**Q1.**

**01.** $\int \sin^2 x \, dx$

$1 - \cos 2x = 2\sin^2 x$

$\sin^2 x = \frac{1 - \cos 2x}{2}$

BACK IN THE SUM

$= \int \frac{1 - \cos 2x}{2} dx$

$= \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + c$

$= \frac{x}{2} - \frac{\sin 2x}{4} + c$

**02.**

$\int \frac{1}{x^2 + 8x + 20} dx$

$\left(\frac{1}{2}(8)\right)^2 = 16$

$= \int \frac{1}{x^2 + 8x + 16 + 20 - 16} dx$

$= \int \frac{1}{(x + 4)^2 + 4} dx$

$= \int \frac{1}{(x + 4)^2 + 2^2} dx$

$= \frac{1}{a} \tan^{-1} \frac{x}{a} + c$

$= \frac{1}{2} \tan^{-1} \frac{x + 4}{2} + c$

**03.**

$\int \frac{\sec x . \tan x}{3\sec x + 5} dx$

LET

$3\sec x + 5 = t$

$3\sec x.\tan x.dx = dt$

$\therefore \sec x.\tan x.dx = \frac{dt}{3}$

NOW THE SUM IS

$= \int \frac{1}{t} \frac{dt}{3}$

$= \frac{1}{3} \int \frac{1}{t} dt$

$= \frac{1}{3} \log | t | + c$

$= \frac{1}{3} \log | 3\sec x + 5 | + c$

**04.**

$\int \frac{1}{\sqrt{4 - 36x^2}} dx$

$= \int \frac{1}{\sqrt{2^2 - (6x)^2}} dx$

$= \sin^{-1} \frac{x}{a} + c$

$= \frac{1}{6} \sin^{-1} \frac{6x}{2} + c$

COEFF. OF X

$= \frac{1}{6} \sin^{-1} 3x + c$

**05.**

$\int e^x \sec x(1 + \tan x) \, dx$

$= \int e^x (\sec x + \sec x.\tan x) \, dx$

$\frac{d}{dx} \sec x = \sec x .\tan x$

HENCE THE SUM IS

$= \int e^x \left(f(x) + f'(x)\right) dx$

$= e^x f(x) + c$

$= e^x \sec x + c$

**06.** $\int x \,.\, \sin x \,.dx$

$$= x\int \sin x \, dx - \int \left[\frac{d}{dx}\, x \int \sin x \, dx\right] dx$$

$$= x.(-\cos x) - \int 1\,.(-\cos x)\; dx$$

$$= -x.\cos x + \sin x + c$$

**07.**

$\int \log x \,.dx$

$$= \int \log x \,.\, 1 \,.dx$$

$$= \log x \int 1\, dx - \int \left[\frac{d}{dx} \log x \int 1\, dx\right] dx$$

$$= \log x \,.\, x - \int \frac{1}{x} \,.\, x \; dx$$

$$= x \log x - \int 1 \; dx$$

$$= x.\log x - x + c$$

**08.** $\int \sqrt{4 - 9x^2}\; dx$

$$= \int \sqrt{(2)^2 - (3x)^2}\; dx$$

$$= \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + c$$

$$= \frac{1}{3}\left[\frac{3x}{2}\sqrt{(2)^2 - (3x)^2} + \frac{2^2}{2} \sin^{-1}\left(\frac{3x}{2}\right)\right]$$

$$= \frac{x}{2}\sqrt{4 - 9x^2} + \frac{2}{3} \sin^{-1}\left(\frac{3x}{2}\right) + c$$

**Q2. A ATTEMPT ANY 2 OUT OF 3**

**01.** $\int \frac{\sin x}{1 - \sin x}\, dx$

$$= \int \frac{\sin x}{1 - \sin x} \times \frac{1 + \sin x}{1 + \sin x}\, dx$$

$$= \int \frac{\sin x + \sin^2 x}{1 - \sin^2 x}\, dx$$

$$= \int \frac{\sin x + \sin^2 x}{\cos^2 x}\, dx$$

$$= \int \left[\frac{\sin x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}\right] dx$$

$$= \int (\sec x \,.\, \tan x + \tan^2 x)\; dx$$

$$= \int (\sec x \,.\tan x + \sec^2 x - 1)\; dx$$

$$= \sec x + \tan x + c$$

**02.** $\int e^x \left[\frac{x}{(x+1)^2}\right] dx$

$$= \int e^x \left[\frac{x + 1 - 1}{(x+1)^2}\right] dx$$

$$= \int e^x \left[\frac{x + 1}{(x+1)^2} - \frac{1}{(x+1)^2}\right] dx$$

$$= \int e^x \left[\frac{1}{x+1} + \frac{-1}{(x+1)^2}\right] dx$$

$$\frac{d}{dx}\,\frac{1}{x+1} = \frac{-1}{(x+1)^2}$$

$$= \int e^x \left[f(x) + f'(x)\right] dx$$

$$= e^x\; f(x) + c$$

$$= e^x \frac{1}{x+1} + c$$

$$= \frac{e^x}{x+1} + c$$

**03.** $\int \frac{\cos x}{\sqrt{\sin^2 x - 2\sin x + 5}}\,dx$

PUT $\sin x = t$

$\cos x \,.\, dx = dt$

THE SUM IS

$$= \int \frac{1}{\sqrt{t^2 - 2t + 5}}\,dt$$

$$= \int \frac{1}{\sqrt{t^2 - 2t + 1 + 5 - 1}}\,dt$$

$$= \int \frac{1}{\sqrt{(t-1)^2 + 4}}\,dt$$

$$= \int \frac{1}{\sqrt{(t-1)^2 + 2^2}}\,dt$$

$$= \log \left| t + \sqrt{t^2 + a^2} \right| + c$$

$$= \log \left| t - 1 + \sqrt{(t-1)^2 + 2^2} \right| + c$$

$$= \log \left| t - 1 + \sqrt{t^2 - 2t + 5} \right| + c$$

$$= \log \left| \sin x - 1 + \sqrt{\sin^2 x - 2\sin x + 5} \right| + c$$

**Q2. B ATTEMPT ANY 2 OUT OF 3**

**01.**

$$\int \frac{\sec^2 x}{\tan^2 x - 4\tan x - 12}\,dx$$

PUT $\tan x = t$

$\sec^2 x \,.\, dx = dt$

THE SUM IS

$$\int \frac{1}{t^2 - 4t - 12}\,dx$$

$$\left(\frac{1}{2}(4)\right)^2 = 4$$

$$= \int \frac{1}{t^2 - 4t + 4 - 12 - 4}\,dt$$

$$= \int \frac{1}{(t-2)^2 - 16}\,dt$$

$$= \int \frac{1}{(t-2)^2 - 4^2}\,dt$$

$$= \frac{1}{2a} \log \left| \frac{t-a}{t+a} \right| + c$$

$$= \frac{1}{2(4)} \log \left| \frac{t-2-4}{t-2+4} \right| + c$$

$$= \frac{1}{8} \log \left| \frac{t-6}{t+2} \right|$$

$$= \frac{1}{8} \log \left| \frac{\tan x - 6}{\tan x + 2} \right|$$

**02.** $\int \log(1 + x^2).dx$

$$= \int \log(1 + x^2).\, 1dx$$

$$= \log(1 + x^2) \int 1\,dx - \int \left( \frac{d}{dx} \log(1 + x^2) \int 1\,dx \right) dx$$

$$= \log(1 + x^2) \,.\, x - \int \frac{2x}{1 + x^2} \,.\, x \,dx$$

$$= x.\log(1 + x^2) - 2 \int \frac{x^2}{1 + x^2}\,dx$$

$$= x.\log(1 + x^2) - 2 \int \frac{1 + x^2 - 1}{1 + x^2}\,.\,dx$$

$$= x.\log(1 + x^2) - 2 \int 1 - \frac{1}{1 + x^2}\,dx$$

$$= x.\log(1 + x^2) - 2 \left( x - \tan^{-1} x \right) + c$$

$$= x.\log(1 + x^2) - 2x + 2\tan^{-1} x + c$$

**03.** $$\int \frac{1 + \log x}{x(2 + \log x)(3 + \log x)}\, dx$$

**SOLUTION**

$$\log x = t \qquad \therefore \frac{1}{x} \cdot dx = dt$$

$$= \int \frac{1 + t}{(2 + t)(3 + t)}\, dt$$

$$\frac{1 + t}{(2 + t)(3 + t)} = \frac{A}{2 + t} + \frac{B}{3 + t}$$

$$1 + t = A(3 + t) + B(2 + t)$$

**Put t = – 3**

$$1 - 3 = B(2 - 3)$$

$$-2 = B(-1) \qquad \therefore B = 2$$

**Put t = – 2**

$$1 - 2 = A(3 - 2)$$

$$-1 = A(1) \qquad \therefore A = -1$$

**HENCE**

$$\frac{1 + t}{(2 + t)(3 + t)} = \frac{-1}{2 + t} + \frac{2}{3 + t}$$

BACK IN THE SUM

$$= \int \frac{-1}{2 + t} + \frac{2}{3 + t}\, dt$$

$$= -\log|2 + t| + 2\log|3 + t| + c$$

RESUBS. $$= -\log|2 + \log x| + 2\log|3 + \log x| + c$$

**Q3. A ATTEMPT ANY 2 OUT OF 3**

**01.**

$$I = \int_0^4 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5} + \sqrt[3]{9-x}}\,dx \quad \text{.....( 1 )}$$

**USING** $\int_0^a f(x)dx = \int_0^a f(a-x)\,dx$

**CHANGE 'x' TO '4 – x'**

$$I = \int_0^4 \frac{\sqrt[3]{4-x+5}}{\sqrt[3]{4-x+5} + \sqrt[3]{9-(4-x)}}\,dx$$

$$I = \int_0^4 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x} + \sqrt[3]{9-4+x}}\,dx$$

$$I = \int_0^4 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x} + \sqrt[3]{5+x}}\,dx \quad \text{.....( 2 )}$$

(1) + (2)

$$2I = \int_0^4 \frac{\sqrt[3]{x+5} + \sqrt[3]{9-x}}{\sqrt[3]{x+5} + \sqrt[3]{9-x}}\,dx$$

$$2I = \int_0^4 1\,dx$$

$$2I = \left(x\right)_0^4$$

$$2I = 4 - 0$$

$$2I = 4$$

$$I = 2$$

**02.**

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt[3]{\cos x}}{\sqrt[3]{\cos x} + \sqrt[3]{\sin x}}\,dx$$

**USING** $\int_a^b f(x)\,dx = \int_b^b f(a+b-x)\,dx$

**WE CHANGE 'x' TO '$\pi/6 + \pi/3$ – x' i.e.**

**WE CHANGE 'x' TO '$\pi/2 - x$'**

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt[3]{\cos(\pi/2-x)}}{\sqrt[3]{\cos(\pi/2-x)} + \sqrt[3]{\sin(\pi/2-x)}}\,dx$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt[3]{\sin x}}{\sqrt[3]{\sin x} + \sqrt[3]{\cos x}}\,dx \quad \text{... ( 2 )}$$

( 1 ) + ( 2 )

$$2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt[3]{\cos x} + \sqrt[3]{\sin x}}{\sqrt[3]{\cos x} + \sqrt[3]{\sin x}}\,dx$$

$$2I = \int_{\pi/6}^{\pi/3} 1\,dx$$

$$2I = \left(x\right)_{\pi/6}^{\pi/3}$$

$$2I = \frac{\pi}{3} - \frac{\pi}{6}$$

$$2I = \frac{2\pi - \pi}{6}$$

$$2I = \frac{\pi}{6}$$

$$I = \frac{\pi}{12}$$

**03.** $$\int_0^4 \frac{1}{\sqrt{4x - x^2}}\,dx$$

$$= \int_0^4 \frac{1}{\sqrt{0 - (x^2 - 4x + 4) + 4}}\,dx$$

$$= \int_0^4 \frac{1}{\sqrt{4 - (x - 2)^2}}\,dx$$

$$= \int_0^4 \frac{1}{\sqrt{2^2 - (x - 2)^2}}\,dx$$

$$= \left\{\sin^{-1}\left(\frac{x - 2}{2}\right)\right\}_0^4$$

$$= \sin^{-1}\left(\frac{4 - 2}{2}\right) - \sin^{-1}\left(\frac{0 - 2}{2}\right)$$

$$= \sin^{-1}\left(\frac{2}{2}\right) - \sin^{-1}\left(\frac{-2}{2}\right)$$

$$= \sin^{-1} 1 - \sin^{-1}(-1)$$

$$= \sin^{-1} 1 + \sin^{-1} 1$$

$$= 2 \sin^{-1} 1$$

$$= 2 . \frac{\pi}{2}$$

$$= \pi$$

**Q3. B ATTEMPT ANY 2 OUT OF 3**

**01.** $$I = \int_0^3 \frac{1}{x + \sqrt{9 - x^2}}\,dx$$

Put $x = 3\sin\theta$

$\frac{dx}{d\theta} = 3\cos\theta$

$dx = 3\cos\theta\, d\theta$

when $x = 3$: $\sin\theta = 1$, $\theta = \pi/2$

when $x = 0$: $\sin\theta = 0$, $\theta = 0$

$$I = \int_0^{\pi/2} \frac{1}{3\sin\theta + \sqrt{9 - 9\sin^2\theta}}\, 3\cos\theta \, d\theta$$

$$I = \int_0^{\pi/2} \frac{3\cos\theta}{3\sin\theta + \sqrt{9(1 - \sin^2\theta)}}\, d\theta$$

$$I = \int_0^{\pi/2} \frac{3\cos\theta}{3\sin\theta + \sqrt{9\cos^2\theta}}\, d\theta$$

$$I = \int_0^{\pi/2} \frac{3\cos\theta}{3\sin\theta + 3\cos\theta}\, d\theta$$

$$I = \int_0^{\pi/2} \frac{\cos\theta}{\sin\theta + \cos\theta}\, d\theta \quad \text{...... ( 1 )}$$

**USING** $\int_0^a f(x)dx = \int_0^a f(a - x)\,dx$

**WE CHANGE** '$\theta$' **TO** '$\pi/2 - \theta$'

$$I = \int_0^{\pi/2} \frac{\cos(\pi/2 - \theta)}{\cos(\pi/2 - \theta) + \sin(\pi/2 - \theta)}\, d\theta$$

$$I = \int_0^{\pi/2} \frac{\sin\theta}{\sin\theta + \cos\theta}\, d\theta \quad \text{...... ( 2 )}$$

(1) + (2)

$$I = \int_0^{\pi/2} \frac{\cos\theta + \sin\theta}{\cos\theta + \sin\theta}\, d\theta$$

$$2I = \int_0^{\pi/2} 1 \; d\theta$$

$$2I = \left[\theta\right]_0^{\pi/2}$$

$$2I = \pi/2 - 0$$

$$2I = \pi/2$$

$$I = \pi/4$$

**02.**

$$I = \int_0^1 x^2(1-x)^{1/2}\, dx$$

**USING** $\int_0^a f(x)dx = \int_0^a f(a-x)\,dx$

**WE CHANGE** 'x' **TO** '1 – x'

$$I = \int_0^1 (1-x)^2 . x^{1/2}\, dx$$

$$I = \int_0^1 (1 - 2x + x^2).\, x^{1/2}\, dx$$

$$I = \int_0^1 (x^{1/2} - 2x^{3/2} + x^{5/2})dx$$

$$I = \left[\frac{x^{3/2}}{\frac{3}{2}} - \frac{2x^{5/2}}{\frac{5}{2}} + \frac{x^{7/2}}{\frac{7}{2}}\right]_0^1$$

$$I = \left[\frac{2}{3}x^{3/2} - \frac{4}{5}x^{5/2} + \frac{2}{7}x^{7/2}\right]_0^1$$

$$I = \frac{2}{3}1^{3/2} - \frac{4}{5}1^{5/2} + \frac{2}{7}1^{7/2}$$

$$I = \frac{2}{3} - \frac{4}{5} + \frac{2}{7}$$

$$I = 2\left(\frac{1}{3} - \frac{2}{5} + \frac{1}{7}\right)$$

$$I = 2\left(\frac{35 - 42 + 15}{105}\right)$$

$$I = 2\ \frac{8}{105}$$

$$I = \frac{16}{105}$$

**03.** `

$$\int_0^{\pi/2} \frac{\sin x.\cos x}{(2\sin x + 1).(\sin x + 1)}\, dx$$

$\sin x = t$ $\quad x = 0$ , $t = \sin 0 = 0$

$\cos x \,.\, dx = dt$ $\quad x = \frac{\pi}{2}$ , $t = \sin\frac{\pi}{2} = 1$

THE SUM IS

$$\int_0^1 \frac{t}{(2t+1)(t+1)}\, dt$$

$$\frac{t}{(2t+1)(t+1)} = \frac{A}{2t+1} + \frac{B}{t+1}$$

$$t = A(t+1) + B(2t+1)$$

Put $t = -1$

$-1 = B(-2+1)$

$-1 = B(-1)$

$1 = B$

Put $t = -1/2$

$-1/2 = A(-1/2 + 1)$

$-1/2 = A(1/2)$

$-1 = A$

HENCE

$$\frac{t}{(2t+1)(t+1)} = \frac{-1}{2t+1} + \frac{1}{t+1}$$

BACK IN THE SUM

$$= \int_0^1 \frac{-1}{2t+1} + \frac{1}{t+1} \, dt$$

$$= \left[ \frac{-1}{2} \log |2t+1| + \log |t+1| \right]_0^1$$

$$= \left( \frac{-1}{2} \log 3 + \log 2 \right) - \left( \frac{-1}{2} \log 1 + \log 1 \right)$$

$$= \log 2 - \frac{1}{2} \log 3 \qquad ( \log 1 = 0 )$$

$$= \frac{2 \log 2 - \log 3}{2}$$

$$= \frac{\log 4 - \log 3}{2} = \frac{1}{2} \log \frac{4}{3}$$