI YEH NAHI KAREGE

WHAT ARE WE SUPPOSE TO DO

$y = u + v$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

STEP 1 – $u = f(x)^{g(x)}$

Use logarithmic differentiation to get to du/dx

STEP 2 – $v = f(x)^{g(x)}$

Use logarithmic differentiation to get to dv/dx

STEP 3 – $y = u + v$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$y = x^x + (7x - 1)^x$

$y = u + v$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$u = x^x$

Taking log on both sides

Log u = x.log x

$$\frac{1}{u}\frac{du}{dx} = x\frac{d}{dx}\log x + \log x\frac{d}{dx}x$$

$$\frac{du}{dx} = u\left[x\frac{1}{x} + \log x\right]$$

$$\frac{du}{dx} = x^x.(1 + \log x)$$

$v = (7x - 1)^x$

Taking log on both sides

Log v = x.log (7x – 1)

$$\frac{1}{v}\frac{dv}{dx} = x\frac{d}{dx}\log(7x-1) + \log(7x-1)\frac{d}{dx}x$$

$$\frac{dv}{dx} = u\left[x\frac{1}{7x-1}\frac{d}{dx}(7x-1) + \log(7x-1)\right]$$

$$\frac{dv}{dx} = (7x-1)^x\left[\frac{7x}{7x-1} + \log(7x-1)\right]$$

HENCE

$$\frac{dy}{dx} = x^x.(1 + \log x) + (7x-1)^x\left[\frac{7x}{7x-1} + \log(7x-1)\right]$$

02.

$y = 10^{x^x} + 10^{x^{10}} + 10^{10^x}$

$y = u + v + w$

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx}$

$u = 10^{x^x}$

Taking log on both sides

$\text{Log } u = x^x . \log 10$

$\frac{1}{u}\frac{du}{dx} = \log 10 \frac{d}{dx} x^x$ …… (1)

let $z = x^x$

taking log

$\log z = x.\log x$

Differetiating wrt x

$\frac{1}{z}\frac{dz}{dx} = x. \frac{d}{dx}\log x + \log x \frac{d}{dx} x$

$\frac{dz}{dx} = z \left(x. \frac{1}{x} + \log x \right)$

$\frac{dz}{dx} = x^x (1 + \log x)$

BACK IN (1)

$\frac{du}{dx} = u.\log 10. x^x(1+\log x)$

$= 10^{x^x}. \log 10 . x^x(1 + \log x)$

$v = 10^{x^{10}}$

Taking log on both sides

$\text{Log } v = x^{10} . \log 10$

$\frac{1}{v}\frac{dv}{dx} = \log 10 \frac{d}{dx} x^{10}$

$\frac{dv}{dx} = v.\log 10 . 10x^9$

$\frac{dv}{dx} = 10^{x^{10}}. \log 10 . 10x^9$

$w = 10^{10^x}$

Taking log on both sides

$\text{Log } w = 10^x . \log 10$

$\frac{1}{w}\frac{dw}{dx} = \log 10 \frac{d}{dx} 10^x$

$\frac{dw}{dx} = w.\log 10 . 10^x. \log 10$

$\frac{dw}{dx} = 10^{x^{10}}. 10^x (\log 10)^2$

FINALLY

$\frac{dy}{dx} = 10^{x^x}. \log 10 . x^x(1 + \log x)$

$+ 10^{x^{10}}. \log 10 . 10x^9$

$+ 10^{x^{10}}. 10^x (\log 10)^2$

Ex 3.4 – DERIVATIVES OF IMPLICIT FUNCTIONS

STUDENTS ARE SUPPOSE TO CHECK THIS BOX BEFORE GOING INTO THE SUMS

$$\frac{d}{dx} y^2 = 2y \frac{dy}{dx}$$

$$\frac{d}{dx} y^3 = 3y^2 \frac{dy}{dx}$$

$$\frac{d}{dx} \sqrt{y} = \frac{1}{2\sqrt{y}} \frac{dy}{dx}$$

$$\frac{d}{dx} \log y = \frac{1}{y} \frac{dy}{dx}$$

$$\frac{d}{dx} e^y = e^y \frac{dy}{dx}$$

BY CHAIN RULE

STEP 1 – Start differentiating wrt x

STEP 2 – Collect dy/dx on one side and other terms on other side

STEP 3 – Get to dy/dx

01. $ax^2 + 2hxy + by^2 = 0$

Differentiating wrt x ,

$$a2x + 2h\left(x\frac{dy}{dx} + y.1\right) + b\,2y\frac{dy}{dx} = 0$$

$$ax + h\left(x\frac{dy}{dx} + y\right) + by\frac{dy}{dx} = 0$$

$$ax + hx\frac{dy}{dx} + hy + by\frac{dy}{dx} = 0$$

$$(hx + by)\frac{dy}{dx} = -(ax + hy)$$

$$\frac{dy}{dx} = -\frac{ax + hy}{hx + by}$$

02. $x^3y^3 = x^2 - y^2$

$$x^3.\frac{d}{dx}y^3 + y^3\frac{d}{dx}x^3 = 2x - 2y\frac{dy}{dx}$$

$$x^3.3y^2\frac{dy}{dx} + y^3.3x^2 = 2x - 2y\frac{dy}{dx}$$

$$3x^3.y^2\frac{dy}{dx} + 3x^2y^3 = 2x - 2y\frac{dy}{dx}$$

$$(3x^3.y^2 + 2y)\frac{dy}{dx} = 2x - 3x^2y^3$$

$$\frac{dy}{dx} = \frac{2x - 3x^2y^3}{3x^3.y^2 + 2y}$$

03. $x^5y^7 = (x + y)^{12}$ Show that $\frac{dy}{dx} = \frac{y}{x}$

$$x^5.y^7 = (x + y)^{12}$$

Taking log on both sides

$$5.\log x + 7.\log y = 12.\log(x + y)$$

$$5\frac{1}{x} + 7\frac{1}{y}\frac{dy}{dx} = 12\frac{1}{x+y}\frac{d}{dx}(x + y)$$

$$\frac{5}{x} + \frac{7}{y}\frac{dy}{dx} = \frac{12}{x+y}\left(1 + \frac{dy}{dx}\right)$$

$$\frac{5}{x} + \frac{7}{y}\frac{dy}{dx} = \frac{12}{x+y} + \frac{12}{x+y}\frac{dy}{dx}$$

$$\left(\frac{7}{y} - \frac{12}{x+y}\right)\frac{dy}{dx} = \frac{12}{x+y} - \frac{5}{x}$$

$$\frac{7x + 7y - 12y}{y(x + y)}\frac{dy}{dx} = \frac{12x - 5x - 5y}{x(x + y)}$$

$$\frac{7x - 5y}{y(x + y)}\frac{dy}{dx} = \frac{7x - 5y}{x(x + y)}$$

$$\frac{dy}{dx} = \frac{y}{x}$$ ……… PROVED

NOTE : In the coming sums , in show that , you will find that dy/dx ka answers are completely in terms of x i.e they are explicit . Such answers are possible only when functions are explicit . Hence student first needs to make the IMPLICIT FUCTIONS EXPLICIT and then start differentiating

01. $x^y = e^{x-y}$ Prove : $\dfrac{dy}{dx} = \dfrac{\log x}{(1+\log x)^2}$

SINCE $^{dy}/_{dx}$ IS EXPLICITLY IN TERMS OF X , CONVERT THE GIVEN IMPLICIT FUNCTION INTO EXPLICIT FUNCTION y = f(x) AND ONLY THEN PROCEED TO DIFFERENTIATE

$x^y = e^{x-y}$

Taking log on both sides

$y \cdot \log x = (x - y) \cdot \log e$

$y \cdot \log x = x - y$

$y \cdot \log x + y = x$

$y(\log x + 1) = x$

$y = \dfrac{x}{1 + \log x}$

Differentiating wrt x ,

$$\frac{dy}{dx} = \frac{(1 + \log x) \cdot \frac{d}{dx}x - x \cdot \frac{d}{dx}(1 + \log x)}{(1 + \log x)^2}$$

$$\frac{dy}{dx} = \frac{(1 + \log x) \cdot 1 - \frac{x \cdot 1}{x}}{(1 + \log x)^2}$$

$$\frac{dy}{dx} = \frac{1 + \log x - 1}{(1 + \log x)^2}$$

$$\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$$

Ex 3.5 – DERIVATIVES OF PARAMETRIC FUNCTIONS

$x = f(t)$, $y = f(t)$

x and y are called PARAMETRIC functions where 't' is called as the parameter

HOW TO FIND $^{dy}/_{dx}$

STEP 1 $x = f(t)$
Differentiate wrt 't' ,
get $^{dx}/_{dt}$

STEP 2 $y = f(t)$
Differentiate wrt 't' ,
get $^{dy}/_{dt}$

STEP 3 $^{dy}/_{dx} = \dfrac{^{dy}/_{dt}}{^{dx}/_{dt}}$

01. $x = 5t^2$; $y = 10t$

✓ $x = 5t^2$

Diff. wrt 't'

$$\frac{dx}{dt} = 5.2t = 10t$$

✓ $y = 10t$

Diff. wrt 't'

$$\frac{dy}{dt} = 10.1 = 10$$

Now

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{10}{10t} = \frac{1}{t}$$

02. $x = e^{3t}$; $y = e^{4t + 5}$

✓ $y = e^{4t + 5}$

Diff wrt t

$$\frac{du}{dt} = e^{4t + 5} \frac{d}{dt}(4t + 5)$$

$$= 4.e^{4t + 5}$$

✓ $x = e^{3t}$

Diff wrt t

$$\frac{dx}{dt} = e^{3t} \frac{d}{dt}(3t) = 3e^{3t}$$

Now

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{4.e^{4t + 5}}{3.e^{3t}}$$

$$= \frac{4.e^{4t + 5 - 3t}}{3} = \frac{4.e^{t + 5}}{3}$$

03. Differentiate $\log t$ with respect to $\log(1+t^2)$

> NOTE : lets consider
> $y = \log t$ and $x = \log(1+t^2)$
> Now when we read the question , it reads
> Differentiate y wrt x i.e. dy/dx
> LETS PROCEED

✓ $y = \log t$, Differentiating wrt 't'

$$\frac{dy}{dt} = \frac{1}{t}$$

✓ $x = \log(1+t^2)$, Differentiating wrt 't'

$$\frac{dx}{dt} = \frac{1}{1+t^2}\frac{d}{dt}(1+t^2) = \frac{2t}{1+t^2}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{1}{t}}{\frac{2t}{(1+t^2)}} = \frac{1+t^2}{2t^2}$$

04.

$x = \sqrt{1+u^2}$, $y = \log(1+u^2)$

✓ $x = \sqrt{1+u^2}$

Differentiate wrt u

$$\frac{dx}{du} = \frac{1}{2\sqrt{1+u^2}}\frac{d}{du}1+u^2$$

$$= \frac{2u}{2\sqrt{1+u^2}} = \frac{u}{\sqrt{1+u^2}}$$

✓ $y = \log(1+u^2)$

Differentiate wrt u

$$\frac{dy}{du} = \frac{1}{1+u^2}\frac{d}{du}(1+u^2) = \frac{2u}{1+u^2}$$

Now

$$\frac{dy}{dx} = \frac{\frac{dy}{du}}{\frac{dx}{du}} = \frac{\frac{2u}{1+u^2}}{\frac{u}{\sqrt{1+u^2}}} = \frac{2}{\sqrt{1+u^2}}$$

05. $x = \frac{4t}{1+t^2}$; $y = 3\,\frac{1-t^2}{1+t^2}$

Show that $\frac{dy}{dx} = -\frac{9x}{4y}$

✓ $x = \frac{4t}{1+t^2}$ ……….. (1)

$$\frac{dx}{dt} = \frac{(1+t^2)\frac{d}{dt}4t - 4t\frac{d}{dt}(1+t^2)}{(1+t^2)^2}$$

$$= \frac{(1+t^2)\,4 - 4t.2t}{(1+t^2)^2}$$

$$= \frac{4+4t^2-8t^2}{(1+t^2)^2}$$

$$= \frac{4-4t^2}{(1+t^2)^2} = \frac{4(1-t^2)}{(1+t^2)^2}$$

✓ $y = 3\,\frac{1-t^2}{1+t^2}$ …………. (2)

$$\frac{dy}{dt} = 3\,\frac{(1+t^2)\frac{d}{dt}(1-t^2) - (1-t^2)\frac{d}{dt}(1+t^2)}{(1+t^2)^2}$$

$$= 3\,\frac{(1+t^2)(-2t) - (1-t^2).2t}{(1+t^2)^2}$$

$$= 3\,\frac{-2t(-1-t^2-1+t^2)}{(1+t^2)^2}$$

$$= \frac{-12t}{(1+t^2)^2}$$

✓ $$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{-12t}{(1+t^2)^2}}{\frac{4(1-t^2)}{(1+t^2)^2}} = \frac{-\frac{3t}{1+t^2}}{\frac{1-t^2}{1+t^2}}$$

$$= -\frac{3x/4}{y/3} = -\frac{9x}{4y}$$ ……. PROVED

FROM (1) & (2)

Ex 3.6 – SECOND ORDER DERIVATIVES

01. $y = x^6$

Differentiating wrt x

$$\frac{dy}{dx} = 6x^5$$

Differentiating once again wrt x

$$\frac{d^2y}{dx^2} = 30x^4$$

02.

$y = \log x$

Differentiating wrt x

$$\frac{dy}{dx} = \frac{1}{x}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{x^2}$$

03.

$y = e^{\log x}$

> NOTE : IN LOGARITHMS
>
> $$a^{\log_a x} = x$$
>
> WE CAN APPLY THIS IN THE ABOVE SUM

$y = e^{\log_e x} = x$

$y = x$

> RECALL
>
> IN CALCULUS LOGs ARE TO THE BASE e

Differentiating wrt x

$$\frac{dy}{dx} = 1$$

Differentiating once again wrt x

$$\frac{d^2y}{dx^2} = 0$$

03. $ax^2 + 2hxy + by^2 = 0$

Differentiating wrt x ,

$$a2x + 2h\left[x\frac{dy}{dx} + y\,.1\right] + b\,2y\frac{dy}{dx} = 0$$

Dividing through out by 2

$$ax + h\left[x\frac{dy}{dx} + y\right] + by\frac{dy}{dx} = 0$$

$$ax + hx\frac{dy}{dx} + hy + by\frac{dy}{dx} = 0$$

$$(hx + by)\frac{dy}{dx} = -(ax + hy)$$

$$\frac{dy}{dx} = -\frac{ax + hy}{hx + by} \quad .....(1)$$

$$ax^2 + 2hxy + by^2 = 0$$

$$ax^2 + hxy + hxy + by^2 = 0$$

$$x(ax + hy) + y(hx + by) = 0$$

$$x(ax + hy) = -y(hx + by)$$

$$\frac{ax + hy}{hx + by} = -\frac{y}{x}$$

subs in (1)

$$\frac{dy}{dx} = \frac{y}{x} \quad ....(2)$$

Differentiating once again wrt x

$$\frac{d^2y}{dx^2} = \frac{x\frac{dy}{dx} - y}{x^2}$$

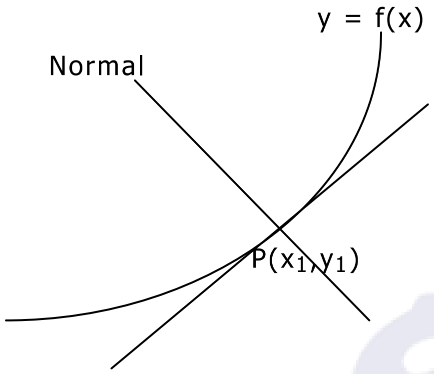
$$= \frac{x\frac{y}{x} - y}{x^2} \quad \text{from (2)}$$

$$= \frac{y - y}{x^2} = 0 \quad ...... \text{ PROVED}$$

PAPER - I
CHAPTER - 4
APPLICATION OF DERIVATIVES

APP. OF DERIVATIVES - 1

TANGENT & NORMAL



✓ Slope of tangent

$$m = \left.\frac{dy}{dx}\right|_{P(x_1,y_1)}$$

✓ Slope of normal $= {}^{-1}/_{m}$

($\perp$ lines , $m_1.m_2 = -1$)

✓ Having found the slopes , form equations of tangent and normal at $P(x_1,y_1)$using

$y - y_1 = m(x - x_1)$

Q1.

Find equation of tangent and normal to the curve $y = x^2 + 4x + 1$ at $P(-1,-2)$

$y = x^2 + 4x + 1$

$\frac{dy}{dx} = 2x + 4$

Slope of tangent

$= \left.\frac{dy}{dx}\right|_{P(-1,-2)}$

$= 2(-) + 4$

$= 2$

Slope of normal $= {}^{-1}/_{2}$ ($\perp$ lines)

Equation of tangent

$m = 2$, $P(-1,-2)$

$y - y_1 = m(x - x_1)$

$y + 2 = 2(x + 1)$

$y + 2 = 2x + 2$

$2x - y = 0$

Equation of normal

$m = {}^{-1}/_{2}$, $P(-1,-2)$

$y - y_1 = m(x - x_1)$

$y + 2 = {}^{-1}/_{2} (x + 1)$

$2y + 4 = -x - 1$

$x + 2y + 5 = 0$

Q2.

Find equation of tangent and normal to the curve $y = x^3 - x^2 - 1$ at point whose abscissa is -2

P(−2,y) lies on the curve $y = x^3 - x^2 - 1$ and hence must satisfy the equation

$\therefore y = (-2)^3 - (-2)^2 - 1$

$y = -8 - 4 - 1$

$y = -13$ Hence P(−2,−13)

$y = x^3 - x^2 - 1$

$\frac{dy}{dx} = 3x^2 - 2x$

Slope of tangent

$= \left.\frac{dy}{dx}\right|_{P(-2,-13)}$

$= 3(4) + 4$

$= 16$

Slope of normal $= {}^{-1}/_{16}$ (⊥ lines)

Equation of tangent

$m = 16$, P(−2,−13)

$y - y_1 = m(x - x_1)$

$y + 13 = 16(x + 2)$

$y + 13 = 16x + 32$

$16x - y + 19 = 0$

Equation of normal

$m = {}^{-1}/_{16}$, P(−2,−13)

$y - y_1 = m(x - x_1)$

$y + 13 = {}^{-1}/_{16}(x + 2)$

$16y + 208 = -x - 2$

$x + 16y + 210 = 0$

Q3.

Find equation of tangent and normal to the curve $y = x^2 + 5$ at point where tangent is parallel to $4x - y + 7 = 0$

$y = x^2 + 5$ $\frac{dy}{dx} = 2x$

Slope of tangent at $P(x_1,y_1)$

$= \left.\frac{dy}{dx}\right|_{P(x_1,y_1)} = 2x_1$ ……… (1)

Slope of line $4x - y + 7 = 0$

$m = -\frac{a}{b} = -\frac{4}{-1} = 4$

Slope of tangent = 4 (// lines) ……… (2)

$2x_1 = 4$ …… From (1) & (2)

$x_1 = 2$

$P(2,y_1)$ lies on the curve $y = x^2 + 5$

$\therefore y = 2^2 + 5 = 9$

P(2,9)

Equation of tangent

$m = 4$, P(2,9)

$y - y_1 = m(x - x_1)$

$y - 9 = 4(x - 2)$

$y - 9 = 4x - 8$

$4x - y + 1 = 0$

Equation of normal

$m = {}^{-1}/_4$, P(2,9)

$y - y_1 = m(x - x_1)$

$y - 9 = {}^{-1}/_4(x - 2)$

$4y - 36 = -x + 2$

$x + 4y - 38 = 0$

APP. OF DERIVATIVES - 2

INCREASING & DECREASING FN'S

NOTE :

- ✓ If the f(x) is increasing at x = c ; $f'(c) > 0$
- ✓ If the f(x) is decreasing at x = c ; $f'(c) < 0$
- ✓ to find for what values of x , the f(x) is increasing : Solve $f'(x) > 0$
- ✓ to find for what values of x , the f(x) is decreasing : Solve $f'(x) < 0$

STUDENTS ARE REQUESTED TO MAKE A NOTE OF THE FOLLOWING TWO EXAMPLES OF ALGEBRA INVOLVED IN INEQUATIONS BEFORE CHECKING INTO TO ANY OF THE FORTHCOMING SOLUTIONS

EXAMPLE – 1

$(x-2)(x-3) > 0$

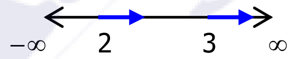
CASE 1

$x - 2 > 0$ & $x - 3 > 0$

$x > 2$ & $x > 3$

$x > 3$

$x \in (3, \infty)$



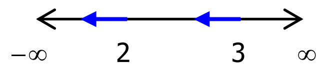
CASE 2

$x - 3 < 0$ & $x - 2 < 0$

$x < 3$ & $x < 2$

$x < 2$

$x \in (-\infty, 2)$



$x \in (-\infty, 2) \cup (3, \infty)$

EXAMPLE – 2

$(x-1)(x-2) < 0$

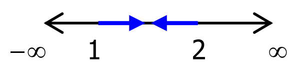
CASE 1

$x - 1 > 0$ & $x - 2 < 0$

$x > 1$ & $x < 2$

$1 < x < 2$

$x \in (1, 2)$



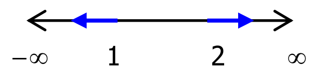
CASE 2

$x - 1 < 0$ & $x - 2 > 0$

$x < 1$ & $x > 2$

NOT POSSIBLE

DISCARD



FINALLY $x \in (1, 2)$

Q1

$f(x) = x^3 + 12x^2 + 36x + 6$

Find the values of x for which the function is increasing

for f(x) increasing

$f'(x) > 0$

$3x^2 + 24x + 36 > 0$

$x^2 + 8x + 12 > 0$

$(x + 6)(x + 2) > 0$

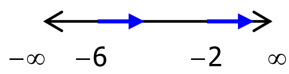
CASE 1

$x + 6 > 0$ & $x + 2 > 0$

$x > -6$ & $x > -2$

$x > -2$

$x \in (-2, \infty)$



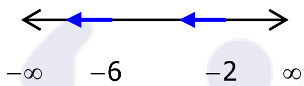
CASE 2

$x + 6 < 0$ & $x + 2 < 0$

$x < -6$ & $x < -2$

$x < -6$

$x \in (-\infty, -6)$



f(x) is increasing in $(-\infty, -6) \cup (-2, \infty)$

Q2

$f(x) = 2x^3 - 15x^2 - 144x - 7$

Find the values of x for which the function is decreasing

for f(x) decreasing

$f'(x) < 0$

$6x^2 - 30x - 144 < 0$

$x^2 - 5x - 24 < 0$

$(x - 8)(x + 3) < 0$

CASE 1

$x - 8 > 0$ & $x + 3 < 0$

$x > 8$ & $x < -3$

NOT POSSIBLE

DISCARD

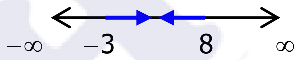
CASE 2

$x - 8 < 0$ & $x + 3 > 0$

$x < 8$ & $x > -3$

$-3 < x < 8$

$x \in (-3, 8)$



f(x) is decreasing in $x \in (-3, 8)$

BEFORE GOING AHEAD ,LETS REHEARSE

✓ FOR f(x) decreasing $f'(x) < 0$

✓ FOR f(x) increasing $f'(x) > 0$

✓ FOR cost C decreasing $\frac{dC}{dx} < 0$

✓ FOR average cost C_A decreasing

$\frac{dC_A}{dx} < 0$

where $C_A = {}^{C}/_{x}$

✓ FOR Revenue increasing $\frac{dR}{dx} > 0$

where R = px ,

p = price per item

x = demand , number of items that can be sold at that price

✓ FOR Profit increasing $\frac{d\pi}{dx} > 0$

where $\pi = R - C$

Q3

the total cost of manufacturing x articles is $c = 47x + 300x^2 - x^4$

Find x for which average cost is decreasing

SOLUTION

$C = 47x + 300x^2 - x^4$

AVERAGE COST

$C_A = \dfrac{C}{x}$

$= \dfrac{47x + 300x^2 - x^4}{x}$

$= 47 + 300x - x^3$

For average cost decreasing ,

$\dfrac{dC_A}{dx} < 0$

$300 - 3x^2 < 0$

$300 < 3x^2$

$100 < x^2$

$x^2 > 100$

$x > 10$

average cost is decreasing for **x > 10**

Q4.

The manufacturing company produces x items at the total cost of Rs (180 + 4x) . The demand function is p = (240 – x).

Find x for which i) Revenue is increasing

ii) Profit is increasing

SOLUTION

a) REVENUE

$R = p.x$

$= (240 - x).x$

$= 240x - x^2$

For Revenue increasing ,

$\dfrac{dR}{dx} > 0$

$240 - 2x > 0$

$240 > 2x$

$120 > x$

$x < 120$

ans : revenue is increasing when $x < 120$

b) PROFIT

$\pi = R - C$

$= 240x - x^2 - (180 + 4x)$

$= 240x - x^2 - 180 - 4x$

$= 236x - x^2 - 180$

For Profit increasing ,

$\dfrac{d\pi}{dx} > 0$

$236 - 2x > 0$

$236 > 2x$

$118 > x$

$x < 118$

ans : Profit is increasing when $x < 118$

Q5.

$f(x) = x^3 - 3x^2 + 3x - 100$

Test whether function is increasing or decreasing

$f'(x) = 3x^2 - 6x + 3$

$= 3(x^2 - 2x + 1)$

$= 3(x - 1)^2$

NOW ,

$(x - 1)^2 > 0 \quad \forall\ x \in R - \{1\}$

$3(x - 1)^2 > 0 \quad \forall\ x \in R - \{1\}$

$f'(x) > 0 \quad \forall\ x \in R - \{1\}$

Hence f(x) is increasing $\forall\ x \in R - \{1\}$

Q6.

$f(x) = 2 - 3x + 3x^2 - x^3$

Test whether function is increasing or decreasing

$f'(x) = -3 + 6x - 3x^2$

$= -3(x^2 - 2x + 1)$

$= -3(x - 1)^2$

NOW ,

$(x - 1)^2 > 0 \quad \forall\ x \in R - \{1\}$

$-3(x - 1)^2 < 0 \quad \forall\ x \in R - \{1\}$

$f'(x) < 0 \quad \forall\ x \in R - \{1\}$

Hence f(x) is decreasing $\forall\ x \in R - \{1\}$

APP. OF DERIVATIVES - 3

MAXIMA & MINIMA

NOTE :

STEPS TO SOLVE A SUM

STEP 1 : form f(x)

STEP 2 : find f′(x) & f″(x)

STEP 3 : Solve f′(x) = 0 , get x = a and x = b

STEP 4 : If $f''(x)\big|_{x = a} < 0$

f(x) is maximum at x = a

If $f''(x)\big|_{x = b} > 0$

f(x) is minimum at x = b

STEP 5 : maximum value of the f(x) , find $f(x)\big|_{x = a}$

maximum value of the f(x) , find $f(x)\big|_{x = b}$

Q1.

$f(x) = x^3 - 9x^2 + 24x$

Examine the f(x) for maxima & minima

SOLUTION

STEP 1 :

$f(x) = x^3 - 9x^2 + 24x$

STEP 2 :

$f'(x) = 3x^2 - 18x + 24$

$f''(x) = 6x - 18$

STEP 3 :

$f'(x) = 0$

$3x^2 - 18x + 24 = 0$

$x^2 - 6x + 8 = 0$

$(x - 2)(x - 4) = 0$

$x = 2 \quad , \quad x = 4$

STEP 4 :

$f''(x)\Big|_{x = 2} = 6(2) - 18$

$= 12 - 18 = -6 < 0$

f is maximum at $x = 2$

$f''(x)\Big|_{x = 4} = 6(4) - 18$

$= 24 - 18 = 6 > 0$

f is minimum at $x = 4$

STEP 5 :

Since f is maximum at $x = 2$

Maximum value of f

$= f(x) \quad x = 2$

$= 2^3 - 9.2^2 + 24.2$

$= 8 - 36 + 48$

$= 20$

Since f is minimum at $x = 4$

minimum value of f

$= f(x)\Big| x = 4$

$= 4^3 - 9.4^2 + 24.4$

$= 64 - 9.16 + 96$

$= 64 - 144 + 96$

$= 16$

Q2

Divide 20 into two parts such that their product is maximum

let 20 be divided into x & $20 - x$

STEP 1 :

$f(x) = \text{product}$

$= x(20 - x)$

$= 20x - x^2$

STEP 2 :

$f'(x) = 20 - 2x$

$f''(x) = -2$

STEP 3 :

$f'(x) = 0$

$20 - 2x = 0$

$x = 10$

STEP 4 :

$f''(x)\Big|_{x = 10} = -2 < 0$

f is maximum at $x = 10$

Hence divide 20 into 10,10

Q3.

a metal wire of 36 cm is bent to form a rectangle . Find its distance if area is maximum

let length = x , breadth = y

$2x + 2y = 36$

$x + y = 18$

STEP 1 :

$f(x) = \text{area}$

$= xy$

$= x(18 - x)$

$= 18x - x^2$

STEP 2 :

$f'(x) = 18 - 2x$

$f''(x) = -2$

STEP 3 :

$f'(x) = 0$

$18 - 2x = 0$

$x = 9$

STEP 4 :

$f''(x)\Big|_{x = 9} = -2 < 0$

f is maximum at $x = 9$

for area to be maximum

dimensions need to be 9cm x 9cm

Q4.

The total cost of producing x units is Rs $(x^2 + 60x + 50)$ and the price per unit is Rs $(180 - x)$. For what units the profit is maximum

SOLUTION

STEP 1 :

$R = p.x$

$= (180 - x).x$

$= 180x - x^2$

PROFIT

$\pi = R - C$

$\pi = 180x - x^2 - (x^2 + 60x + 50)$

$\pi = 180x - x^2 - x^2 - 60x - 50$

$\pi = 120x - 2x^2 - 50$ ...... say f(x)

STEP 2 :

$f'(x) = 120 - 4x$

$f''(x) = -4$

STEP 3 :

$f'(x) = 0$

$120 - 4x = 0$

$4x = 120$

$x = 30$

STEP 4 :

$f''(x)\Big|_{x = 30} = -4 < 0$

Profit is maximum at x = 30

ELASTICITY OF DEMAND

$$\eta = -\frac{P}{D}\frac{dD}{dP}$$

$-\frac{dD}{dp} < \frac{D}{P} \quad \therefore -\frac{P}{D}\frac{dD}{dp} < 1 \quad \therefore \eta < 1$

Demand is Relatively Inelastic

Demand is less sensitive to price

$-\frac{dD}{dp} = \frac{D}{P} \quad \therefore -\frac{P}{D}\frac{dD}{dp} = 1 \quad \therefore \eta = 1$

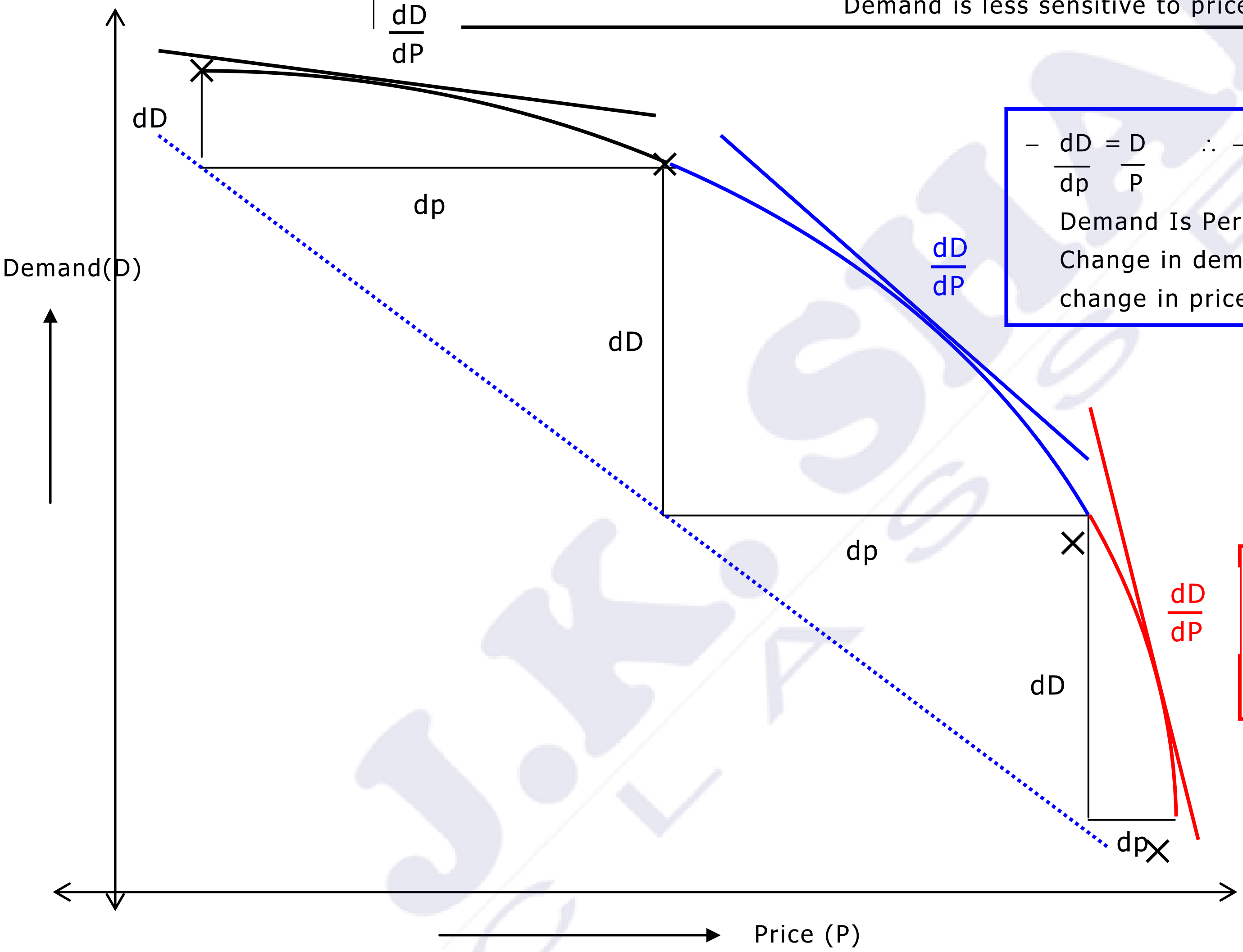
Demand Is Perfect Elastic (Unitary Elastic).

Change in demand is in proportion to change in price

$-\frac{dD}{dp} > \frac{D}{P} \quad \therefore -\frac{P}{D}\frac{dD}{dp} = 1 \quad \therefore \eta > 1$

Demand Is Relatively Elastic .

Demand is highly sensitive to price



Q1.

if the demand function is $D = 50 - 3p - p^2$.

Find the elasticity of demand at

i) $p = 5$ ii) $p = 2$ Also comment on the result

SOLUTION

STEP 1 : $D = 50 - 3p - p^2$.

$$\frac{dD}{dp} = -3 - 2p$$

STEP 2 : $\eta = \frac{-P}{D} \cdot \frac{dD}{dp}$

$$= -\frac{p}{50 - 3p - p^2} \cdot (-3 - 2p)$$

$$= \frac{3p + 2p^2}{50 - 3p - p^2}$$

STEP 3 : $\eta \Big|_{p = 5}$

$$= \frac{3(5) + 2(5)^2}{50 - 3(5) - (5)^2}$$

$$= \frac{15 + 2(25)}{50 - 15 - 25}$$

$$= \frac{65}{10}$$

$$= 6.5 > 1 .$$

Demand is relatively elastic

STEP 4 : $\eta \Big|_{p = 2}$

$$= \frac{3(2) + 2(2)^2}{50 - 3(2) - (2)^2}$$

$$= \frac{6 + 8}{50 - 6 - 4}$$

$$= \frac{14}{40}$$

$$= \frac{7}{20} < 1$$

Demand is relatively inelastic

Q2.

Find the price for the demand function

$$D = \frac{p + 6}{p - 8} ;$$

when elasticity of Demand is 14⁄11

SOLUTION

STEP 1 :

$$D = \frac{p + 6}{p - 8}$$

$$\frac{dD}{dp} = \frac{(p-8)\frac{d}{dp}(p+6) - (p+6)\frac{d}{dp}(p - 8)}{(p - 8)^2}$$

$$= \frac{(p - 8).1 - (p + 6)(1)}{(p - 8)^2}$$

$$= \frac{p - 8 - p - 6}{(p - 8)^2}$$

$$= \frac{-14}{(p - 8)^2}$$

STEP 2 :

$$\eta = \frac{-P}{D} \cdot \frac{dD}{dp}$$

$$\frac{14}{11} = \frac{-p}{\frac{p + 6}{p - 8}} \cdot \frac{-14}{(p - 8)^2}$$

$$\frac{1}{11} = \frac{p}{(p + 6)(p - 8)}$$

$$(p + 6)(p - 8) = 11p$$

$$p^2 - 2p - 48 = 11p$$

$$p^2 - 13p - 48 = 0$$

$$(p - 16)(p + 3) = 0$$

$$p = 16 \quad ; \quad p \neq -3$$

Q3.

Demand function x , for a certain commodity is given as $x = 200 - 4p$, where p is the price Find

i) elasticity of demand as function of p

ii) elasticity of demand when p = 10 ; p = 30 . Interpret the results

iii) the price p for which elasticity of demand is equal to one

STEP 1 : $x = 200 - 4p$

$$\frac{dx}{dp} = -4$$

STEP 2 : $\eta = \frac{-P}{D} \cdot \frac{dD}{dp}$

$$= \frac{-p}{x} \cdot \frac{dx}{dp}$$

IN THIS SUM DEMAND 'D' IS DENOTED AS 'X'

$$= \frac{-p}{200-4p} \cdot -4$$

$$= \frac{p}{50 - p}$$

STEP 3 : $\eta \Big|_{p = 10} = \frac{10}{50 - 10}$

$$= \frac{10}{40} = 0.25 < 1$$

Demand is relatively inelastic

STEP 4 : $\eta \Big|_{p = 30} = \frac{30}{50 - 30}$

$$= \frac{30}{20} = 1.5 > 1$$

Demand is relatively elastic

STEP 5 : $\eta = \frac{p}{50 - p}$

$$1 = \frac{p}{50 - p}$$

$$50 - p = p$$

$$50 = 2p \qquad \therefore p = 25$$

RELATIONSHIP BETWEEN :

Marginal Revenue (Rm); Average Revenue (R_A); Elasticity of Demand (η)

$$Rm = \frac{dR}{dD} = \frac{d}{dD}(p.D)$$

$$= p\frac{dD}{dD} + D\frac{dp}{dD}$$

$$= p + p\frac{D}{p}\frac{dp}{dD}$$

$$= p\left[1 - \frac{-D}{p}\frac{dD}{dP}\right]$$

$$= p\left[1 - \frac{1}{-\frac{P}{D}\frac{dD}{dp}}\right]$$

$$Rm = R_A\left[1 - \frac{1}{\eta}\right]$$

NOTE : average revenue is the revenue received by selling one item which is the price (p) of the item

Hence $p = R_A$

Q4.

if the avg revenue R_A is 50 and elasticity of demand $\eta = 5$, find the marginal revenue

SOLUTION

$$Rm = R_A\left(1 - \frac{1}{\eta}\right)$$

$$= 50\left(1 - \frac{1}{5}\right)$$

$$= 50 \times \frac{4}{5}$$

$$= 40$$

APP. OF DERIVATIVES - 5

INCOME – EXPENDITURE – SAVINGS

For any person with income x ,
his consumption expenditure (Ec) depends on x

$$Ec = f(x)$$

Marginal Propensity to consume

$$MPC = \frac{dEc}{dx}$$

Marginal Propensity to save

$$MPS = 1 - MPC$$

NOTE

$$Ec + S = x$$

$$\frac{dEc}{dx} + \frac{dS}{dx} = 1$$

$$MPC + MPS = 1$$

Average Propensity to consume

$$APC = \frac{Ec}{x}$$

NOTE : average consumption is the amount spent (consumed) in a rupee . Hence if a persons income is 100 and he spends/ consumes 80 then his average consumption will be 80/100 = 0.80 i.e. in a rupee person spends 0.80
In that case , his average saving s will be = 1 – 0.80 = 0.20

Average Propensity to save

$$APS = 1 - APC$$

Q1 Find MPC ; MPS ; APC ; APS if the expenditure E_c of a person with income I is given as

$Ec = 0.0003 I^2 + 0.075 I$

when I = 1000

$$E_c = 0.0003I^2 + 0.075I$$

$$APC\Big|_{I=1000} = \frac{Ec}{I}$$

$$= \frac{0.0003I^2 + 0.075 I}{I}$$

$$= 0.0003 I + 0.075$$

$$= 0.0003(1000) + 0.075$$

$$= 0.3 + 0.075$$

$$= 0.375$$

$$APS\Big|_{I=1000} = 1 - APC\Big|_{I=1000}$$

$$= 1 - 0.375$$

$$= 0.625$$

$$MPC\Big|_{I=1000} = \frac{dEc}{dI}$$

$$= \frac{d\ 0.0003 I^2 + 0.075 I}{dI}$$

$$= 0.0006 I + 0.075$$

$$= 0.0006(1000) + 0.075$$

$$= 0.6 + 0.075$$

$$= 0.675$$

$$MPS\Big|_{I=1000} = 1 - MPC\Big|_{I=1000}$$

$$= 1 - 0.675$$

$$= 0.325$$

PAPER - I CHAPTER - 5

INDEFINITE INTEGRATION

$$\int 1 \, dx = x + C$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \frac{1}{\sqrt{x}} \, dx = 2\sqrt{x} + C$$

$$\int \frac{1}{x^2} \, dx = -\frac{1}{x} + C$$

$$\int \frac{1}{x} \, dx = \log |x| + C$$

$$\int e^x \, dx = e^x + C$$

$$\int a^x \, dx = \frac{a^x}{\log a} + C$$

$$\int \frac{1}{x^2 - a^2} \, dx = \frac{1}{2a} \log \left| \frac{x - a}{x + a} \right| + c$$

$$\int \frac{1}{a^2 - x^2} \, dx = \frac{1}{2a} \log \left| \frac{a + x}{a - x} \right| + c$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \log \left| x + \sqrt{x^2 + a^2} \right| + C$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \log \left| x + \sqrt{x^2 - a^2} \right| + C$$

$$\int \sqrt{x^2 + a^2} \, dx$$

$$= \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2} \log \left| x + \sqrt{x^2 + a^2} \right| + C$$

$$\int \sqrt{x^2 - a^2} \, dx$$

$$= \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2} \log \left| x + \sqrt{x^2 - a^2} \right| + C$$

$$\int \frac{f'(x)}{f(x)} \, dx = \log |f(x)| + C$$

$$\int \frac{f'(x)}{\sqrt{f(x)}} \, dx = 2\sqrt{f(x)} + c$$

$$\int e^x [f(x) + f'(x)] \, dx = ex.f(x) + C$$

$$\int \frac{ae^x + b}{ce^x + d} \, dx$$

EXPRESS

NUMERATOR

= A(DENOMINATOR)

$+ B \frac{d}{dx}$ (DENOMINATOR)

$$\int u.v.dx$$

$$= u \int v dx - \int \frac{d}{dx} u \int v \, dx \, dx$$

EXERCISE - 5.1

01. $\int (3x^2 - 5)^2 \, dx$

$= \int (9x^4 - 30x^2 + 25) \, dx$

$= \frac{9x^5}{5} - \frac{30x^3}{3} + 25x + C$

$= \frac{9x^5}{5} - 10x^3 + 25x + C$

02. $\int \frac{x^3 + 4x^2 - 6x + 5}{x} \, dx$

$= \int \left(x^2 + 4x - 6 + \frac{5}{x} \right) dx$

$= \frac{x^3}{3} + \frac{4x^2}{2} - 6x + 5 \log |x| + C$

$= \frac{x^3}{3} + 2x^2 - 6x + 5 \log |x| + C$

03. $\int x^3 \left(2 - \frac{3}{x} \right)^2$

$= \int x^3 \left(4 - \frac{12}{x} + \frac{9}{x^2} \right) dx$

$= \int (4x^3 - 12x^2 + 9x) \, dx$

$= \frac{4x^4}{4} - \frac{12x^3}{3} + \frac{9x^2}{2} + C$

$= x^4 - 4x^3 + \frac{9x^2}{2} + C$

04. $\int \left(x + \frac{1}{x} \right)^3 dx$

$= \int \left(x^3 + 3x^2 . \frac{1}{x} + 3x . \frac{1}{x^2} + \frac{1}{x^3} \right) dx$

$= \int \left(x^3 + 3x + \frac{3}{x} + x^{-3} \right) dx$

$= \frac{x^4}{4} + \frac{3x^2}{2} + 3 \log|x| + \frac{x^{-2}}{-2} + C$

$= \frac{x^4}{4} + \frac{3x^2}{2} + 3 \log|x| - \frac{1}{2x^2} + C$

05. $\int \frac{-2}{\sqrt{5x-4} - \sqrt{5x-2}} \, dx$

$= \int \frac{-2}{\sqrt{5x-4} - \sqrt{5x-2}} \cdot \frac{\sqrt{5x-4} + \sqrt{5x-2}}{\sqrt{5x-4} + \sqrt{5x-2}} \, dx$

$= \int \frac{-2}{(5x-4) - (5x-2)} \cdot \frac{\sqrt{5x-4} + \sqrt{5x-2}}{1} \, dx$

$= \int \frac{-2}{5x-4 - 5x+2} \cdot \frac{\sqrt{5x-4} + \sqrt{5x-2}}{1} \, dx$

$= \int \frac{-2}{-2} \cdot \frac{\sqrt{5x-4} + \sqrt{5x-2}}{1} \, dx$

$= \int (5x-4)^{1/2} + (5x-2)^{1/2} \, dx$

$= \frac{(5x-4)^{3/2}}{5 . 3/2} + \frac{(5x-2)^{3/2}}{5 . 3/2} + C$

$= \frac{2}{15} \left[(5x-4)^{3/2} + (5x-2)^{3/2} \right] + C$

06. $\displaystyle\int \frac{5(x^6 + 1)}{x^2 + 1}\,dx$

$$
\begin{array}{r|l}
 & x^4 - x^2 + 1 \\
\hline
x^2 + 1 & x^6 + \qquad\qquad 1 \\
 & \underline{-\;x^6 \;\underline{+}\; x^4} \\
 & \quad - x^4 \\
 & \quad \underline{\mp x^4 \mp x^2} \\
 & \qquad + x^2 + 1 \\
 & \qquad \underline{+ x^2 + 1} \\
 & \qquad\qquad 0
\end{array}
$$

BACK INTO THE SUM

$= 5\int (x^4 - x^2 + 1)\,dx$

$= 5\left(\frac{x^5}{5} - \frac{x^3}{3} + x\right) + C$

$= x^5 - \frac{5x^3}{3} + 5x + C$

07. if $f'(x) = 4x^3 - 3x^2 + 2x + k$, $f(0) = 1$, $f(1) = 4$, then find $f(x)$

$f(x) = \int f'(x)\,dx$

$= \int 4x^3 - 3x^2 + 2x + k\,dx$

$f(x) = \frac{4x^4}{4} - \frac{3x^3}{3} + \frac{2x^2}{2} + kx + C$

$f(x) = x^4 - x^3 + x^2 + kx + C$

GIVEN : $f(0) = 1$

$0 - 0 + 0 + 0 + C = 1$

$C = 1$

GIVEN : $f(1) = 4$

$1 - 1 + 1 + k + C = 4$

$1 + K + 1 = 4$

$k = 2$

Hence ,

$f(x) = x^4 - x^3 + x^2 + 2x + 1$

EXERCISE - 5.2

01. $\int \frac{(\log x)^7}{x} dx$

PUT $\log x = t$

$\frac{1}{x} dx = dt$

BACK INTO THE SUM

$= \int t^7 \, dt$

$= \frac{t^8}{8} + C$

$= \frac{(\log x)^8}{8} + C$

02. $\int \frac{e^x + 1}{e^x + x} dx$

PUT $e^x + x = t$

$e^x + 1 \; dx = dt$

BACK INTO THE SUM

$= \int \frac{1}{t} dt$

$= \log | t | + C$

$= \log | e^x + x | + C$

03. $\int \frac{1}{1 + e^{-x}} dx$

$= \int \frac{1}{1 + \frac{1}{e^x}} dx$

$= \int \frac{e^x}{e^x + 1} dx$

PUT $e^x + 1 = t$

$e^x \, dx = dt$

BACK INTO THE SUM

$= \int \frac{1}{t} dt$

$= \log | t | + C$

$= \log | e^x + 1 | + C$

04. $\int \frac{(x + 1)(x + \log x)^4}{-3x} dx$

$= \frac{1}{-3} \int \frac{(x + 1)(x + \log x)^4}{x} dx$

PUT $x + \log x = t$

$1 + \frac{1}{x} dx = dt$

$\frac{x + 1}{x} dx = dt$

BACK INTO THE SUM

$= \frac{1}{-3} \int t^4 \, dt$

$= \frac{1}{-3} \int \frac{t^5}{5} + C$

$= \frac{(x + \log x)^5}{-15} + C$

05. $\int \frac{3x^2}{\sqrt{1 + x^3}} dx$

PUT $1 + x^3 = t$

$3x^2 . dx = dt$

BACK INTO THE SUM

$= \int \frac{1}{\sqrt{t}} dt$

$= 2\sqrt{t} + C$

$= 2\sqrt{1 + x^3} + C$

06. $\int (x + 1)(x + 2)^7(x + 3) dx$

Put $x + 2 = t$ $\therefore x = t - 2$

$1.dx = dt$

BACK INTO THE SUM

$= \int (t - 2 + 1) . t^7.(t - 2 + 3) dt$

$= \int (t - 1).t^7.(t + 1) dt$

$= \int (t^2 - 1).t^7 dt$

$= \int (t^9 - t^7). dt$

$= \frac{t^{10}}{10} - \frac{t^8}{8} + c$

$= \frac{(x + 2)^{10}}{10} - \frac{(x + 2)^8}{8} + C$

07. $\int (2x + 1) \sqrt{x - 4} dx$

Put $x - 4 = t$

$1.dx = dt$

BACK INTO THE SUM

$= \int [2(t+4) + 1] \sqrt{t} dt$

$= \int (2t + 9).t^{1/2} dt$

$= \int 2t^{1+1/2} + 9t^{1/2} dt$

$= \int 2t^{3/2} + 9t^{1/2} dt$

$= \frac{2t^{5/2}}{5/2} + \frac{9 t^{3/2}}{3/2} + C$

$= \frac{4t^{5/2}}{5} + 6 t^{3/2} + C$

$= \frac{4(x-4)^{5/2}}{5} + 6 (x-4)^{3/2} + C$

EXERCISE - 5.3

$$\int \frac{ae^x + b}{ce^x + d}\,dx$$

EXPRESS

NUMERATOR = A(DENOMINATOR) + B $\frac{d}{dx}$ (DENOMINATOR)

BACK INTO THE SUM

$$\int \frac{A(ce^x + d) + B\frac{d}{dx}(ce^x + d)}{ce^x + d}\,dx$$

$$\int \left(\frac{A(ce^x + d)}{ce^x + d} + \frac{B\frac{d}{dx}(ce^x + d)}{ce^x + d} \right) dx$$

$$\int \left(A + B\frac{f'(x)}{f(x)} \right) dx$$

$$= Ax + B.\log|f(x)| + c$$

$$= Ax + B.\log|ce^x + d| + c$$

01. $\int \frac{4e^x - 25}{2e^x - 5}\,dx$

$$4e^x - 25 = A(2e^x - 5) + B\frac{d(2e^x - 5)}{dx}$$

$$4e^x - 25 = A(2e^x - 5) + B(2e^x)$$

$$4e^x - 25 = 2Ae^x - 5A + 2Be^x$$

$$4e^x - 25 = (2A + 2B)e^x - 5A$$

ON COMPARING ,

$-5A = -25 \quad \therefore A = 5$

$2A + 2B = 4$

PUT $A = 5 \quad \therefore B = -3$

HENCE ,

$$4e^x - 25 = 5(2e^x - 5) - 3(2e^x)$$

BACK INTO THE SUM

$$= \int \frac{5(2e^x - 5) - 3(2e^x)}{2e^x - 5}\,dx$$

$$= \int \left(\frac{5(2e^x - 5)}{2e^x - 5} - \frac{3(2e^x)}{2e^x - 5} \right) dx$$

$$= \int \left(5 - 3\frac{2e^x}{2e^x - 5} \right) dx$$

$$= 5x - 3\log|2e^x - 5| + C$$

Using $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + C$

EXERCISE - 5.4(A)

FORMULA

$$\int \frac{1}{x^2 - a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$$

$$\int \frac{1}{a^2 - x^2}\,dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$$

Q1

01.

$$\int \frac{1}{9x^2 - 4}\,dx$$

$$= \int \frac{1}{(3x)^2 - (2)^2}\,dx$$

$$= \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$$

$$= \frac{1}{3}\,\frac{1}{2(2)}\log\left|\frac{3x-2}{3x+2}\right| + c$$

COEF. OF X

$$= \frac{1}{12}\log\left|\frac{3x-2}{3x+2}\right| + c$$

02.

$$\int \frac{1}{16 - 9x^2}\,dx$$

$$= \int \frac{1}{(4)^2 - (3x)^2}\,dx$$

$$= \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$$

$$= \frac{1}{3}\,\frac{1}{2(4)}\log\left|\frac{4+3x}{4-3x}\right| + c$$

COEF. OF X

$$= \frac{1}{24}\log\left|\frac{4+3x}{4-3x}\right| + c$$

03.

$$\int \frac{1}{x^2 + 4x - 5}\,dx$$

$$\left(\frac{1}{2}(4)\right)^2 = 4$$

$$= \int \frac{1}{x^2 + 4x + 4 - 5 - 4}\,dx$$

$$= \int \frac{1}{(x+2)^2 - 9}\,dx$$

$$= \int \frac{1}{(x+2)^2 - 3^2}\,dx$$

$$= \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$$

$$= \frac{1}{2(3)}\log\left|\frac{x+2-3}{x+2+3}\right| + C$$

$$= \frac{1}{6}\log\left|\frac{x-1}{x+5}\right| + C$$

04.

$$\int \frac{1}{3 - 2x - x^2}\,dx$$

$$= \int \frac{1}{3 - (x^2 + 2x)}\,dx$$

$$= \int \frac{1}{3 - (x^2 + 2x + 1) + 1}\,dx$$

$$= \int \frac{1}{2^2 - (x+1)^2}\,dx$$

$$= \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$$

$$= \frac{1}{2(2)}\log\left|\frac{2+x+1}{2-x-1}\right| + c$$

$$= \frac{1}{4}\log\left|\frac{3+x}{1-x}\right| + c$$

12.

$$\int \frac{1}{4x^2 - 20x + 17}\,dx$$

$$\frac{1}{4}\int \frac{1}{x^2 - 5x + \frac{17}{4}}\,dx$$

$$\left(\frac{1}{2}\,5\right)^2 = \frac{25}{4}$$

$$= \frac{1}{4}\int \frac{1}{x^2 - 5x + \frac{25}{4} + \frac{17}{4} - \frac{25}{4}}\,dx$$

$$= \frac{1}{4}\int \frac{1}{\left(x - \frac{5}{2}\right)^2 - \frac{8}{4}}\,dx$$

$$= \frac{1}{4}\int \frac{1}{\left(x - \frac{5}{2}\right)^2 - 2}\,dx$$

$$= \frac{1}{4}\int \frac{1}{\left(x - \frac{5}{2}\right)^2 - \sqrt{2}^2}\,dx$$

$$= \frac{1}{2a}\log\left|\frac{a + x}{a - x}\right| + c$$

$$= \frac{1}{4}\,\frac{1}{2\sqrt{2}}\log\left|\frac{x - \frac{5}{2} - \sqrt{2}}{x - \frac{5}{2} + \sqrt{2}}\right| + C$$

$$= \frac{1}{8\sqrt{2}}\log\left|\frac{2x - 5 - 2\sqrt{2}}{2x - 5 + 2\sqrt{2}}\right| + C$$

Q2

DECEPTION (DHOKA)

BY APPROPRIATE SUBSTITUTIONS , YOU CAN CONVERT THE SUM INTO

$$\int \frac{1}{at^2 + bt + c}\,dt$$

AND THEN FOLLOW Q1

01.

$$\int \frac{1}{x\sqrt{(\log x)^2 - 4}}\,dx$$

PUT $\log x = t$

$$\frac{1}{x}\cdot dx = dt$$

THE SUM IS

$$= \int \frac{1}{t^2 - 4}\,dt$$

$$= \int \frac{1}{t^2 - 2^2}\,dt$$

$$= \frac{1}{2a}\log\left|\frac{t - a}{t + a}\right| + c$$

$$= \frac{1}{2(2)}\log\left|\frac{t - 2}{t + 2}\right| + c$$

$$= \frac{1}{4}\log\left|\frac{t - 2}{t + 2}\right| + c$$

Resubs.

$$= \frac{1}{4}\log\left|\frac{\log x - 2}{\log x + 2}\right| + c$$

02.

$$\int \frac{x^3}{16x^8 - 25}\, dx$$

$$= \int \frac{x^3}{16(x^4)^2 - 25}\, dx$$

PUT $x^4 = t$

$4x^3 . dx = dt$

$$= \frac{1}{4}\int \frac{4x^3}{16(x^4)^2 - 25}\, dx$$

THE SUM IS

$$= \frac{1}{4}\int \frac{1}{16t^2 - 25}\, dt$$

$$= \int \frac{1}{(4t)^2 - 5^2}\, dt.$$

$$= \frac{1}{2a}\log\left|\frac{t - a}{t + a}\right| + c$$

$$= \frac{1}{4}\ \frac{1}{4}\ \frac{1}{2(5)}\ \log\left|\frac{4t - 5}{4t + 5}\right| + c$$

$$= 1 \quad \log\left|4x^4 - 5\right| + c$$

03.

$$\int \frac{1}{x\left[(\log x)^2 + 4\log x - 1\right]}\, dx$$

PUT $\log x = t$

$\frac{1}{x} . dx = dt$

THE SUM IS

$$= \int \frac{1}{t^2 + 4t - 1}\, dt$$

$$\left(\frac{1}{2}(4)\right)^2 = 4$$

$$= \int \frac{1}{t^2 + 4t + 4 - 1 - 4}\, dt$$

$$= \int \frac{1}{(t + 2)^2 - 5}\, dt$$

$$= \int \frac{1}{(t + 2)^2 - \sqrt{5}^2}\, dt$$

$$= \frac{1}{2a}\log\left|\frac{t - a}{t + a}\right| + C$$

$$= \frac{1}{2(\sqrt{5})}\log\left|\frac{t + 2 - \sqrt{5}}{t + 2 + \sqrt{5}}\right| + C$$

$$= \frac{1}{2(\sqrt{5})}\log\left|\frac{\log x + 2 - \sqrt{5}}{\log x + 2 + \sqrt{5}}\right| + C$$

FORMULA

$$\int \frac{1}{\sqrt{x^2 + a^2}}\, dx = \log \left| x + \sqrt{x^2 + a^2} \right| + C$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\, dx = \log \left| x + \sqrt{x^2 - a^2} \right| + C$$

Q1

01.

$$\int \frac{1}{\sqrt{9x^2+25}}\, dx$$

$$= \int \frac{1}{\sqrt{(3x)^2 + 5^2}}\, dx$$

$$= \frac{1}{3} \log \left| 3x + \sqrt{(3x)^2 + 5^2} \right| + c$$

COEFF. OF X

$$= \frac{1}{3} \log \left| 3x + \sqrt{9x^2+25} \right| + c$$

02.

$$\int \frac{1}{\sqrt{4x^2 - 9}}\, dx$$

$$= \int \frac{1}{\sqrt{(2x)^2 - 3^2}}\, dx$$

$$= \frac{1}{2} \log \left| 2x + \sqrt{(2x)^2 - 3^2} \right| + c$$

COEFF. OF X

$$= \frac{1}{2} \log \left| 2x + \sqrt{4x^2 - 9} \right| + c$$

EXERCISE - 5.4(B)

03.

$$\int \frac{1}{\sqrt{x^2 + 4x + 13}}\, dx$$

$$= \int \frac{1}{\sqrt{x^2 + 4x + 4 + 13 - 4}}\, dx$$

$$= \int \frac{1}{\sqrt{(x + 2)^2 + 3^2}}\, dx$$

$$= \log \left| x + \sqrt{x^2 - a^2} \right| + c$$

$$= \log \left| x + 2 + \sqrt{(x+2)^2 + 3^2} \right| + c$$

$$= \log \left| x + 2 + \sqrt{x^2 + 4x + 13} \right| + c$$

04.

$$\int \frac{1}{\sqrt{(x - 2).(x - 3)}}\, dx$$

$$= \int \frac{1}{\sqrt{x^2 - 5x + 6}}\, dx$$

$$= \int \frac{1}{\sqrt{x^2 - 5x + \frac{25}{4} + 6 - \frac{25}{4}}}\, dx$$

$$= \int \frac{1}{\sqrt{\left(x - \frac{5}{2}\right)^2 - \frac{1}{4}}}\, dx$$

$$= \int \frac{1}{\sqrt{\left(x - \frac{5}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}\, dx$$

$$= \log \left| x - \frac{5}{2} + \sqrt{\left(x - \frac{5}{2}\right)^2 - \left(\frac{1}{2}\right)^2} \right| + c$$

$$= \log \left| x - \frac{5}{2} + \sqrt{x^2 - 5x + 6} \right| + c$$

Q2

DECEPTION (DHOKA)

BY APPROPRIATE SUBSTITUTIONS , YOU CAN CONVERT THE SUM INTO

$$\int \frac{1}{\sqrt{at^2 + bt + c}}\, dt$$

AND THEN FOLLOW Q1

01. $\int \frac{x^2}{\sqrt{x^6 + 2x^3 + 3}}\, dx$

PUT $x^3 = t$

$3x^2 \cdot dx = dt$

$x^2 \cdot dx = \frac{dt}{3}$

THE SUM IS

$$= \frac{1}{3} \int \frac{1}{\sqrt{t^2 + 2t + 3}}\, dt$$

$$= \frac{1}{3} \int \frac{1}{\sqrt{t^2 + 2t + 1 + 3 - 1}}\, dt$$

$$= \frac{1}{3} \int \frac{1}{\sqrt{(t+1)^2 + \sqrt{2}^2}}\, dt$$

$$= \frac{1}{3} \log \left| t + \sqrt{t^2 + a^2} \right| + c$$

$$= \frac{1}{3} \log \left| t + 1 + \sqrt{(t+1)^2 + \sqrt{2}^2} \right| + c$$

$$= \frac{1}{3} \log \left| t + 1 + \sqrt{t^2 + 2t + 3} \right| + c$$

$$= \frac{1}{3} \log \left| x^3 + 1 + \sqrt{x^6 + 2x^3 + 3} \right| + c$$

03. $\int \frac{e^x}{\sqrt{e^{2x} + 4e^x + 13}}\, dx$

PUT $e^x = t$

$e^x \cdot dx = dt$

THE SUM IS

$$\int \frac{1}{\sqrt{t^2 + 4t + 13}}\, dt$$

$$= \int \frac{1}{\sqrt{t^2 + 4t + 4 + 13 - 4}}\, dt$$

$$= \int \frac{1}{\sqrt{(t+2)^2 + 9}}\, dx$$

$$= \int \frac{1}{\sqrt{(t+2)^2 + 3^2}}\, dt$$

$$= \log \left| t + \sqrt{t^2 + a^2} \right| + c$$

$$= \log \left| t + 2 + \sqrt{(t+2)^2 + 3^2} \right| + C$$

$$= \log \left| t + 2 + \sqrt{t^2 + 4t + 13} \right| + C$$

$$= \log \left| e^x + 2 + \sqrt{e^{2x} + 4e^x + 13} \right| + C$$

EXERCISE - 5.4(C)

$$\int \frac{px + q}{\sqrt{ax^2 + bx + c}}\, dx$$

REMEMBER THIS EXERCISE AS

$$\frac{\text{LINEAR}}{\sqrt{\text{QUADRATIC}}}$$

Express $px + q = A \frac{d}{dx}(ax^2 + bx + c) + B$

$$= \int \frac{A \frac{d}{dx}(ax^2 + bx + c) + B}{\sqrt{ax^2 + bx + c}}\, dx$$

$$= A \int \frac{\frac{d}{dx}(ax^2 + bx + c)}{\sqrt{ax^2 + bx + c}}\, dx + B \int \frac{1}{\sqrt{ax^2 + bx + c}}\, dx$$

$$= I_1 + I_2$$

Where

$$I_1 = A \int \frac{\frac{d}{dx}(ax^2 + bx + c)}{\sqrt{ax^2 + bx + c}}\, dx$$

Using $\int \frac{f'(x)}{\sqrt{f(x)}}\, dx = 2\sqrt{f(x)} + c$

$$= A \cdot 2\sqrt{ax^2 + bx + c} + c_1$$

$$I_2 = B \int \frac{1}{\sqrt{ax^2 + bx + c}}\, dx$$

REFER : EX 5.4(B) Q1

EXERCISE - 5.4(C)

01.

$$I = \int \frac{2x + 1}{\sqrt{x^2 + 2x + 3}}\, dx$$

$$I = \int \frac{1\,(2x + 2) - 1}{\sqrt{x^2 + 2x + 3}}\, dx$$

$$I = \int \frac{2x + 2}{\sqrt{x^2 + 2x + 3}}\, dx - \int \frac{1}{\sqrt{x^2 + 2x + 3}}\, dx$$

$$I = I_1 - I_2$$

Now

$$I_1 = \int \frac{2x + 2}{\sqrt{x^2 + 2x + 3}}\, dx$$

$$I_1 = \int \frac{f'(x)}{\sqrt{f(x)}}\, dx$$

$$I_1 = 2\sqrt{f(x)} + c_1$$

$$I_1 = 2\sqrt{x^2 + 2x + 3} + c_1$$

Now

$$I_2 = \int \frac{1}{\sqrt{x^2 + 2x + 3}}\, dx$$

$$= \int \frac{1}{\sqrt{x^2 + 2x + 1 + 3 - 1}}\, dx$$

$$= \int \frac{1}{\sqrt{(x + 1)^2 + 2}}\, dx$$

$$= \int \frac{1}{\sqrt{(x + 1)^2 + \sqrt{2}^2}}\, dx$$

$$= \log \left| x + \sqrt{x^2 - a^2} \right| + c_2$$

$$= \log \left| x + 1 + \sqrt{(x + 1)^2 + \sqrt{2}^2} \right| + c_2$$

$$= \log \left| x + 1 + \sqrt{x^2 + 2x + 3} \right| + c_2$$

FINALLY

$$I = 2\sqrt{x^2 + 2x + 3} - \log \left| x + 1 + \sqrt{x^2 + 2x + 3} \right| + c$$

where $c = c_1 - c_2$

02. $\int \sqrt{\frac{x+1}{x+3}}\,dx$

$$= \int \sqrt{\frac{x+1}{x+3} \cdot \frac{x+1}{x+1}}\,dx$$

$$= \int \frac{x+1}{\sqrt{x^2+4x+3}}\,dx$$

$$= \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+4x+3}}\,dx$$

$$= \frac{1}{2}\int \frac{2x+4-2}{\sqrt{x^2+4x+3}}\,dx$$

$$I = \frac{1}{2}\int \frac{2x+4}{\sqrt{x^2+4x+3}}\,dx - \int \frac{1}{\sqrt{x^2+4x+3}}\,dx$$

$$I = I_1 - I_2$$

Now

$$I_1 = \frac{1}{2}\int \frac{2x+4}{\sqrt{x^2+4x+3}}\,dx$$

$$= \frac{1}{2}\int \frac{f'(x)}{\sqrt{f(x)}}\,dx$$

$$= \frac{1}{2}\, 2\sqrt{f(x)} + c_1$$

$$= \sqrt{x^2+4x+3} + c_1$$

Now

$$I_2 = \int \frac{1}{\sqrt{x^2+4x+3}}\,dx$$

$$= \int \frac{1}{\sqrt{x^2+4x+4+3-4}}\,dx$$

$$= \int \frac{1}{\sqrt{(x+2)^2-1}}\,dx$$

$$= \int \frac{1}{\sqrt{(x+2)^2-1^2}}\,dx$$

$$= \log\left|x + \sqrt{x^2-a^2}\right| + c$$

$$= \log\left|x + 2 + \sqrt{(x+2)^2-1^2}\right| + c$$

$$= \log\left|x + 2 + \sqrt{x^2+4x+3}\right| + c$$

FINALLY

$$I = \sqrt{x^2+4x+3}$$

$$- \log\left|x + 2 + \sqrt{x^2+4x+3}\right| + c$$

where $c = c_1 - c_2$

EXERCISE - 5.5(A)

$$\int e^x[f(x) + f'(x)]\,dx = e^x.f(x) + c$$

01. $$\int e^x \frac{x \log x + 1}{x}\,dx$$

$= \int e^x \left[\log x + \frac{1}{x}\right] dx$

$= \int e^x[f(x) + f'(x)]\,dx$

$= e^x.f(x) + c$

$= e^x.\log x + c$

02. $$\int e^x \left[\frac{1}{x} - \frac{1}{x^2}\right] dx$$

$= \int e^x \left[\frac{1}{x} + \frac{-1}{x^2}\right] dx$

$= \int e^x[f(x) + f'(x)]\,dx$

$= e^x.f(x) + c$

$= e^x.\frac{1}{x} + c$

03. $$\int e^x \left[(\log x)^2 + \frac{2\log x}{x}\right] dx$$

$\frac{d}{dx}(\log x)^2 = 2 \log x \frac{d}{dx}\log x$

$= \frac{2\log x}{x}$

$= \int e^x[f(x) + f'(x)]\,dx$

$= e^x.f(x) + c$

$= e^x.(\log x)^2 + c$

04. $$\int e^x \frac{x}{(x + 1)^2}\,dx$$

$= \int e^x \frac{x + 1 - 1}{(x + 1)^2}\,dx$

$= \int e^x \left[\frac{x + 1}{(x + 1)^2} - \frac{1}{(x + 1)^2}\right] dx$

$= \int e^x \left[\frac{1}{x + 1} + \frac{-1}{(x + 1)^2}\right] dx$

$\frac{d}{dx}\frac{1}{x+1} = \frac{-1}{(x+1)^2}\frac{d(x+1)}{dx} = \frac{-1}{(x+1)^2}$

$= \int e^x[f(x) + f'(x)]\,dx$

$= e^x.f(x) + c$

$= e^x.\frac{1}{x+1} + c$

05.

$$\int \frac{\log x}{(1 + \log x)^2} dx$$

$\log_e x = t \quad \Rightarrow \quad x = e^t$

(EXPONENTIAL FORM)

$\frac{1}{x} dx = dt$

BACK INTO THE SUM

$$= \int x \frac{\log x}{(1 + \log x)^2} \frac{1}{x} dx$$

$$= \int e^t \frac{t}{(1 + t)^2} dt$$

$$= \int e^t \frac{1 + t - 1}{(1 + t)^2} dt$$

$$= \int e^t \left[\frac{1 + t}{(1 + t)^2} - \frac{1}{(1 + t)^2}\right] dt$$

$$= \int e^t \left[\frac{1}{1 + t} + \frac{-1}{(1 + t)^2}\right] dt$$

$$\frac{d}{dt} \frac{1}{1+t} = \frac{-1}{(1+t)^2} \frac{d(1+t)}{dt} = \frac{-1}{(1+t)^2}$$

$$= \int e^t [f(t) + f'(t)] \; dt$$

$$= e^t.f(t) + C$$

$$= e^t.\frac{1}{1+t} + C$$

$$= \frac{x}{1 + \log x} + C$$

EXERCISE - 5.5(B)

$\int uv\,dx = u\int v\,dx - \int\left(\frac{du}{dx}\int v\,dx\right)dx$, LIATE , L – LOG , I – INVERSE , A - ALGEBRAIC T – TRIGNOMETRIC , E – EXPONENTIAL WILL DECIDE THE FIRST FUNCTION

01. $\int x \,.\, \log x \,.dx$

$= \int \log x \,.\, x \,.dx$

$= \log x\int x\,dx - \int\left(\frac{d}{dx}\log x\int x\,dx\right)dx$

$= \log x\,.\frac{x^2}{2} - \int \frac{1}{x}.\frac{x^2}{2}\,dx$

$= \frac{x^2}{2}\log x - \frac{1}{2}\int x\,dx$

$= \frac{x^2}{2}\log x - \frac{x^2}{4} + c$

$= \frac{x^2}{4}\left(2.\log x - 1\right) + c$

02. $\int x^3.\log x\,dx$

$= \int \log x \,.\, x^3\,.dx$

$= \log x\int x^3\,dx - \int\left(\frac{d}{dx}\log x\int x^3\,dx\right)dx$

$= \log x\,.\frac{x^4}{4} - \int \frac{1}{x}.\frac{x^4}{4}\,dx$

$= \frac{x^4}{4}\log x - \frac{1}{4}\int x^3\,dx$

$= \frac{x^4}{4}\log x - \frac{x^4}{16} + c$

$= \frac{x^4}{16}\left(4.\log x - 1\right) + c$

03. $\int x^2.e^{3x}\,dx$

$= x^2\int e^{3x}\,dx - \int \frac{d}{dx}x^2\int e^{3x}\,dx\,dx$

$= \frac{x^2\,e^{3x}}{3} - \frac{2}{3}\int x\,e^{3x}\,dx$

$= \frac{x^2\,e^{3x}}{3} - \frac{2}{3}\left(x\int e^{3x}\,dx - \int\frac{d}{dx}x\int e^{3x}\,dx\,dx\right)$

$= \frac{x^2\,e^{3x}}{3} - \frac{2}{3}\left(\frac{x\,e^{3x}}{3} - \int\frac{1.e^{3x}}{3}\,dx\right)$

$= \frac{x^2\,e^{3x}}{3} - \frac{2}{3}\left(\frac{x\,e^{3x}}{3} - \frac{1}{3}\,\frac{e^{3x}}{3}\right) + C$

$= \frac{x^2\,e^{3x}}{3} - \frac{2}{9}x\,e^{3x} + \frac{2}{27}e^{3x} + C$

$= e^{3x}\left(\frac{x^2}{3} - \frac{2x}{9} + \frac{2}{27}\right) + c$

TYPE – 1 $\dfrac{px+q}{(x-a)(x-b)} = \dfrac{A}{x-a} + \dfrac{B}{x-B}$

EXERCISE - 5.6(A)

01. $\displaystyle\int \frac{2x+1}{(x+1)(x-2)}\,dx$

$$\frac{2x+1}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$$

$$2x+1 = A(x-2) + B(x+1)$$

Put $x = 2$ $\quad 2(2)+1 = B(2+1) \quad \therefore B = {}^5/_3$

Put $x = -1$ $\quad 2(-1)+1 = A(-1-2) \quad \therefore A = {}^1/_3$

BACK IN THE SUM

$$= \int \frac{\frac{1}{3}}{x+1} + \frac{\frac{5}{3}}{x-2}\,dx$$

$$= \frac{1}{3}\log|x+1| + \frac{5}{3}\log|x-2| + C$$

02. $\displaystyle\int \frac{2x+1}{x(x-1)(x-4)}\,dx$

$$\frac{2x+1}{x(x-1)(x-4)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-4}$$

$$2x+1 = A(x-1)(x-4) + Bx(x-4) + Cx(x-1)$$

Put $x = 1$ $\quad 3 = B1(1-4) \quad B = -1$

Put $x = 4$ $\quad 9 = C4(4-1) \quad C = {}^3/_4$

Put $x = 0$ $\quad 1 = A(0-1)(0-4) \quad A = {}^1/_4$

BACK IN THE SUM

$$= \int \frac{\frac{1}{4}}{x} - \frac{1}{x-1} + \frac{\frac{3}{4}}{x-4}\,dx$$

$$= \frac{1}{4}\log|x| - \log|x-2| + \frac{3}{4}\log|x-4| + c$$

03. $\displaystyle\int \frac{x^3 - 4x^2 + 3x + 11}{x^2 - 5x + 6}\,dx$ → IMPROPER FRACTION : DEGREEE OF NUMERATOR ≥ DEGREE OF DENOMINATOR

NOTE : ONLY PROPER FRACTIONS CAN ENTER PARTIAL FRACTIONS

∴ DIVIDE & MAKE IT PROPER FRACTION BEFORE GOING INTO PARTIAL FRACTION

$$\begin{array}{r|l} & x + 1 \\ \hline x^2 - 5x + 6 & x^3 - 4x^2 + 3x + 11 \\ & x^3 - 5x^2 + 6x \\ & \quad - \quad + \quad - \\ \hline & \quad x^2 - 3x + 11 \\ & \quad x^2 - 5x + 6 \\ & \quad - \quad + \quad - \\ \hline & \qquad 2x + 5 \end{array}$$

BACK INTO THE SUM

$$= \int \left(x + 1 + \frac{2x + 5}{x^2 - 5x + 6} \right) dx$$

$$= \int \left(x + 1 + \frac{2x + 5}{(x - 3)(x - 2)} \right) dx$$

$$\frac{2x + 5}{(x - 3)(x - 2)} = \frac{A}{x - 3} + \frac{B}{x - 2}$$

$$2x + 5 = A(x - 2) + B(x - 3)$$

Put $x = 2$ $\quad 9 = B(2 - 3) \qquad \therefore B = -9$

Put $x = 3$ $\quad 11 = A(3 - 2) \qquad \therefore A = 11$

BACK IN THE SUM

$$= \int \left(x + 1 + \frac{11}{x - 3} - \frac{9}{x - 2} \right) dx$$

$$= \frac{x^2}{2} + x + 11\log|x - 3| - 9\log|x - 2| + C$$

Q2

TYPE– 2 $\dfrac{px^2 + qx + r}{(x-a)(x-b)^2} = \dfrac{A}{x-a} + \dfrac{B}{x-b} + \dfrac{C}{(x-b)^2}$

01. $\displaystyle\int \frac{3x + 1}{(x-2)^2(x+2)}\,dx$

$$\frac{3x + 1}{(x-2)^2(x+2)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+2}$$

$$3x + 1 = A(x-2)(x+2) + B(x+2) + C(x-2)^2$$

Put $x = 2$ $\quad 3(2) + 1 = B(2 + 2)$

$7 = B(4)$ $\quad \therefore B = \dfrac{7}{4}$

Put $x = -2$ $\quad 3(-2) + 1 = C(-2-2)^2$

$-5 = C(16)$

$\therefore C = \dfrac{-5}{16}$

Put $x = 0$ $\quad 3(0) + 1 = A(-2)(2) + B(2) + C(-2)^2$

$1 = -4A + 2B + 4C$

$1 = -4A + 2.\dfrac{7}{4} + 4.\dfrac{-5}{16}$

$4A = \dfrac{7}{2} - \dfrac{5}{4} - 1$

$4A = \dfrac{14 - 5 - 4}{4}$

$4A = \dfrac{5}{4}$ $\quad \therefore A = \dfrac{5}{16}$

HENCE

$$\frac{3x + 1}{(x-2)^2(x+2)} = \frac{\frac{5}{16}}{x-2} + \frac{\frac{7}{4}}{(x-2)^2} - \frac{\frac{5}{16}}{x+2}$$

BACK IN THE SUM

$$= \int \frac{\frac{5}{16}}{x-2} + \frac{\frac{7}{4}}{(x-2)^2} - \frac{\frac{5}{16}}{x+2}\,dx$$

$$= \frac{5}{16}\log|x-2| + \frac{7}{4}\,\frac{-1}{x-2} - \frac{5}{16}\log|x+2| + c$$

$$= \frac{5}{16}\log\left|\frac{x-2}{x+2}\right| - \frac{7}{4(x-2)} + c$$

Q3

01. $\int \frac{1}{x(x^3+1)}\,dx$

$= \int \frac{x^3 + 1 - x^3}{x(x^3+1)}\,dx$

$= \int \left(\frac{x^3+1}{x(x^3+1)} - \frac{x^3}{x(x^3+1)} \right) dx$

$= \int \left(\frac{1}{x} - \frac{x^2}{x^3+1} \right) dx$

$= \int \left(\frac{1}{x} - \frac{1}{3} \frac{3x^2}{x^3+1} \right) dx$

its $\int \frac{f'(x)dx}{f(x)} = \log|f(x)| + C$

$= \log|x| - \frac{1}{3}\log|x^3+1| + c$

$= \frac{3\log|x| - \log|x^3+1|}{3} + c$

$= \frac{\log|x^3| - \log|x^3+1|}{3} + c$

$= \frac{1}{3}\log\left|\frac{x^3}{x^3+1}\right| + c$

02. $\int \frac{1}{x(x^n+1)}\,dx$

$= \int \frac{x^n + 1 - x^n}{x(x^n+1)}\,dx$

$= \int \left(\frac{x^n+1}{x(x^n+1)} - \frac{x^n}{x(x^n+1)} \right) dx$

$= \int \left(\frac{1}{x} - \frac{x^{n-1}}{x^n+1} \right) dx$

$= \int \left(\frac{1}{x} - \frac{1}{n} \frac{nx^{n-1}}{x^n+1} \right) dx$

$= \log|x| - \frac{1}{n}\log|x^n+1| + c$

$= \frac{n\log|x| - \log|x^n+1|}{n} + c$

$= \frac{\log|x^n| - \log|x^n+1|}{n} + c$

$= \frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right| + c$

SOLVED EXAMPLES ON PG 131 , 132 , MISC Pg 139

$$\int \sqrt{x^2 + a^2}\, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + c$$

$$\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + c$$

01. $\int \sqrt{9x^2 - 4}\, dx$

$= \int \sqrt{(3x)^2 - 2^2}\, dx$

$= \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + c$

$= \frac{1}{3}\left[\frac{3x\sqrt{(3x)^2 - 2^2}}{2} - \frac{2^2}{2}\log\left|3x + \sqrt{(3x)^2 - 2^2}\right|\right]$

COEFF OF X

$= \frac{x}{2}\sqrt{9x^2 - 4} - \frac{2}{3}\log\left|3x + \sqrt{9x^2 - 4}\right| + c$

02. $\int \sqrt{x^2 + 2x + 5}\, dx$

$= \int \sqrt{x^2 + 2x + 1 + 5 - 1}\, dx$

$= \int \sqrt{(x + 1)^2 + 2^2}\, dx$

$= \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + c$

$= \frac{x + 1}{2}\sqrt{(x+1)^2 + 2^2} + \frac{2^2}{2}\log\left|x + 1 + \sqrt{(x+1)^2 + 2^2}\right| + c$

$= \frac{x + 1}{2}\sqrt{x^2 + 2x + 5} + 2\log\left|x + 1 + \sqrt{x^2 + 2x + 5}\right| + c$

03. $\int \sqrt{x^2 - 8x + 7}\ dx$

$= \int \sqrt{x^2 - 8x + 16 + 7 - 16}\ dx$

$= \int \sqrt{(x-4)^2 - 9}\ dx$

$= \int \sqrt{(x-4)^2 - 3^2}\ dx$

$= \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + c$

$= \frac{x-4}{2}\sqrt{(x-4)^2 - 3^2} - \frac{3^2}{2}\log\left|x - 4 + \sqrt{(x-4)^2 - 3^2}\right| + c$

$= \frac{x-4}{2}\sqrt{x^2 - 8x + 7} - \frac{9}{2}\log\left|x - 4 + \sqrt{x^2 - 8x + 7}\right| + c$

PAPER - I

CHAPTER - 6

DEFINITE INTEGRATION

EXERCISE 6.1

01.

$$\int_2^4 \frac{x}{1+x^2}\,dx$$

$$= \frac{1}{2}\int_2^4 \frac{2x}{1+x^2}\,dx$$

$$= \frac{1}{2}\left[\log(1+x^2)\right]_2^4$$

$$= \frac{1}{2}\left[\log(1+16) - \log(1+4)\right]$$

$$= \frac{1}{2}\log\left(\frac{17}{5}\right)$$

02.

$$\int_0^1 \frac{x^2+3x+2}{\sqrt{x}}\,dx$$

$$= \int_0^1 \left[\frac{x^2}{\sqrt{x}} + \frac{3x}{\sqrt{x}} + \frac{2}{\sqrt{x}}\right]dx$$

$$= \int_0^1 \left[x^{2-1/2} + 3\sqrt{x} + \frac{2}{\sqrt{x}}\right]dx$$

$$= \int_0^1 \left[x^{3/2} + 3x^{1/2} + \frac{2}{\sqrt{x}}\right]dx$$

$$= \left[\frac{x^{5/2}}{5/2} + \frac{3x^{3/2}}{3/2} + 2.2\sqrt{x}\right]_0^1$$

$$= \left[\frac{2x^{5/2}}{5} + 2x^{3/2} + 4\sqrt{x}\right]_0^1$$

$$= \left(\frac{2}{5} + 2 + 4\right) - 0$$

$$= 32/5$$

03.

$$\int_3^5 \frac{1}{\sqrt{x+4}+\sqrt{x-2}}\,dx$$

$$= \int_3^5 \frac{1}{\sqrt{x+4}+\sqrt{x-2}} \cdot \frac{\sqrt{x+4}+\sqrt{x-2}}{\sqrt{x+4}+\sqrt{x-2}}\,dx$$

$$= \int_3^5 \frac{\sqrt{x+4}+\sqrt{x-2}}{(x+4)-(x-2)}\,dx$$

$$= \int_3^5 \frac{(x+4)^{\frac{1}{2}} + (x-2)^{\frac{1}{2}}}{6}\,dx$$

$$= \frac{1}{6}\left[\frac{(x+4)^{\frac{3}{2}}}{\frac{3}{2}} + \frac{(x-2)^{\frac{3}{2}}}{\frac{3}{2}}\right]_3^5$$

$$= \frac{1}{6}\,\frac{2}{3}\left[(x+4)^{\frac{3}{2}} + (x-2)^{\frac{3}{2}}\right]_3^5$$

$$= \frac{1}{9}\left[(x+4)^{\frac{3}{2}} + (x-2)^{\frac{3}{2}}\right]_3^5$$

$$= \frac{1}{9}\left[(9^{3/2} - 3^{3/2}) - (7^{3/2} - 1^{3/2})\right]$$

$$= \frac{1}{9}\left[(9\sqrt{9} - 3\sqrt{3}) - (7\sqrt{7} - 1)\right]$$

$$= \frac{1}{9}(27 - 3\sqrt{3} - 7\sqrt{7} + 1)$$

$$= \frac{1}{9}(28 - 3\sqrt{3} - 7\sqrt{7})$$

04.

$$\int_1^3 x^2 \log x \; dx$$

$$\int_1^3 \log x \, . \, x^2 \; dx$$

$$\left\{\log x \, . \int x^2 dx - \int\left(\frac{d}{dx}\log x \int x^2.dx\right)dx\right\}_1^2$$

$$\left\{\log x \, . \, \frac{x^3}{3} - \int \frac{1}{x} \, . \, \frac{x^3}{3}dx\right\}_1^3$$

$$\left\{\frac{x^3}{3}\log x - \frac{1}{3}\int x^2 dx\right\}_1^3$$

$$\left(\frac{x^3}{3}\log x - \frac{x^3}{9}\right)_1^3$$

$$\left(\frac{27}{3}\log 3 - \frac{27}{9}\right) - \left(\frac{1}{9}\log 1 - \frac{1}{9}\right)$$

$$9\log 3 - 3 + \frac{1}{9}$$

Log 1 = 0

$$9\log 3 - \frac{26}{9}$$

05.

$$\int_2^3 \frac{x}{(x+2)(x+3)} \; dx$$

$$\frac{x}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$$

$$x = A(x+3) + B(x+2)$$

Put x = – 3

$$-3 = B(-3+2)$$

$$-3 = B(-1)$$

$$3 = B$$

Put x = – 2

$$-2 = A(-2+3)$$

$$-2 = A(1)$$

$$-2 = A$$

BACK IN THE SUM

$$= \int_2^3 \left(\frac{-2}{x+2} + \frac{3}{x+3}\right) dx$$

$$= \left[-2\log|x+2| + 3\log|x+3|\right]_2^3$$

$$= \left[-2\log 5 + 3\log 6\right] - \left[-2\log 4 + 3\log 5\right]$$

$$= -2\log 5 + 3\log 6 + 2\log 4 - 3\log 5$$

$$= 2\log 4 + 3\log 6 - 5\log 5$$

$$= \log 4^2 + \log 6^3 - \log 5^5$$

$$= \log 16 + \log 216 - \log 3125$$

$$= \log\left(\frac{16 \times 216}{3125}\right)$$

$$= \log\left(\frac{3456}{3125}\right)$$

06.

$$\int_1^2 \frac{1}{x^2 + 6x + 5} dx$$

$$\left(\frac{1}{2}(6)\right)^2 = 9$$

$$= \int_1^2 \frac{1}{x^2 + 6x + 9 + 5 - 9} dx$$

$$= \int_1^2 \frac{1}{(x + 3)^2 - 4} dx$$

$$= \int_1^2 \frac{1}{(x + 3)^2 - 2^2} dx$$

$$= \frac{1}{2a} \log\left|\frac{x - a}{x + a}\right| + c$$

$$= \frac{1}{2(2)} \left(\log\left|\frac{x + 3 - 2}{x + 3 + 2}\right|\right)_1^2$$

$$= \frac{1}{4}\left(\log \left|\frac{x + 1}{x + 5}\right|\right)_1^2$$

$$= \frac{1}{4}\left\{\left(\log \frac{3}{7}\right) - \left(\log \frac{2}{6}\right)\right\}$$

$$= \frac{1}{4}\left\{\left(\log \frac{3}{7}\right) - \left(\log \frac{1}{3}\right)\right\}$$

$$= \frac{1}{4} \log\left(\frac{\frac{3}{7}}{\frac{1}{3}}\right)$$

$$= \frac{1}{4} \log \frac{9}{7}$$

EXERCISE 6.2

SUMS ON PROPERTIES

PROPERTY – 1

$$\int_{a}^{b} f(x)\ dx = \int_{a}^{b} f(t)\ dt$$

PROPERTY – 2

$$\int_{a}^{b} f(x)\ dx = -\int_{b}^{a} f(x)\ dx$$

PROPERTY – 3

$$\int_{a}^{b} f(x)\ dx = \int_{a}^{c} f(x)\ dx + \int_{c}^{b} f(x)\ dx$$

PROPERTY – 4

$$\int_{0}^{a} f(x)\ dx = \int_{0}^{a} f(a-x)\ dx$$

PROPERTY – 5

$$\int_{a}^{b} f(x)\ dx = \int_{a}^{b} f(a+b-x)\ dx$$

PROPERTY – 6

$$\int_{0}^{2a} f(x)\ dx = \int_{0}^{a} \left[f(x) + f(2a-x)\right]\ dx$$

PROPERTY – 7

$$\int_{a}^{-a} f(x)\ dx = 2\int_{0}^{a} f(x)\ dx \quad \ldots\ldots\ldots\ldots \quad \text{IF THE F(X) IS EVEN}$$

$$= 0 \quad \ldots\ldots\ldots\ldots \quad \text{IF THE F(X) IS ODD}$$

01.

$$I = \int_0^5 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{5-x}}\,dx \quad \text{.......... (1)}$$

USING $\int_0^a f(x)dx = \int_0^a f(a-x)\,dx$

CHANGE 'x' TO '5 – x'

$$I = \int_0^5 \frac{\sqrt{5-x}}{\sqrt{5-x} + \sqrt{5-(5-x)}}\,dx$$

$$I = \int_0^5 \frac{\sqrt{5-x}}{\sqrt{5-x} + \sqrt{x}}\,dx \quad \text{.......... (2)}$$

(1) + (2)

$$2I = \int_0^5 \frac{\sqrt{x} + \sqrt{5-x}}{\sqrt{x} + \sqrt{5-x}}\,dx$$

$$2I = \int_0^5 1\,dx$$

$$2I = \left[x\right]_0^5$$

$$2I = 5 - 0$$

$$2I = 5$$

$$I = \frac{5}{2}$$

02.

$$I = \int_2^5 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{7-x}}\,dx \quad \text{.......... (1)}$$

USING $\int_a^b f(x)dx = \int_b^a f(a+b-x)\,dx$

CHANGE 'x' TO '2+5 – x'

$$I = \int_2^5 \frac{\sqrt{7-x}}{\sqrt{7-x} + \sqrt{7-(7-x)}}\,dx$$

$$I = \int_2^5 \frac{\sqrt{7-x}}{\sqrt{7-x} + \sqrt{x}}\,dx \quad \text{.......... (2)}$$

(1) + (2)

$$2I = \int_2^5 \frac{\sqrt{x} + \sqrt{7-x}}{\sqrt{x} + \sqrt{7-x}}\,dx$$

$$2I = \int_2^5 1\,dx$$

$$2I = \left[x\right]_2^5$$

$$2I = 5 - 2$$

$$2I = 3$$

$$I = \frac{3}{2}$$

03.

$$I = \int_1^3 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5} + \sqrt[3]{9-x}}\,dx \quad \text{.....(1)}$$

USING $\int_a^b f(x)dx = \int_b^a f(a+b-x)\,dx$

CHANGE 'x' TO '1+3−x' i.e '4−x'

$$I = \int_1^3 \frac{\sqrt[3]{4-x+5}}{\sqrt[3]{4-x+5} + \sqrt[3]{9-(4-x)}}\,dx$$

$$I = \int_1^3 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x} + \sqrt[3]{9-4+x}}\,dx$$

$$I = \int_1^3 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x} + \sqrt[3]{5+x}}\,dx \quad \text{.....(2)}$$

(1) + (2)

$$2I = \int_1^3 \frac{\sqrt[3]{x+5} + \sqrt[3]{9-x}}{\sqrt[3]{x+5} + \sqrt[3]{9-x}}\,dx$$

$$2I = \int_1^3 1\,dx$$

$$2I = \left[x\right]_1^3$$

$$2I = 3 - 1$$

$$2I = 2$$

$$I = 1$$

04.

$$I = \int_0^a x^2(a-x)^{3/2}\,dx$$

USING $\int_0^a f(x)dx = \int_0^a f(a-x)\,dx$

$$I = \int_0^a (a-x)^2 . x^{3/2}\,dx$$

$$= \int_0^a (a^2 - 2ax + x^2).\,x^{3/2}\,dx$$

$$= \int_0^a (a^2x^{3/2} - 2ax^{5/2} + x^{7/2})dx$$

$$= \left[\frac{a^2x^{5/2}}{\frac{5}{2}} - \frac{2ax^{5/2}}{\frac{7}{2}} + \frac{x^{9/2}}{\frac{9}{2}}\right]_0^a$$

$$= \left[\frac{2a^2x^{5/2}}{5} - \frac{4ax^{7/2}}{7} + \frac{2x/^{9/2}}{9}\right]_0^a$$

$$= \frac{2a^{2+5/2}}{5} - \frac{4a^{1+7/2}}{7} + \frac{2a^{9/2}}{7}$$

$$= \frac{2a^{9/2}}{5} - \frac{4a^{9/2}}{7} + \frac{2a^{9/2}}{9}$$

$$= a^{9/2}\left[\frac{2}{5} - \frac{4}{7} + \frac{2}{9}\right]$$

$$= 2a^{9/2}\left[\frac{1}{5} - \frac{2}{7} + \frac{1}{9}\right]$$

$$= 2a^{9/2}\left[\frac{63 - 90 + 35}{315}\right]$$

$$= \frac{16a^{9/2}}{315}$$

05.

$$I = \int_{-9}^{9} \frac{x^3}{4 - x^2}\, dx$$

$$f(x) = \frac{x^3}{4 - x^2}$$

$$f(-x) = \frac{(-x)^3}{4 - (-x)^2}$$

$$f(-x) = \frac{-x^3}{4 - x^2}$$

$f(-x) = -f(x)$, f(x) is ODD

Hence $I = 0$

USING

$\int_{-a}^{a} f(x)\, dx = 0$ when f(x) is ODD

06.

$$I = \int_{-7}^{7} \frac{x^3}{x^2 + 7}\, dx$$

$$f(x) = \frac{x^3}{x^2 + 7}$$

$$f(-x) = \frac{(-x)^3}{(-x)^2 + 7}$$

$$f(-x) = \frac{-x^3}{x^2 + 7}$$

$f(-x) = -f(x)$, f(x) is ODD

Hence $I = 0$

USING

$\int_{-a}^{a} f(x)\, dx = 0$ when f(x) is ODD

PAPER - I

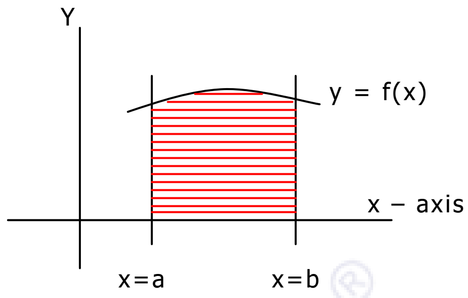
CHAPTER - 7

APPLICATION OF DEFINITE INTEGRATION

(AREA UNDER THE CURVE)

NOTES

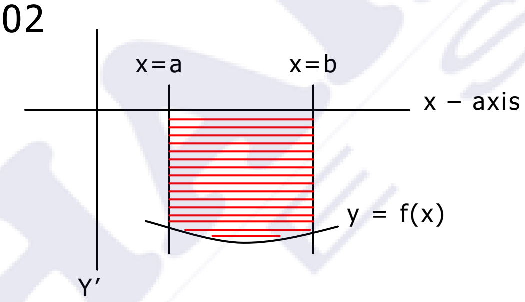
01



area of the shaded region

$$A = \int_a^b y \, dx$$

02

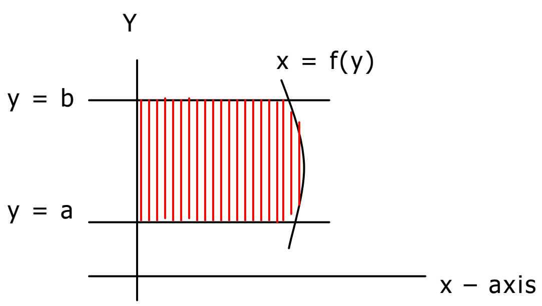


If the curve under consideration is below x − axis , then the area bounded by the curve , x − axis and lines x = a , x = b is negative . In such cases we consider the absolute value . Hence

area of the shaded region

$$A = \left| \int_a^b y \, dx \right|$$

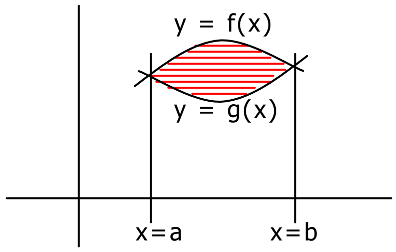
03



area of the shaded region

$$A = \int_a^b x \, dy$$

04



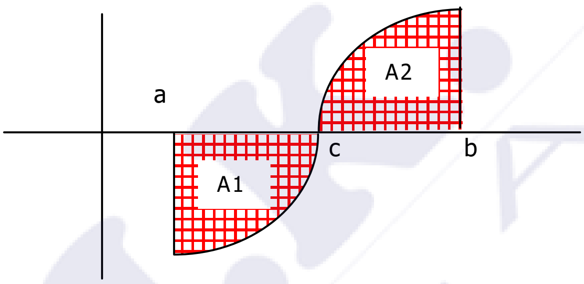
area of the shaded region bounded by two curves $y = f(x)$, $y = g(x)$ is obtained by

$$A = \left| \int_a^b f(x)dx - \int_a^b g(x)\,dx \right|$$

05

If the curve under consideration lies above as well as below the x – axis , say A_1 lies below the x – axis and A_2 lies above the x – axis as shown in the diagram , then A , the area of the region is given by

$A = A_1 + A_2$

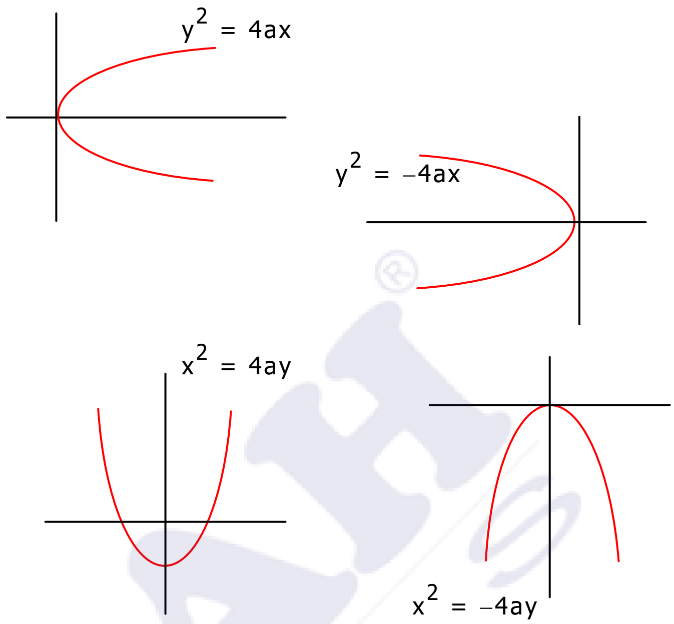


where

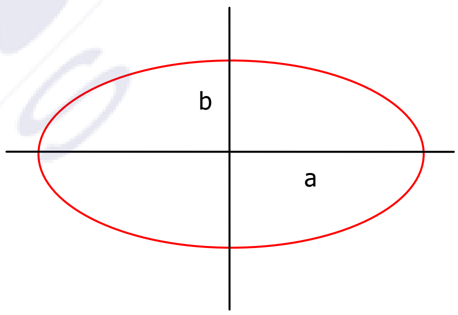
$$A_1 = \left| \int_a^c y\,dx \right|$$

$$A_2 = \int_c^b y\,dx$$

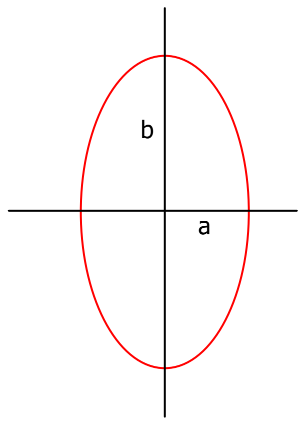
06 STANDARD FORMS OF PARABOLA



07 STANDARD FORMS OF ELLIPSE



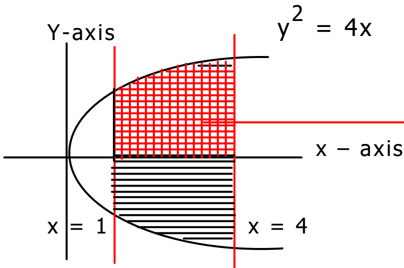
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad , \quad a > b$$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad , \quad a < b$$

01.

Find the area of the region bounded by the curve $y^2 = 4x$ & lines $x = 1$; $x = 4$



USING

$$A = \int_a^b y \, dx$$

$$A = 2\int_1^4 y \, dx \quad \text{..... (BY SYMMETRY)}$$

$$= 2\int_1^4 \sqrt{4x}\, dx$$

$$= 2\int_1^4 2\sqrt{x}\, dx$$

$$= 4\int_1^4 x^{1/2}\, dx$$

$$= 4\left[\frac{x^{3/2}}{\frac{3}{2}}\right]_1^4$$

$$= \frac{4}{3}\left[x^{3/2}\right]_1^4$$

$$= \frac{8}{3}\left[4^{3/2} - 1^{3/2}\right]$$

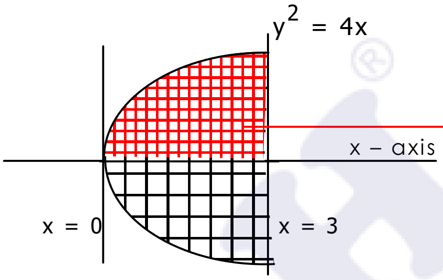
$$= \frac{8}{3}\left[4\sqrt{4} - 1\right]$$

$$= \frac{8}{3}(8 - 1)$$

$$= \frac{56}{3} \text{ sq. units}$$

02.

Find the area of the region bounded by parabola $y^2 = 4x$ and $x = 3$



USING

$$A = \int_a^b y \, dx$$

$$A = 2\int_0^3 y \, dx \quad \text{..... (BY SYMMETRY)}$$

$$= 2\int_0^3 \sqrt{4x}\, dx$$

$$= 2\int_0^3 2\sqrt{x}\, dx$$

$$= 4\int_0^3 x^{1/2}\, dx$$

$$= 4\left[\frac{x^{3/2}}{\frac{3}{2}}\right]_0^3$$

$$= \frac{8}{3}\left[x^{3/2}\right]_0^3$$

$$= \frac{8}{3}\left[3^{3/2} - 0^{3/2}\right]$$

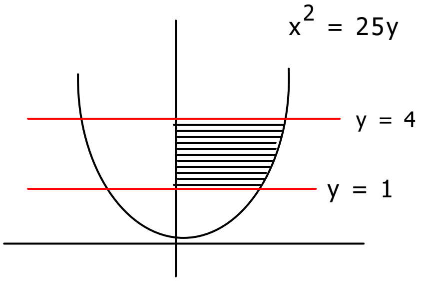
$$= \frac{8}{3}\left[3\sqrt{3}\right]$$

$$= 8\sqrt{3} \text{ sq. units}$$

3

03.

Find the area of the region bounded by the curve $x^2 = 25y$; $y = 1$; $y = 4$ and the y – axis lying in the I quadrant



USING

$$A = \int_b^a x \, dy$$

$$A = \int_1^4 x \, dy$$

$$= \int_1^4 \sqrt{25y} \, dy$$

$$= \int_1^4 5\sqrt{y} \, dy$$

$$= 5 \int_1^4 y^{1/2} \, dy$$

$$= 5 \left[\frac{y^{3/2}}{\frac{3}{2}} \right]_1^4$$

$$= \frac{10}{3} \left(y^{3/2} \right)_1^4$$

$$= \frac{10}{3} \left(4^{3/2} - 1^{3/2} \right)$$

$$= \frac{10}{3} \left(4\sqrt{4} - 1 \right)$$

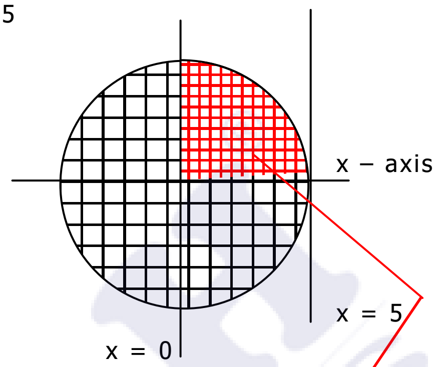
$$= \frac{10}{3} (8 - 1)$$

$= 70$ sq. units

04.

Find the area of the circle $x^2 + y^2 = 25$

$r = 5$



$$x^2 + y^2 = 25$$

$$y^2 = 25 - x^2$$

$$y = \sqrt{25 - x^2}$$

Area of Circle

$$= 4 \int_0^5 y \, dx \quad \text{....... BY SYMMETRY}$$

$$= 4 \int_0^5 \sqrt{5^2 - x^2} \, dx$$

$$= 4 \left[\frac{x}{2}\sqrt{5^2 - x^2} + \frac{5^2}{2} \sin^{-1}\left(\frac{x}{5}\right) \right]_0^5$$

$$= 4 \left\{ \left[\frac{5}{2}\sqrt{5^2 - 5^2} + \frac{25}{2} \sin^{-1}\left(\frac{5}{5}\right) \right] - \left[\frac{0}{2}\sqrt{5^2 - 0^2} + \frac{25}{2} \sin^{-1}\left(\frac{0}{5}\right) \right] \right\}$$

$$= 4 \left\{ \left(0 + \frac{25}{2}\sin^{-1}(1) \right) - \left(0 + \frac{25}{2}\sin^{-1}(0) \right) \right\}$$

$$= 4 \left(\frac{25}{2} \times \frac{\pi}{2} \right)$$

$= 25\pi$ sq. units

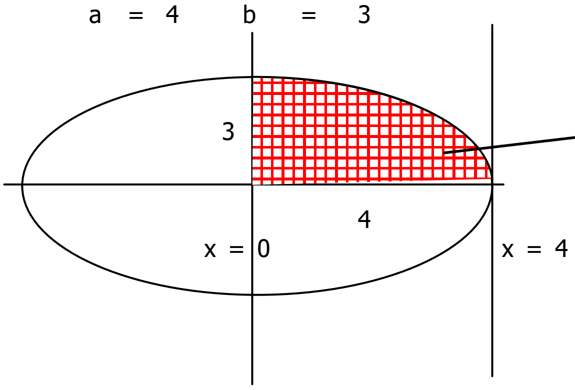
05.

Find the area of the ellipse :

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

$a^2 = 4 \qquad b^2 = 9$

$a = 4 \qquad b = 3$



$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

$$\frac{y^2}{9} = 1 - \frac{x^2}{16}$$

$$\frac{y^2}{9} = \frac{16 - x^2}{16}$$

$$y^2 = \frac{9}{16}(16 - x^2)$$

$$y = \frac{3}{4}\sqrt{16 - x^2}$$

Area of Ellipse

$$= 4\int_0^4 y\, dx \quad \text{....... BY SYMMETRY}$$

$$= 4\int_0^4 \frac{3}{4}\sqrt{16 - x^2}\, dx$$

$$= 3\int_0^4 \sqrt{4^2 - x^2}\, dx$$

$$= 3\left[\frac{x}{2}\sqrt{4^2 - x^2} + \frac{4^2}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$$

$$= 3\left[\frac{x}{2}\sqrt{4^2 - x^2} + 8\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$$

$$= 3\left\{\left[\frac{4}{2}\sqrt{4^2 - 4^2} + 8\sin^{-1}\left(\frac{4}{4}\right)\right] - \left[\frac{0}{2}\sqrt{4^2 - 0^2} + 8\sin^{-1}\left(\frac{0}{4}\right)\right]\right\}$$

$$= 3\left[0 + 8\sin^{-1}(1)\right] - \left[0 + 8\sin^{-1}(0)\right]$$

$$= 3\left(8 \times \frac{\pi}{2}\right)$$

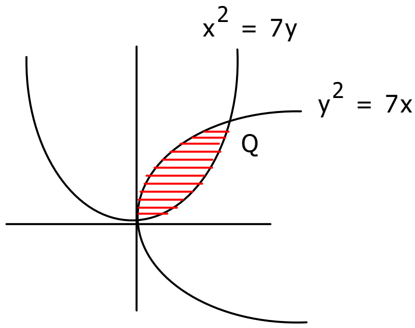
$$= 12\pi \text{ sq. units}$$

06.

Find the area between the parabolas

$y^2 = 7x$ & $x^2 = 7y$

REQUIRED AREA



STEP 1 : FOR Q SOLVE

$x^2 = 7y \quad y = \dfrac{x^2}{7}$ ….. (1)

$y^2 = 7x$ ….. (2)

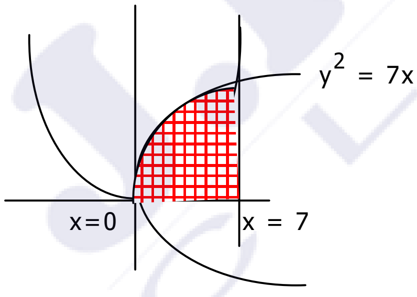
$\dfrac{x^4}{49} = 7x$

$x^3 = 49 \times 7 = 7^3$

$x = 7$ subs in (1)

$y = 7 \quad Q(7,7)$

STEP 2 : A_1



$A_1 = \displaystyle\int_0^7 y \, dx$

$= \displaystyle\int_0^7 \sqrt{7x} \, dx$

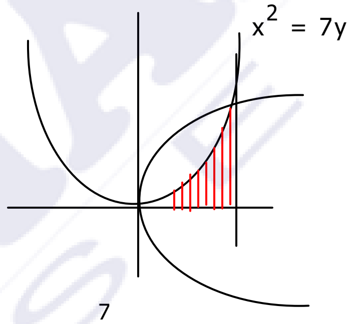
$= \sqrt{7} \displaystyle\int_0^7 x^{1/2} \, dx$

$= \sqrt{7} \left[\dfrac{x^{3/2}}{\frac{3}{2}} \right]_0^7$

$= \dfrac{2\sqrt{7}}{3} \left[x^{3/2} \right]_0^7$

$= \dfrac{2\sqrt{7}}{3} \left[7\sqrt{7} \right] = \dfrac{98}{3}$ sq. units

STEP 3 : A_2



$A_2 = \displaystyle\int_0^7 y \, dx$

$= \displaystyle\int_0^7 \dfrac{x^2}{7} \, dx$

$= \left[\dfrac{x^3}{21} \right]_0^7$

$= \dfrac{7^3}{21}$

$= \dfrac{49}{3}$ sq. units

STEP 4 : REQUIRED AREA

$A = A_1 - A_2$

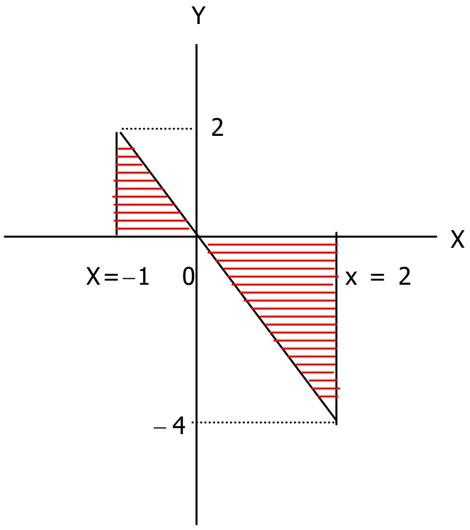
$= \dfrac{98}{3} - \dfrac{49}{3}$

$= \dfrac{49}{3}$ sq. units

07.

Find area of the region bounded by

$y = -2x$, $x = -1$, $x = 2$, x – axis



NOTE : In general when all 4 things are given i.e. y , lines x = a , x = b and the x- axis , we happen to find the area directly using

$$A = \int_b^a y\,dx$$

But in the above sum , as seen A_2 goes below the x – axis . Area of these region will be negative . Hence if we use the above , A_2 will cut A_1 and the area obtained will be much less than what is the actual area .

Plan is , we find A_1 and A_2 seperately and then add to get the required area . Also pl note , while finding area A_2 we use modulus to keep the area positive

$$A_1 = \int_{-1}^{0} y\,dx$$

$$= \int_{-1}^{0} -2x\,dx$$

$$= \left[\frac{-2x^2}{2}\right]_{-1}^{0}$$

$$= -\left[x^2\right]_{-1}^{0}$$

$$= -[(0) - (1)]$$

$$= 1 \text{ sq . unit}$$

$$A_2 = \left|\int_0^2 y\,dx\right|$$

$$= \left|\int_0^2 -2x\,dx\right|$$

$$= \left|\left[\frac{-2x^2}{2}\right]_0^2\right|$$

$$= \left[x^2\right]_0^2$$

NOTE : MODULUS WAS EXECUTED TO GET RID OF – SIGN

$$= [(4) - (0)]$$

$$= 4 \text{ sq . units}$$

REQUIRED AREA

$$A = A_1 + A_2 = 5 \text{ sq units}$$

PAPER - I

CHAPTER - 8

DIFFERENTIAL EQUATIONS

EXERCISE - 8.1

ORDER AND DEGREE OF A DIFFERENTIAL EQUATION

EXERCISE - 8.2

FORM D.E. BY ELIMINATING THE ARBITRARY CONSTANTS

EXERCISE - 8.3

SOLVE D.E. USING VARIABLE SEPARATION METHOD

EXERCISE - 8.4

HOMOGENOUS DIFFERENTIAL EQUATIONS

EXERCISE - 8.5

LINEAR DIFFERENTIAL EQUATIONS

EXERCISE - 8.6

APPLICATION OF DIFFERENTIAL EQUATION

EXERCISE - 8.1

ORDER AND DEGREE OF A DIFFERENTIAL EQULATION

ORDER OF DIFFERENTIAL EQUATION

HIGHEST ORDER DERIVATIVE IN THE DIFFERENTIAL EQUATION BECOMES THE ORDER OF D.E.

DEGREE OF A DIFFERENTIAL EQUATION

IT IS THE POWER OF THE HIGHEST ORDER DERIVATIVE IN D.E. WHEN ALL THE DERIVATIVES ARE MADE FREE FROM FRACTIONAL INDICES AND –VE SIGN

	ORDER OF D.E.	DEGREE OF D.E.
1. $\frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)^2 + 8 = 0$	2	1
2. $\frac{d^4y}{dx^4} + \left(1 + \left(\frac{dy}{dx}\right)^2\right)^3$	4	1
3. $\frac{d^2y}{dx^2} = \sqrt[3]{1 + \left(\frac{dy}{dx}\right)^2}$ CUBING $\left(\frac{d^2y}{dx^2}\right)^3 = 1 + \left(\frac{dy}{dx}\right)^2$	2	3
4. $\sqrt{1 + \frac{1}{\left(\frac{dy}{dx}\right)^2}} = \left(\frac{dy}{dx}\right)^{3/2}$ SQUARING $1 + \frac{1}{\left(\frac{dy}{dx}\right)^2} = \left(\frac{dy}{dx}\right)^3$ $\left(\frac{dy}{dx}\right)^2 + 1 = \left(\frac{dy}{dx}\right)^5$	1	5
5. $\left\{1+\left(\frac{dy}{dx}\right)^3\right\}^{2/3} = 8\frac{d^3y}{dx^3}$ CUBING $\left\{1+\left(\frac{dy}{dx}\right)^3\right\}^2 = 8^3\left(\frac{d^3y}{dx^3}\right)^3$	3	3

EXERCISE - 8.2

FORM D.E. BY ELIMINATING THE ARBITRARY CONSTANTS

01. $x^2 + y^2 = 2ax$

$x^2 + y^2 = 2ax$ ...... (1)

Differentiating wrt x

$2x + 2y\frac{dy}{dx} = 2a$ ...... (2)

subs (2) in (1)

$x^2 + y^2 = \left(2x + 2y\frac{dy}{dx}\right)x$

$x^2 + y^2 = 2x^2 + 2xy\frac{dy}{dx}$

$x^2 + 2xy\frac{dy}{dx} - y^2 = 0$ .... REQD D.E.

02. $y = Ae^{3x} + Be^{-3x}$

$y = Ae^{3x} + Be^{-3x}$

Differentiating wrt x ,

$\frac{dy}{dx} = Ae^{3x}(3) + Be^{-3x}(-3)$

$\frac{dy}{dx} = 3Ae^{3x} - 3Be^{-3x}$

Differentiating once again wrt x ,

$\frac{d^2y}{dx^2} = 3Ae^{3x}(3) - 3Be^{-3x}(-3)$

$\frac{d^2y}{dx^2} = 9Ae^{3x} + 9Be^{-3x}$

$\frac{d^2y}{dx^2} = 9(Ae^{3x} + Be^{-3x})$

$\frac{d^2y}{dx^2} = 9y$ ...... REQD. D.E.

03. EXERCISE 8.2 Q1(iv) , Pg 163

Obtain D.E. by eliminating arbitrary constants from the following equation

$y = c_1e^{3x} + c_2e^{2x}$

$y = c_1e^{3x} + c_2e^{2x}$ → ISOLATE c_2

Dividing throughout by e^{2x}

$\frac{y}{e^{2x}} = \frac{c_1e^{3x}}{e^{2x}} + \frac{c_2e^{2x}}{e^{2x}}$

$ye^{-2x} = c_1e^{x} + c_2$

Differentiating wrt x

$y\frac{d}{dx}e^{-2x} + e^{-2x}\frac{dy}{dx} = c_1e^{x}$

$ye^{-2x}(-2) + e^{-2x}\frac{dy}{dx} = c_1e^{x}$

$e^{-2x}\left(\frac{dy}{dx} - 2y\right) = c_1e^{x}$ → ISOLATE C_1

Dividing throughout by e^{x}

$\frac{e^{-2x}}{e^{x}}\left(\frac{dy}{dx} - 2y\right) = c_1$

$e^{-3x}\left(\frac{dy}{dx} - 2y\right) = c_1$

Differentiating once again wrt x ,

$e^{-3x}\frac{d}{dx}\left(\frac{dy}{dx} - 2y\right) + \left(\frac{dy}{dx} - 2y\right)\frac{d}{dx}e^{-3x} = 0$

$e^{-3x}\left(\frac{d^2y}{dx^2} - \frac{2dy}{dx}\right) + \left(\frac{dy}{dx} - 2y\right)e^{-3x}(-3) = 0$

$\frac{d^2y}{dx^2} - \frac{2dy}{dx} - 3\frac{dy}{dx} + 6y = 0$

$\frac{d^2y}{dx^2} - \frac{5dy}{dx} + 6y = 0$

FROM 02 WE COME TO CONCLUSION ,

IF THE POWERS OF E ARE SAME AND OF OPPOSITE SIGN , WE JUST NEED TO DIFFERENTIATE 2 TIMES AND IN THE END REPLACE WITH Y TO ELIMINATE THE ARBITARY CONSTANTS

FROM 03 WE COME TO CONCLUSION

IF THE POWERS OF E ARE NOT SAME , THEN ISOLATE THE CONSTANT AND DIFFERENTIATE TO ELIMINATE THEM

EXERCISE - 8.3

SOLVE D.E. USING VARIABLE SEPARATION METHOD

01. $y - x\frac{dy}{dx} = 0$

$$y = x\frac{dy}{dx}$$

$$ydx = xdy$$

$$\frac{dx}{x} = \frac{dy}{y} \quad \text{.... VARIABLES SEPARATED}$$

INTEGRATING BOTH SIDES

$$\int \frac{1}{x}\,dx = \int \frac{1}{y}\,dy$$

$$\log x = \log y + \log c$$

$$\log x = \log cy$$

$$x = cy \quad \text{...... REQD SOLUTION}$$

02.

$$\frac{dy}{dx} = \frac{1+y}{1+x}$$

$$dy = \frac{1+y}{1+x}\,dx$$

$$\frac{dy}{1+y} = \frac{dx}{1+x} \quad \text{.... VARIABLES SEPARATED}$$

INTEGRATING BOTH SIDES

$$\int \frac{1}{1+y}\,dy = \int \frac{1}{1+x}\,dx$$

$$\log|1+y| = \log|1+x| + \log c$$

$$\log|1+y| = \log c|1+x|$$

$$1+y = c(1+x) \quad \text{...... REQD SOLUTION}$$

03.

Find the particular solution when x = 2 and y = 0

$(x+y^2x)dx + (y + x^2y)dy = 0$

$x(1+y^2)dx + y(1+x^2)dy = 0$

$\frac{x}{1+x^2}dx + \frac{y}{1+y^2}dy = 0$ .... VARIABLES SEPARATED

INTEGRATING BOTH SIDES

$\int \frac{x}{1+x^2}dx + \int \frac{y}{1+y^2}dy = c$

MULTIPLYING THROUGH OUT BY 2

$\int \frac{2x}{1+x^2}dx + \int \frac{2y}{1+y^2}dy = c$

NOTE : 2 GOT ADJUSTED IN C

USING $\int \frac{f'(x)}{f(x)}dx = \log|f(x)| + c$

$\log|1+x^2| + \log|1+y^2| = \log C$

$\log|1+x^2||1+y^2| = \log C$

$(1+x^2)(1+y^2) = C$ ....GENERAL SOLN

Put x = 2 , y = 0

$(1+4)(1 + 0) = C$

$C = 5$

$(1+x^2)(1+y^2) = 5$ ....PARTICULAR SOLN

04. EXERCISE 8.3 – 2(iv) , Pg 165

SOLVE

$\frac{dy}{dx} = 4x + y + 1$ when y = 1 & x = 0

NOTE : WHEN dx IS PUSHED ON THE FLOOR , 4X + Y + 1 GETS HOOKED WITH dx . Y CAN THEN NOT BE SEPERATED FROM dx AND HENCE WE DECIDE TO GO FOR SUBSTITUTION

PUT $4x + y + 1 = u$

$4 + \frac{dy}{dx} = \frac{du}{dx}$

$\frac{dy}{dx} = \frac{du}{dx} - 4$

BACK INTO THE SUM

$\frac{du}{dx} - 4 = u$

$\frac{du}{dx} = u + 4$

$\frac{du}{u+4} = dx$ .... VARIABLES SEPERATED

INTEGRATING BOTH SIDES

$\int \frac{1}{u+4}du = \int dx$

$\log|u+4| = x + c$

RESUBS

$\log|4x+y+5| = x + c$ .... GENERAL SOLN

PUT x = 0 , y = 1

$\log 6 = c$

SUBS IN THE GENERAL SOLUTION

$\log|4x+y+5| = x + \log 6$

$x = \log\left|\frac{4x+y+5}{6}\right|$ ... PARTICULAR SOLN

EXERCISE - 8.4

HOMOGENOUS DIFFERENTIAL EQUATIONS

STEP 1 MOVE THE TERMS TO GET TO

$$\frac{dy}{dx} = f(y/x)$$

STEP 2 SUBSTITUTE

$$\frac{y}{x} = u$$

$$y = ux$$

$$\frac{dy}{dx} = u + x\frac{du}{dx}$$

STEP 3 USE VARIABLE SEPERATION METHOD AND INTEGRATE TO REACH TO THE SOLUTION

STEP 4 RESUBSTITUTE

SOLVE THE D.E. OR FIND THE SOLUTION TO THE GIVEN D.E.

01.

$$xy\frac{dy}{dx} = x^2 + 2y^2$$

$$\frac{dy}{dx} = \frac{x^2 + 2y^2}{xy}$$

$$\frac{dy}{dx} = \frac{x}{y} + 2\frac{y}{x}$$

SUBS $\frac{y}{x} = u$

$$y = ux$$

$$\frac{dy}{dx} = u + x\frac{du}{dx}$$

$$u + x\frac{du}{dx} = \frac{1}{u} + 2u$$

$$x\frac{du}{dx} = \frac{1}{u} + 2u - u$$

$$x\frac{du}{dx} = \frac{1}{u} + u$$

$$x\frac{du}{dx} = \frac{1 + u^2}{u}$$

$$x\,du = \frac{1 + u^2}{u}\,dx$$

$$\frac{u}{1+u^2}\,du = \frac{dx}{x} \quad \text{.... VARIABLES SEPARATED}$$

INTEGRATING BOTH SIDES

$$\int \frac{u\,du}{1+u^2} = \int \frac{1}{x}\,dx$$

MULTIPLYING BOTH SIDES WITH '2'

$$\int \frac{2u}{1+u^2}\,du = 2\int \frac{1}{x}\,dx$$

USING $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + c$

$$\log|1+u^2| = 2\log|x| + \log c$$

$$\log|1+u^2| = \log|x^2| + \log c$$

$$\log|1+u^2| = \log|cx^2|$$

$$1+u^2 = cx^2$$

RESUBS

$$1+\frac{y^2}{x^2} = cx^2$$

$$x^2 + y^2 = cx^4 \quad \text{....... GENERAL SOLN}$$

02.

$x^2y\,dx - (x^3 + y^3)\,dy = 0$

$x^2y\,dx = (x^3 + y^3)\,dy$

$\frac{dy}{dx} = \frac{x^2y}{x^3+y^3}$

$\frac{dy}{dx} = \frac{1}{\frac{x^3+y^3}{x^2y}}$

$\frac{dy}{dx} = \frac{1}{\frac{x}{y} + \frac{y^2}{x^2}}$

SUBS $\frac{y}{x} = u$

$y = ux$

$\frac{dy}{dx} = u + x\frac{du}{dx}$

$u + x\frac{du}{dx} = \frac{1}{\frac{1}{u} + u^2}$

$u + x\frac{du}{dx} = \frac{u}{1 + u^3}$

$x\frac{du}{dx} = \frac{u}{1 + u^3} - u$

$x\frac{du}{dx} = \frac{u - u - u^4}{1 + u^3}$

$x\frac{du}{dx} = \frac{-u^4}{1 + u^3}$

$x\,du = \frac{-u^4\,dx}{1 + u^3}$

$\frac{1 + u^3}{u^4}\,du = -\frac{1}{x}\,dx$

.... VARIABLES SEPARATED

INTEGRATING BOTH SIDES

$\int \frac{1 + u^3}{u^4}\,du = -\int \frac{1}{x}\,dx$

$\int \left(\frac{1}{u^4} + \frac{u^3}{u^4}\right) du = -\int \frac{1}{x}\,dx$

$\int \left(u^{-4} + \frac{1}{u}\right) du = -\int \frac{1}{x}\,dx$

$\frac{u^{-3}}{-3} + \log|u| = -\log|x| + C$

$\frac{1}{-3u^3} + \log|u| + \log|x| = C$

$\log|ux| + \frac{1}{-3u^3} = C$

RESUBS

$\log\left(\frac{yx}{x}\right) - \frac{x^3}{3y^3} = C$

$\log y - \frac{x^3}{3y^3} = C$ .... GENERAL SOLN

EXERCISE - 8.5

LINEAR DIFFERENTIAL EQUATIONS

TYPE – I

1. $\frac{dy}{dx} + Py = Q$, P & Q are f(x) or constants

2. FIND IF = $e^{\int}$

3. MULTIPLY THE D.E. WITH THE IF

4. ON DOING SO , THE LHS OF THE D.E. WILL GET ADJUSTED TO $\frac{d}{dx}(y.IF)$

5. OUR D.E. NOW LOOKS
$\frac{d}{dx}(y.IF) = Q.IF$

6. FINALLY TO REACH THE SOLUTION INTEGRATE BOTH SIDES WRT X
$\int \frac{d}{dx}(y.IF)dx = \int Q.IF\, dx + C$
$y.IF = \int Q.IF\, dx + C$

TYPE – II

1. $\frac{dx}{dy} + Px = Q$, P & Q are f(y) or constants

2. FIND IF = $e^{\int}$

3. MULTIPLY THE D.E. WITH THE IF

4. ON DOING SO , THE LHS OF THE D.E. WILL GET ADJUSTED TO $\frac{d}{dy}(x.IF)$

5. OUR D.E. NOW LOOKS
$\frac{d}{dy}(x.IF) = Q.IF$

6. FINALLY TO REACH THE SOLUTION INTEGRATE BOTH SIDES WRT Y
$\int \frac{d}{dy}(x.IF)dy = \int Q.IF\, dy + C$
$x.IF = \int Q.IF\, dy + C$

SOLVE THE D.E. OR FIND THE SOLUTION TO THE GIVEN D.E.

01. $\frac{dy}{dx} + y = e^{-x}$

$\frac{dy}{dx} + Py = Q$, $P = 1$

$IF = e^{\int P dx} = e^{\int 1 dx} = e^{x}$

MULTIPLYING THE D.E. BY THE IF

$e^{x}\frac{dy}{dx} + y\,e^{x} = e^{x}.\,e^{-x}$

$\frac{d}{dx}(y.IF)$

$\frac{d}{dx}(y\,e^{x}) = 1$

INTEGRATING BOTH SIDES wrt X

$\int \frac{d}{dx}(y\,e^{x}) = x + C$

$y\,e^{x} = x + C$ ……. GENERAL SOLN

02.

$\frac{dy}{dx} = x + y$

$\frac{dy}{dx} - y = x$

$\frac{dy}{dx} + Py = Q$, $P = -1$

$IF = e^{\int P dx} = e^{\int -1 dx} = e^{-x}$

MULTIPLYING THE D.E. BY THE IF

$e^{-x}\frac{dy}{dx} + y\,e^{-x} = xe^{-x}$

$\frac{d}{dx}(y.IF)$

$\frac{d}{dx}(y\,e^{-x}) = xe^{-x}$

INTEGRATING BOTH SIDES wrt X

$ye^{-x} = x\int e^{-x}dx - \int\left[\frac{d}{dx}x.\int e^{-x}\,dx\right]dx + C$

$ye^{-x} = \frac{xe^{-x}}{-1} - \int\frac{e^{-x}}{-1}\,dx + C$

$ye^{-x} = -xe^{-x} + \int e^{-x}\,dx + C$

$ye^{-x} = -xe^{-x} + \frac{e^{-x}}{-1} + C$

$ye^{-x} = -xe^{-x} - e^{-x} + C$

$ye^{-x} + xe^{-x} + e^{-x} = C$

$e^{-x}(y + x + 1) = C$

$x + y + 1 = ce^{x}$ ……. GENERAL SOLN

03.

$$\frac{dy}{dx} + \frac{y}{x} = x^3 - 3$$

$$\frac{dy}{dx} + Py = Q \quad , \quad P = \frac{1}{x}$$

$$IF = e^{\int P dx} = e^{\int {}^{1}/_{x}\, dx} = e^{\log_e x}$$

$$= x$$

MULTIPLYING THE D.E. BY THE IF

$$x\frac{dy}{dx} + y = x^4 - 3x$$

${}^{d}/_{dx}(y.IF)$

$$\frac{d}{dx}(xy) = x^4 - 3x$$

INTEGRATING BOTH SIDES wrt X

$$\int \frac{d}{dx}(xy)\ dx = \int (x^4 - 3x)\ dx + C$$

$$xy = x^5 - 3x^2 + C$$

04.

$$x\frac{dy}{dx} + 2y = x^2 \log x$$

$$\frac{dy}{dx} + \frac{2y}{x} = \frac{x^2 \log x}{x}$$

$$\frac{dy}{dx} + \frac{2y}{x} = x\log x$$

$$\frac{dy}{dx} + Py = Q \quad , \quad P = \frac{2}{x}$$

$$IF = e^{\int P dx} = e^{\int {}^{2}/_{x}\, dx} = e^{2\log_e x}$$

$$= e^{\log_e x^2}$$

$$= x^2$$

MULTIPLYING THE D.E. BY THE IF

$$x^2\frac{dy}{dx} + 2xy = x^3 \log x$$

${}^{d}/_{dx}(y.IF)$

$$\frac{d}{dx}(y.x^2) = x^3 \log x$$

INTEGRATING BOTH SIDES wrt X

$$\int \frac{d}{dx}(y.x^2)dx = \int x^3 \log x \ .\ dx + C$$

LIATE

$$yx^2 = \int \log x . x^3\ dx + C$$

$$yx^2 = \log x \int x^3\ dx - \int\left[\frac{d}{dx}\log x \int x^3\ dx\right]dx$$

$$yx^2 = \log x\ \frac{x^4}{4} - \int \frac{1}{x}\ \frac{x^4}{4}\ dx + C$$

$$yx^2 = \frac{x^4}{4}\log x - \int \frac{x^3}{4}\ dx + C$$

$$yx^2 = \frac{x^4}{4}\log x - \frac{x^4}{16} + C$$

Q - 2

01.

$$(x+y)\frac{dy}{dx} = 1$$

$$\frac{dx}{dy} = x + y$$

$$\frac{dx}{dy} - x = y$$

$$\frac{dx}{dy} + Px = Q \quad , \quad P = -1$$

$$IF = e^{\int P dy} = e^{\int -1 dy} = e^{-y}$$

MULTIPLYING THE D.E. BY THE IF

$$e^{-y}\frac{dx}{dy} - x\,e^{-y} = ye^{-y}$$

$\frac{d}{dy}(x.IF)$

$$\frac{d}{dy}(x\,e^{-y}) = ye^{-y}$$

INTEGRATING BOTH SIDES wrt y

$$xe^{-y} = y\int e^{-y}dy - \int\left[\frac{d}{dy}y.\int e^{-y}\,dy\right]dy + C$$

$$xe^{-y} = \frac{ye^{-y}}{-1} - \int \frac{e^{-y}}{-1}\,dy + C$$

$$xe^{-y} = -ye^{-y} + \int e^{-y}\,dy + C$$

$$xe^{-y} = -ye^{-y} + \frac{e^{-y}}{-1} + C$$

$$xe^{-y} = -ye^{-y} - e^{-y} + C$$

$$xe^{-y} + ye^{-y} + e^{-y} = C$$

$$e^{-y}(x + y + 1) = C$$

$x + y + 1 = ce^{y}$ …….. GENERAL SOLN

02.

$$(x + 2y^3)\frac{dy}{dx} = y$$

$$\frac{dx}{dy} = \frac{x + 2y^3}{y}$$

$$\frac{dx}{dy} = \frac{x}{y} + 2y^2$$

$$\frac{dx}{dy} - \frac{x}{y} = 2y^2$$

$$\frac{dx}{dy} + Px = Q \quad , \quad P = \frac{-1}{y}$$

$$IF = e^{\int P dy} = e^{\int {}^{-1}/_{y}\, dy} = e^{-\log_e y}$$

$$= e^{\log_e y^{-1}}$$

$$= y^{-1} = \frac{1}{y}$$

MULTIPLYING THE D.E. BY THE IF

$$\frac{1}{y}\frac{dx}{dy} - \frac{x}{y^2} = 2y$$

$\frac{d}{dy}(x.IF)$

$$\frac{d}{dy}\left[x.\frac{1}{y}\right] = 2y$$

INTEGRATING BOTH SIDES wrt y

$$\int\frac{d}{dy}\left[x.\frac{1}{y}\right] = \int 2y.dy + C$$

$$\frac{x}{y} = \frac{2y^2}{2} + C$$

$$\frac{x}{y} = y^2 + C$$

$x = y(y^2 + C)$ …….. GENERAL SOLN

EXERCISE - 8.6

APPLICATION OF DIFFERENTIAL EQUATION

01.

Bacteria increased at the rate proportional to the number of bacteria present . If original number N doubles in 3 hours , find in how many hours , the number of bacteria will be 4N

Let x be the number of bacteria present at any time 't'

$\frac{dx}{dt} \propto x$

$\frac{dx}{dt} = kx$

$\frac{dx}{x} = kdt$ ……. VARIABLES SEPARATED

$\int \frac{dx}{x} = k\int dt$

$\boxed{\log x = kt + C}$

at t = 0 , x = N

$\log N = C$

$\boxed{\log x = kt + \log N}$

at t = 3 , x = 2N

$\log 2N = k(3) + \log N$

$\log 2N - \log N = 3k$

$\log\left(\frac{2N}{N}\right) = 3k$

$k = \frac{1}{3}\log 2$

$\boxed{\log x = \frac{t}{3}\log 2 + \log N}$

x = 4N , t = ?

$\log 4N = \frac{t}{3}\log 2 + \log N$

t →	0	3	6
x →	N	2N	4N

WOW ! YOU HAVE CRACKED THE ANSWER ORALLY

$\log 4N - \log N = \frac{t}{3}\log 2$

$\log\left(\frac{4N}{N}\right) = \frac{t}{3}\log 2$

$\log 4 = \frac{t}{3}\log 2$

$\log 2^2 = \frac{t}{3}\log 2$

$2\log 2 = \frac{t}{3}\log 2$

$2 = \frac{t}{3}$

$t = 6$ hrs

Bacteria will become 4N in 6 hrs

02.

In a certain culture of bacteria , rate of increase is proportional to number present . If it is found that number doubles in 4 hrs , find the number of times the bacteria are increased in 12 hrs

t	→	0	4	8	12
x	→	N	2N	4N	8N

WOW ! YOU HAVE CRACKED THE ANSWER ORALLY

Let x be the number of bacteria present at any time 't'

$$\frac{dx}{dt} \propto x$$

$$\frac{dx}{dt} = kx$$

$$\frac{dx}{x} = kdt$$ ……. VARIABLES SEPARATED

$$\int \frac{dx}{x} = k\int dt$$

$$\boxed{\log x = kt + C}$$

at $t = 0$, $x = N$

$$\log N = C$$

$$\boxed{\log x = kt + \log N}$$

at $t = 4$, $x = 2N$

$$\log 2N = k(4) + \log N$$

$$\log 2N - \log N = 3k$$

$$\log \left(\frac{2N}{N}\right) = 4k$$

$$k = \frac{1}{4} \log 2$$

$$\boxed{\log x = \frac{t}{4} \log 2 + \log N}$$

$t = 12$, $x = ?$

$$\log x = \frac{12}{4} \log 2 + \log N$$

$$\log x = 3 \log 2 + \log N$$

$$\log x = \log 2^3 + \log N$$

$$\log x = \log 8N$$

$$x = 8N$$

Bacteria will become 8N in 12 hrs

03.

Population of a town increases at a rate proportional to the population at that time . IF the population increases from 40 thousand to 60 thousand in 40 years , what will be the population in another 20 years (GIVEN $\sqrt{3/2} = 1.2247$)

Let P be the population at any time 't'

$$\frac{dP}{dt} \propto P$$

$$\frac{dP}{dt} = kP$$

$$\frac{dP}{P} = k\,dt$$

.... VARIABLES SEPARATED

$$\int \frac{dP}{P} = k\int dt$$

$$\log P = kt + C$$

at t = 0 , P = 40000

$$\log 40000 = C$$

$$\log P = kt + \log 40000$$

at t = 40 , P = 60000

$$\log 60000 = k(40) + \log 40000$$

$$\log 60000 - \log 40000 = 40k$$

$$\log\left(\frac{60000}{40000}\right) = 40k$$

$$k = \frac{1}{40}\log\left(\frac{3}{2}\right)$$

$$\log P = \frac{t}{40}\log\left(\frac{3}{2}\right) + \log 40000$$

t = 60 , P = ?

$$\log P = \frac{60}{40}\log\left(\frac{3}{2}\right) + \log 40000$$

$$\log P = \frac{3}{2}\log\left(\frac{3}{2}\right) + \log 40000$$

$$\log P = \log\left(\frac{3}{2}\right)^{3/2} + \log 40000$$

$$\log P = \log\left\{\left(\frac{3}{2}\right)^{3/2} . 40000\right\}$$

$$P = \left(\frac{3}{2}\right)^{3/2} . 40000$$

$$P = \frac{3}{2} \times \sqrt{\frac{3}{2}} \times 40000$$

$$P = 60000 \times 1.2247$$

$$P = 60000 \times \frac{12247}{10000}$$

$$P = 73,482$$

04.

The rate of disintegration of a radioactive element at time 't' is proportional to its mass at that time . The original mass of 800 grams will disintegrate into its mass of 400 grams after 5 days . Find mass remaining after 30 days

Let x be the number of bacteria present at any time 't'

$$\frac{dx}{dt} \alpha \; x$$

$$\frac{dx}{dt} = -kx$$

$$\frac{dx}{x} = -kdt$$ .... VARIABLES SEPARATED

$$\int \frac{dx}{x} = -k\int dt$$

$$\boxed{\log x = -kt + C}$$

at t = 0 , x = 800

$$\log 800 = C$$

$$\boxed{\log x = -kt + \log 800}$$

at t = 5 , x = 400

$$\log 400 = -k(5) + \log 1000$$

$$\log 400 - \log 800 = -5k$$

$$\log\left(\frac{400}{800}\right) = -5k$$

$$-k = \frac{1}{5}\log\left(\frac{1}{2}\right)$$

$$\boxed{\log x = \frac{t}{5} \log\left(\frac{1}{2}\right) + \log 800}$$

t = 30 , x = ?

$$\log x = \frac{30}{5} \log\left(\frac{1}{2}\right) + \log 800$$

$$\log x = 6 \log\left(\frac{1}{2}\right) + \log 800$$

$$\log x = \log\left(\frac{1}{2}\right)^6 + \log 800$$

$$\log x = \log\left(\frac{1}{2}\right)^6 .800$$

$$x = \left(\frac{1}{2}\right)^6 .800$$

$$x = \frac{800}{64}$$

$$x = \frac{100}{8}$$

$$x = 12.5 \text{ grams}$$

HSC RESULT 19-20

Name	Percentage
Khushbu Mali	95.54
Priyanka Udeshi	94.92
Smruti Suresh Jagdale	94.92
Nidhi Dhanani	94.77
Ishika Pravin Sanghavi	94.62
Vansh Vora	94.46
Aishwarya Vijay Badhe	94.46
Khushi Vipul Darji	94.46
Kushal Thakkar	94.46
Sampreeth Jayantha Poojary	94.31
Janahvi Bharat Dayare	94.31
Kriti Khatri	94.31
Sindhu Umesh Gawde	94.31
Dhruvi Sanghvi	94.31
Gautami Taggerse	94.15
Sudhanshu Singh	94.15
Komal Jitesh Gandhi	94.15
Vedika Mediboina	94.15
Sharvari Dilip Sawant	94.15
Rashi Sanjay Jain	94.15
Saniya Kulkarni	94.15
Rochelle Menezes	94.15
Aditi Mogaveera	94.00
Arundatii Singh	94.00
Yukta Sukerkar	94.00
Megha J Hinduja	94.00
Shreya Harlalka	93.85
Mansi Kadian	93.85
Sakshi Shankar Sudrik	93.85
Ridhi Ajit Rikame	93.85
Rutika Vartak	93.69
Vaedik Khatod	93.69
Bhavya Bhandari	93.69
Vidisha Shetty	93.69
Parth Dubey	93.54
Rohan Subramanian	93.54
Kervi Singhvi	93.54
Diya Khaturia	93.54
Hetal Poonamchand Hingad	93.54
Anushka M Dalvi	93.54
Jay Singh	93.54
Saras Sali	93.54
Yashasvi Maheshwari	93.38
Livya Noronha	93.38
Ishita Kute	93.38
Khushi Agrawal	93.38
Khushboo Shah	93.38
Khushee Shah	93.23
Deep Jayesh Gada	93.23
Siddharth Manoj Sethia	93.23
Aditya Kanal	93.23
Kosha Shah	93.23
Roshni Keshav Iddya	93.23
Neha Motwani	93.23
Parth Agarwal	93.23

Name	Percentage
Netri Shah	93.23
Sakshi Navin Shetty	93.23
Parth Patki	93.23
Pratham Shah	93.08
Prishita Shah	93.08
Prachi Parkar	93.08
Pratishta Pravin Shetty	93.08
Pallavi Jha	93.08
Nameera Ahmed	93.08
Shreya Bharat Jain	93.08
Yashvi	93.08
Sakshi Kothari	92.92
Khushi Nayan Makadia	92.92
Kashti Mehta	92.92
Kevin Patel	92.77
Priyanshi Mihir Shah	92.77
Prabhankit Shinde	92.77
Krupa Bidye	92.77
Prasham Gandhi	92.77
Nisha Surendra Rai	92.77
Devank S. Mayekar	92.77
Abhishek Dhuri	92.77
Shravani Wabekar	92.77
Shreya Niranjan Bhorawat	92.77
Tithi Parmar	92.62
Kamlesh Suthar	92.62
Akshat Choudhary	92.62
Khushal Parihar	92.62
Devanshi Kapadia	92.62
Amey Mhaskar	92.62
Keya Trivedi	92.62
Neer Shah	92.62
Yashvi Shah	92.62
Soham Angre	92.62
Ayush Ajay Sawant	92.62
Ankita Kewalramani	92.62
Deepam	92.62
Prasanna	92.62
Prasanna Suresh	92.62
Hrishita Raghu Poojari	92.46
Devdas Ranjeet Patole	92.46
Anannya Mhatre	92.46
Sanskruti Shashikant Phavade	92.46
Sanskar Maheshwari	92.46
Neeti Vakharia	92.46
Payas Mehta	92.46
Shobhit Maliwal	92.46
Leesha Gupta	92.46
Nikunj Jain	92.46
Siddhant Hemant Avhad	92.46
Khushi Maheshwari	92.46
Chaitra Billava	92.31
Hitakshi Mehta	92.31
Smit Manish Fofaria	92.31
Khushi Varaiya	92.31

Name	Percentage
Riya Mahyavanshi	92.31
Ayush Agrawal	92.31
Gautam Bhavesh Shah	92.31
Bhumit Mehta	92.31
Saakshi Deepak Karia	92.31
Palak Jaitly	92.31
Prerna Rajen Vora	92.31
Manasvi Patankar	92.31
Hetavi Shah	92.15
Bansi Madlani	92.15
Deeksha Kapoor	92.15
Yash Nautiyal	92.15
Shruti Jain	92.15
Mahek Payak	92.15
Raksha Shekhar Shetty	92.15
Dev Shah	92.15
Aditi Ashok Shetty	92.15
Athira Vaipur	92.15
Jahnavi	92.15
Manas Shetty	92.00
Neeraj Shah	92.00
Yash Shah	92.00
Yash Divyank Dhah	92.00
Aditya Kandari	92.00
Isha Chotai	92.00
Breanna Fernandes	92.00
Kashish Bhargava	92.00
Krishna Bharat Bhanushali	92.00
Keni Mehta	92.00
Khushi Kanodia	91.85
Shreya Tatke	91.85
Pratyush Deepak Rajgor	91.85
Bhaktee Shah	91.85
Bhargavraju Veerla	91.85
Krupa Rakesh Gajre	91.85
Swizal Gomes	91.85
Heli Sanjay Dhruv	91.85
Parth Upadhyay	91.85
Vinit	91.85
Cheryl Andrade	91.85
Yash Thakare	91.69
Vitrag Singhi	91.69
Radhika Dabholkar	91.69
Aastha Hari Chand	91.69
Dhruvi Desai	91.69
Rohit Baviskar	91.69
Bhinde Parth Mahendra	91.69
Parth Mahendra bhinde	91.69
Tanish Agarwal	91.69
Mokshitha Sherty	91.69
Sanchit Jain	91.69
Samiksha Bhatt	91.69
Sejal Phapale	91.69
Isha Bathia	91.69
Radhika Garg	91.69

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Name	Percentage
Om Kedia	91.69
Esha Trisom Sonkusale	91.69
Samarth	91.54
Viraj Mehta	91.54
Mangesh Gadewar	91.54
Murtaza Saria	91.54
Disha Mody	91.54
Samma Naresh Kewlani	91.54
Ayush Panchamiya	91.54
Priya Rao	91.54
Kiara Xavier	91.54
Hansika Gupte	91.54
Deval Mehta	91.38
Nagesh Banne	91.38
Ojas	91.38
Tanaya	91.38
Jhanvi	91.38
Aparna Ramanathan	91.38
Mahek Shah	91.38
Niel Patade	91.38
Harshi Kothari	91.38
Aryan Karnawat	91.38
Ananya Akerkar	91.38
Aabeid Shaikh	91.38
Shubham Modi	91.38
Isha Shah	91.23
Neeraj Kishore Udasi	91.23
Honey Waghela	91.23
Vanshita Devadiga	91.23
Kavish Garg	91.23
Mit Shah	91.23
Ayushi Dhruva	91.23
Cheril Nitin Shah	91.23
Mohak Savla	91.23
Bhakti Deshmukh	91.23
Kaivan Dhruval Doshi	91.23
Shweta Lackdivey	91.23
Shreyas Badiger	91.08
Sneha Chavan	91.08
Sathvika Shetty	91.08
Ankita Joshi	91.08
Mansi Lad	91.08
Nitansh Shah	91.08
Shree Joshi	91.08
Zubiya Ansari	91.08
Mitali Shetty	91.08
Ashmita Devadiga	91.08
Vidhi Shah	91.08
Diya Chheda	91.08
Dimple Dangi	91.08
Chandan Tiwari	90.92
Disha N Shah	90.92
Gauri Ojha	90.92
Tanish Dhami	90.92
Arishit Shetty	90.92

Name	Percentage
Swastik	90.92
Shayan Sadik Desai	90.92
Khushi Rakesh Chordia	90.92
Krish Parmar	90.92
Vidhi Singh	90.92
Saloni	90.92
Shreya Reddy	90.92
Diya Dedhia	90.92
Shambhavi Pai	90.92
Vrunda Atul Mehta	90.77
Parikshit Vanjara	90.77
Khushi Soni	90.77
Esha Hingarh	90.77
Merill D'souza	90.77
Riya Patel	90.77
Poojan Sanghavi	90.77
Maurya Borse	90.77
Ashi Devang Dhruva	90.77
Heena	90.77
Khush Agarwal	90.77
Siddhi Panchal	90.77
Siddhi	90.77
Tania	90.77
Srivatsa Patil	90.77
Rahul Medda	90.77
Nishi Jagdish Punmiya	90.77
Tanushree Yadav	90.77
Vedant Keluskar	90.77
Nishtha jain	90.62
Krish Shah	90.62
Amisha Mehta	90.62
Fenil Soneji	90.62
Richa Pravin Naik	90.62
Jhanvi Joshi	90.62
Smriti Jain	90.62
Priya Mangesh Jagtap	90.62
Sakshi Kalpesh Shah	90.62
Rajlaxmi Magadum	90.62
Devanshi Vira	90.62
Kashissh Singhania	90.62
Disha Shah	90.62
Vidhi	90.62
Kaushik K Bhartiya	90.62
Krina Satra	90.62
Vedant Shriyan	90.46
Krish Jain	90.46
Lokesh M Jain	90.46
Sanskar Agarwal	90.46
Narayani Gaur	90.46
Jahnvi Shah	90.46
Shreeya Deorukhkar	90.46
Aryaa Punyarthi	90.46
Sneha Ashok Shinde	90.46
Sneha Shinde	90.46
Yuvraj Abhaykumar Gandhi	90.46

Name	Percentage
Tanvi Rasal	90.46
Hatim Sonkachwala	90.46
Meet Paresh Kanakia	90.46
Meet Kanakia	90.46
Jaineel Dalal	90.46
Disha Biyani	90.46
Vishakha Ranga	90.31
Devesh Dilip Pimpale	90.31
Khushi Vinod Bhanushali	90.31
Vini Desai	90.31
Pauravi Nitin Baikar	90.31
Sharvari Deshpande	90.31
Nisha Rajesh Rao	90.31
Vanshita Vora	90.31
Sayed Mohammed Junaid	90.31
Anish	90.31
Shubham Vora	90.31
Harshit Kedia	90.31
Kirti Balu Hase	90.31
Deepal Vikas Gohel	90.31
Deepal Gohel	90.31
Gautam Kothari	90.15
Preksha Patel	90.15
Pratik Dattu Koakte	90.15
Sanika Shivaji Varal	90.15
Gandhali Sumukh Desai	90.15
Surabhi Sonar	90.15
Jainam Swayam Shah	90.15
Siddhi Tiwari	90.15
Het Fariya	90.15
Nemin Doshi	90.15
Jeni Shah	90.15
Hayyan Badamia	90.15
Arushi Keniya	90.15
Roshan Jain	90.00
Shruti Shetty	90.00
Ramnek Chhipa	90.00
Riya Shirvaikar	90.00
Ayush Barbhaya	90.00
Pranav	90.00
Riddhi	90.00
Sakshi Raut	90.00
Manjiri Parab	90.00
Pal Shah	90.00
Yash Ganesh Khanolkar	90.00
Prasesh Mehta	90.00
Disha Bucha	90.00
Tanish Dharmendra Parmar	90.00
Palak Jain	90.00