J. K. SHAH CLASSES
FYJC - TEST - 02

BRANCH : BORIVALI & ANDHERI(A1 & B2) **MARKS : 40**
SUB : MATHS - SET 1 **Time : 1 Hour 30 min.**
Date : 2 / 10 /2016

SOLUTION

MEASURES OF DISPERSION

1. For the following frequency distribution the value of Q_1 is 33. Find the missing frequency and then calculate quartile deviation. (4 Marks)

Wages (₹ per hour)	30 – 32	32 – 34	34 – 36	36 – 38	38 – 40	40 – 42	42 - 44
No of workers	?	18	16	14	12	8	4

Ans: Let x be the missing frequency. Now we shall form 1.c.f. table as follows :

Class Interval	**Frequency**	**l.c.f**
30 - 32	x	x
32 - 34	18	18 + x
34 – 36	16	34 + x
36 – 38	14	48 + x
38 – 40	12	48 + x
40 – 42	8	68 + x
42 - 44	4	72 + x

Given that, Q_1 = 33, 32 -34 is the first quartile class.

∴ L = 32, c.f. = x, h = 2, f = 18

∴ $Q_1 = L + \frac{h}{f}\left(\frac{N}{4} - c.f.\right)$

∴ $33 = 32 + \frac{2}{18}\left(\frac{(72+x)}{4} - x\right)$

∴ $9 \times (33 - 32) = \left(\frac{(72+x)}{4} - x\right)$

∴ $9 + x = \frac{(72+x)}{4}$

∴ 36 + 4x = 72 + x

∴ 3x = 36

∴ x = 12

Now, N = 72 + 12 = **84**

After getting missing frequency, we can form l.c.f. table as follows :

Class Interval	Frequency	l.c.f.
30 – 32	12	12
32 – 34	18	30
34 – 36	16	46
36 – 38	14	60
38 – 40	12	72
40 – 42	8	80
42 - 44	4	84

For Q_3 consider $\left(\frac{3N}{4}\right) = 63$

∴ Q_3 lies in the class 38 -40

∴ L = 38, c.f. = 60, h = 2, f = 12

∴ $Q_3 = L + \frac{h}{f}\left(\frac{3N}{4} - c.f.\right)$

$= 38 + \frac{2}{12}(63 - 60)$

$= 38 + \frac{6}{12}$

$= 38 + 0.5$

∴ $Q_3 = 38.5$

∴ Q.D. $= \frac{Q_3 - Q_1}{2}$

$= \frac{38.5-33}{2}$

$\boxed{\text{Q.d.} = 2.75}$

2. Compute the mean deviation about (i) the mean and (ii) the median for the price of share of particular company in a week 40,47,38,41,49. (3 Marks)

Ans: First we find the mean and the median of given data.

Mean $= \frac{\sum x}{n} = \frac{215}{5} = 43$

Now we shall find median of the observation. For this we arrange the observation in ascending order as 38,40,41,47,49.

By definition, Median = Me = 41

x_i	$\lvert x_i - \text{Mean}\rvert = \lvert x_i - 43\rvert$	$\lvert x_i$ Me$\rvert = \lvert x_i - 41\rvert$
40	3	1
47	4	6
38	5	3
41	2	0
49	6	8
Total	**20**	**18**

From table

$\sum_{i=1}^{n} \lvert x_i - \text{Mean}\rvert = 20,\qquad \sum_{i=1}^{n} \lvert x - \text{Me}\rvert = 18$

M.D. about mean $= \frac{\sum \lvert x - \text{Mean}\rvert}{n} = \frac{20}{5} = 4$

M.D. about median $= \frac{\sum \lvert x - \text{Median}\rvert}{n} = \frac{18}{5} = 3.6$

3. Prices of a particular commodity in five years in two cities are as follows : (4 Marks)

Price in city A	20	22	19	23	26
Price in city B	10	20	18	12	15

Ans: Let price in city A be x and price in city B be y

x	x^2	**y**	y^2
20	400	10	100
22	484	20	400
19	361	18	324
23	529	12	144
26	676	15	225
110	**2450**	**75**	**1193**

Step 1 :

$$\bar{x} = \frac{\Sigma\ x}{n} = \frac{110}{5} = 22$$

Step 2 :

$$\sigma_x = \sqrt{\frac{\Sigma\ x^2}{n} - (\bar{x})^2}$$

$$= \sqrt{\frac{2450}{5} - (22)^2}$$

$$= \sqrt{490 - 484}$$

$$= \sqrt{6}$$

= 2.45 [Applying log]

Step 3 :

$$\text{C.}V_A = \frac{\sigma_x}{\bar{x}} \text{ x } 100$$

$$= \frac{2.45}{22} \text{ x } 100$$

= 11.136 %

Step 4 :

$$\bar{y} = \frac{\Sigma\ y}{n} = \frac{75}{5} = 15$$

Step 5 :

$$\sigma_y = \sqrt{\frac{\Sigma\ y^2}{n} - (\bar{y})^2}$$

$$= \sqrt{\frac{1193}{5} - (15)^2}$$

$$= \sqrt{238.6 - 225}$$

$$= \sqrt{13.6}$$

= 3.69 [Applying log]

Step 6 :

$$= \text{C.}V_B = \frac{\sigma_y}{\bar{y}} \text{ x } 100$$

$$= \frac{3.69}{15} \text{ x } 100$$

= 24.59 %

Step 7 : Since

$\text{C.}V_A < \text{C.}V_B$

Since the C.V corresponding to city A is less than that corresponding to city B, the prices show more stability in city A.

MOMENTS

4. The first four raw moments of a distribution are 2,20,40 and 50 respectively. Find the first four central moments of the distribution. (3 Marks)

Ans: The first four raw moments of the distribution are

$\mu_1' = 2$, $\mu_2' = 20$, $\mu_3' = 40$ and $\mu_4' = 50$

We want to find first four central moments μ_1, μ_2, μ_3 and μ_4.

We know that $\mu_1 = 0$.

Using relations between raw moments and central moments, We get

$\mu_2 = \mu_2' - \mu_1'^2$

= 20-4 = 16

$\mu_3 = \mu_3' - 3\mu_2' \mu_1' + 2\mu_1'^3$

= 40 − 3(20)(2) + 2(8)

= 40 − 120 + 16

= − 64

$\mu_4 = \mu_4' - 4\mu_3' \mu_1' + 6\mu_2' \mu_1^2 - 3\mu_1'^4$

$= 50 - 4(40)(2) + 6(20)(4) - 3(16)$

$= 50-320+480-48$

$= 162$

Thus, the first four central moments are $\mu_1 = 0$, $\mu_2 = 16$, $\mu_3 = -64$ and $\mu_4 = 162$

ANGLE AND ITS MEASUREMENT

5. One angle of a quadrilateral has measure $\frac{2\pi}{9}$ radians and the measures of the other three angles are in the ratio 3 :5 : 8, find the measures in radians. (3 Marks)

Ans: The sum of angles of a quadrilateral is 360°.
One of the angles is given to be
$\frac{2\pi}{9} = \frac{2\pi}{9} \times \frac{180}{\pi} = 40°$

$\therefore$ Sum of the remaining three angles is 360° − 40° = 320°

Since these three angles are in the ratio 3 : 5 :8,

degree measures of these angles are $3k$, $5k$, $8k$, where k is constant.

$\therefore\ 3k + 5k + 8k = 320°$

$\therefore 16k = 320°$

$\therefore\ K = 20°$

$\therefore$ The measures of three angles are

$(3k)° = (3 \times 20)° = 60°$

$(5k)° = (5 \times 20)° = 100°$

and $(8k)° = (8 \times 20)° = 160°$

$\therefore$ Three angles are 30o, 100o , 160o.

These three angles in radians are

$60° = \left[60\, x\, \frac{\pi}{180}\right]^c = \frac{\pi^c}{3}$

$100° = \left[100\, x\, \frac{\pi}{180}\right]^c = \frac{5\pi^c}{9}$

$160° = \left[160\, x\, \frac{\pi}{180}\right]^c = \frac{8\pi^c}{9}$

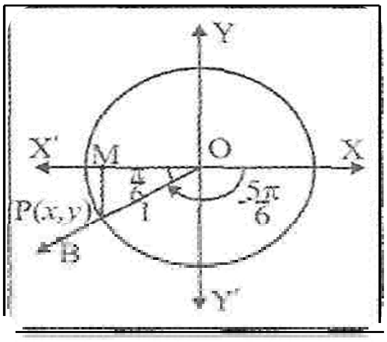
TRIGONOMETRY

6. Find the trigonometric function of - $\frac{5\pi^c}{6}$ (3 Marks)

Ans: Let m $\angle$XOB = $-\frac{5\pi^c}{6}$

Draw a unit circle with centre at the origin. Let ray OB meet the standard unit circle in P (x, y).

$l(\text{OP}) = 1$
Draw segment PM perpendicular to X − axis
In Δ POM, m $\angle$POM = $\frac{\pi^c}{6}$ = 30°



In Δ POM, m $\angle$POM = $\frac{\pi^c}{6}$ = 30°

$l(OM) = \frac{\sqrt{3}}{2}.\ l(\text{OP}) = \frac{\sqrt{3}}{2}.\ 1 = \frac{\sqrt{3}}{2}$

Since P lies in third quadrant,

P ≡ (x,y) ≡ $\left[-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right]$

∴ by definition

sin $\left(-\frac{5\pi}{6}\right)^c$ = y = $-\frac{1}{2}$

∴ cosec $\left(-\frac{5\pi}{6}\right)^c$ = - 2

cos$\left(-\frac{5\pi}{6}\right)$ = x =$-\frac{\sqrt{3}}{2}$

∴ sec $\left(-\frac{5\pi}{6}\right)^c$ = $-\frac{2}{\sqrt{3}}$

tan$\left(-\frac{5\pi}{6}\right)^c$ = $\frac{y}{x}\ \frac{(-1/2)}{(-\sqrt{3/2})}$ = $\frac{1}{\sqrt{3}}$

∴ cot $\left(-\frac{5\pi}{6}\right)^c$ = $\sqrt{3}$

7. Eliminateθ, if x = asecθ + b tan θ; y =asec θ- b tanθ **(2 Marks)**

Ans: x = a sec θ + b tan θ …………………(I)

Y = a sec θ - b tan θ ……………….(II)

On adding (I) and (II), on subtracting (II) from (I)

x + y = 2asec θ, x − y= 2b tan θ

∴ sec θ = $\frac{x+y}{2a}$; tan θ = $\frac{x-y}{2b}$

We have, $sec^2\ \theta - \tan^2\theta = 1$

$$\left(\frac{x+y}{2a}\right)^2 - \left(\frac{x-y}{2b}\right)^2 = 1$$

$$\frac{(x+y)^2}{4a^2} - \frac{(x-y)^2}{4b^2} = 1.$$

8. **If Sin A = $\frac{4}{5}$, $\frac{\pi^c}{2} < A < \pi^c$** **(4 Marks)**

Cos B = $\frac{5}{13}$, $\frac{3\pi^c}{2} < B < 2\pi^c$. **Find (i) Sin(A + B), (ii) Cos (A − B) (iii) Tan (A-B)**

Ans. We have,

$$cos^2\ A = 1 - sin^2\ A$$

$$= 1 - \left(\frac{4}{5}\right)^2$$

$$= 1 - \frac{16}{25}$$

$$= \frac{9}{25}$$

$$\therefore \cos A = \pm \frac{3}{5}$$

Since, $\lfloor A$ lies in the second quadrant.

$$\therefore \cos A = -\frac{3}{5}$$

$$\text{Tan } A = \frac{\sin A}{\cos A}$$

$$= \left(\frac{4/5}{-3/5}\right)$$

$$= -\frac{4}{3}$$

$$sin^2\ B = 1 - cos^2\ B$$

$$= 1 - \left(\frac{5}{13}\right)^2$$

$$= 1 - \frac{25}{169}$$

$$= \frac{144}{169}$$

$$\therefore \sin B = \pm \frac{12}{13}$$

Since, $\lfloor B$ line in the fourth quadrant.

$$\therefore \sin B = -\frac{12}{13}$$

$$\tan B = \frac{\sin B}{\cos B}$$

$$= \left(\frac{-12/13}{5/13}\right)$$

$$= \frac{-12}{5}$$

(i) Sin (A+B) = sin A . cos B + cos A . sin B

$$= \left(\frac{4}{5}\right)\left(\frac{5}{13}\right) + \left(\frac{-3}{5}\right)\left(\frac{-12}{13}\right)$$

$$= \frac{20}{65} + \frac{36}{65}$$

$$= \frac{56}{65}$$

(ii) cos (A-B) = cos A . cos B + sin A . sin B

$$= \left(\frac{-3}{5}\right)\left(\frac{5}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{-12}{13}\right)$$

$$= \frac{-15}{65} + \frac{48}{65}$$

$$= \frac{-63}{65}$$

(iii) $\tan (A-B) = \frac{\tan A - \tan B}{1 + \tan A . \tan B}$

$$= \frac{\left(\frac{-4}{3}\right) - \left(\frac{-12}{5}\right)}{1 + \left(\frac{-4}{3}\right) . \left(\frac{-12}{5}\right)}$$

$$= \frac{\frac{-4}{3} + \frac{12}{5}}{1 + \frac{48}{15}}$$

$$= \frac{\frac{16}{15}}{\frac{63}{15}}$$

$$= \frac{16}{63}$$

LOCUS

9. A (2,5) and B (9,-14), are the vertices of ΔABC. The third vertex C lies on the locus whose equation is 3x + 4y + 5 = 0. Find the locus of the centroid of ΔABC. (3 Marks)

Ans: Let G (x,y) be the centroid of Δ ABC. Let C(h,k) be any point on the locus

$3x + 4y + 5 = 0$

$\therefore 3h + 4k + 5 = 0$ ……………….(i)

G(x,y) is the centroid of ABC.

$\therefore G(x,y) = G\left[\frac{2+9+h}{3}, \frac{5-14+k}{3}\right]$

(By centroid formula)

$\therefore x = \frac{11+h}{3}$, and $y = \frac{-9+k}{3}$

$\therefore h = 3x - 11$ and $k = 3y + 9$

Putting the values of h and k in equation (i),
we get

$3(3x - 11) + 4(3y + 9) + 5 = 0$

$\therefore 9x + 12y + 8 = 0$

This is the required equation of the locus of centroid of ΔABC.

LOGARITHM

10. If $\log\left(\frac{x-y}{5}\right) = \frac{1}{2}\log x + \frac{1}{2}\log y$, Show that $x^2 + y^2 = 27xy$ (3 Marks)

Ans.

$\log\left(\frac{x-y}{5}\right) = \frac{1}{2}\log x + \frac{1}{2}\log y$,

$\log\left(\frac{x-y}{5}\right) = \frac{1}{2}[\log x + \log y]$,

$\log\left(\frac{x-y}{5}\right) = \frac{1}{2}\log(xy)$,

$\log\left(\frac{x-y}{5}\right) = \log(xy)^{1/2}$,

$\log\left(\frac{x-y}{5}\right) = \log\sqrt{xy}$,

Dropping log :

$\frac{x-y}{5} = \sqrt{xy}$,

Squaring

$\left(\frac{X-y}{5}\right)^2 = xy$

$\frac{(x-y)^2}{25} = xy$

$x^2 - 2xy + y^2 = 25xy$

$\boxed{x^2 + y^2 = 27xy}$

COMPLEX NUMBER (3 Marks)

11. If a and b are real and $(i^4 + 3i)$ a + $(i-1)$b + $5i^3 = 0$, Find a and b

Ans. $(i^4 + 3i)$ a + $(i-1)$b + $5i^3 = 0 = 0 + 0\,i$

$\therefore$ $(1+ 3i)$ a + $(i-1)$b + $5i^3 = 0 + 0\,i$

$\therefore$ a + 3a i + b i −b −5 i = 0 + 0 i

(a-b) + (3a+b −5) i = 0 + 0 i

Equating real and imaginary parts on both sides, we get,

a – b = 0 and 3a + b-5 = 0

$\therefore$ **a = b and 3a + b = 5**

$\therefore$ 3a + a = 5

4a = 5

a = $\frac{5}{4}$

$\therefore$ **a = b =** $\frac{5}{4}$

SEQUENCE AND SERIES (AP & GP) (2 Marks)

12. For a G.P if a = 5 and $t_7 = \frac{1}{3125}$, find r and t_9

Ans. a = 5 , $t_7 = \frac{1}{3125}$

t_n = a $(r)^{n-1}$,

t_7 = 5 $(r)^{7-1}$,

$\frac{1}{3125}$ = 5 $(r)^6$

$\frac{1}{3125\,x\,5} = r^6$

$\boxed{r = \frac{1}{5}}$

t_9 = a $(r)^{9-1}$

$= 5\left(\frac{1}{5}\right)^8$

$= \cancel{5}\left(\frac{1}{\cancel{5}}\right)\left(\frac{1}{5}\right)^7$

$= \frac{1}{78125}$

13. Find four Numbers in G.P such that their product is 729 and the sum of second and third number is 12. (3 Marks)

Ans. **Let 4 Numbers in G.P be** $\frac{a}{r^3}$, $\frac{a}{r}$ **, or,** ar^3

First Condition :

Product = 729

$\frac{a}{\cancel{r^3}}$, $\frac{a}{\cancel{r}}$, **a$\cancel{r}$**, $a\cancel{r^3}$ **= 729**

a^4 **= 729**

a = $\sqrt[4]{729}$

a = $\sqrt{27}$

a = 3 $\sqrt{3}$

$\sqrt{3}\,r^2$ **- 4**r **+** $\sqrt{3}$ **= 0**

Second Condition :

Sum of 2nd & 3rd = 12

$\frac{a}{r}$ **+ ar = 12**

a $\left(\frac{1}{r}+r\right)$ **= 12**

$\cancel{3}\sqrt{3}\left(\frac{1+r^2}{r}\right) = \cancel{12}\,4$

$\sqrt{3}\,(1+r^2)$ **= 4**r

$\sqrt{3}\,r^2$ **+** $\sqrt{3}$ **= 4**r

$$\sqrt{3}\, r^2 - 3r - r + \sqrt{3} = 0$$

$$\sqrt{3r}\,(r - \sqrt{3}) - 1\,(r - \sqrt{3}) = 0$$

$$(r - \sqrt{3})\ \left(\sqrt{3}\, r - 1\right) = 0$$

$$r = \sqrt{3} \text{ or } \frac{1}{\sqrt{3}}$$

If $r = \sqrt{3}$, $a = 3\sqrt{3}$

$$\frac{a}{r^3} = \frac{3\sqrt{3}}{\left(\sqrt{3}\right)^3} = 1$$

$$\frac{a}{r} = \frac{3\sqrt{3}}{\sqrt{3}} = 3$$

$$ar = \left(3\sqrt{3}\right) . \sqrt{3} = 9$$

$$ar^3 = 3\sqrt{3} . \left(\left(\sqrt{3}\right)^3\right) = 27$$

If $r = \frac{1}{\sqrt{3}}$, $a = 3\sqrt{3}$

$$\frac{a}{r^3} = \frac{3\sqrt{3}}{\left(\frac{1}{\sqrt{3}}\right)^3} = 27$$

$$\frac{a}{r} = \frac{3\sqrt{3}}{\left(\frac{1}{\sqrt{3}}\right)} = 9$$

$$ar = 3\sqrt{3}\ \left(\frac{1}{\sqrt{3}}\right) = 3$$

$$ar^3 = 3\sqrt{3}\ \left(\frac{1}{\sqrt{3}}\right)^3 = 1$$

(1,3,9,27) OR (27,9,3,1)