J. K. SHAH CLASSES
FYJC - TEST - 02

BRANCH : BORIVALI & ANDHERI(A2+ B2) & VASAI **MARKS : 40**
SUB : MATHS - SET 2 **TIME : 1 Hour 30 min.**
Date : 2 /10 /2016

SOLUTION

MEASURES OF DISPERSION

1. For the following frequency distribution the value of Q_1 is 33. Find the missing frequency and then calculate quartile deviation. (4 Marks)

Wages (₹ per hour)	30 – 32	32 – 34	34 – 36	36 – 38	38 – 40	40 – 42	42 - 44
No of workers	?	18	16	14	12	8	4

Ans: Let x be the missing frequency. Now we shall form 1.c.f. table as follows :

Class Interval	Frequency	l.c.f
30 - 32	x	x
32 - 34	18	18 + x
34 – 36	16	34 + x
36 – 38	14	48 + x
38 – 40	12	48 + x
40 – 42	8	68 + x
42 - 44	4	72 + x

Given that, Q_1 = 33, 32 -34 is the first quartile class.

∴ L = 32, c.f. = x, h = 2, f = 18

∴ $Q_1 = L + \frac{h}{f}\left(\frac{N}{4} - c.f.\right)$

∴ $33 = 32 + \frac{2}{18}\left(\frac{(72+x)}{4} - x\right)$

∴ $9 \times (33 - 32) = \left(\frac{(72+x)}{4} - x\right)$

∴ $9 + x = \frac{(72+x)}{4}$

∴ 36 + 4x = 72 + x

∴ 3x = 36

∴ x = 12

Now, N = 72 + 12 = 84

After getting missing frequency, we can form l.c.f. table as follows :

Class Interval	Frequency	l.c.f.
30 – 32	12	12
32 – 34	18	30
34 – 36	16	46
36 – 38	14	60
38 – 40	12	72
40 – 42	8	80
42 - 44	4	84

For Q_3 consider $\left(\frac{3N}{4}\right)$ = 63

∴ Q_3 lies in the class 38 -40

∴ L = 38, c.f. = 60, h = 2, f = 12

∴ $Q_3 = L + \frac{h}{f}\left(\frac{3N}{4} - c.f.\right)$

$= 38 + \frac{2}{12}(63 - 60)$

$= 38 + \frac{6}{12}$

$= 38 + 0.5$

∴ $Q_3 = 38.5$

∴ Q.D. $= \frac{Q_3 - Q_1}{2}$

$= \frac{38.5-33}{2}$

∴ $\boxed{\text{Q.d.} = 2.75}$

2. Compute the mean deviation about
(i) the mean and (ii) the median for the price of share of particular company in a week 40,47,38,41,49. (3 Marks)

Ans: First we find the mean and the median to be given data.

Mean = $\frac{\sum x}{n} = \frac{215}{5}$ = 43

Now we shall find median of the observation. For this we arrange the observation in ascending order as 38,40,41,47,49.

By definition, Median = Me = 41

x_i	$\|x_i - \text{Mean}\| = \|x_i - 43\|$	$\|x_i \ \text{Me}\| = \|x_i - 41\|$
40	3	1
47	4	6
38	5	3
41	2	0
49	6	8
Total	**20**	**18**

From table

$\sum_{i=1}^{n} |x_i - \text{Mean}| = 20,$ $\qquad \sum_{i=1}^{n} |x_i - \text{Me}| = 18$

M.D. about mean $= \frac{\sum |x - \text{Mean}|}{n} = \frac{20}{5} = 4$

M.D. abut median $= \frac{\sum |x - \text{Median}|}{n} = \frac{18}{5} = 3.6$

3. Prices of a particular commodity in five years in two cities are as follows : (4 Marks)

Price in city A	20	22	19	23	26
Price in city B	10	20	18	12	15

Ans: Let price in city a be x and price in city B be y

x	x^2	y	y^2
20	400	10	100
22	484	20	400
19	361	18	324
23	529	12	144
26	676	15	225
110	**2450**	**75**	**1193**

Step 1 :

$$\bar{x} = \frac{\Sigma\ x}{n} = \frac{110}{5} = 22$$

Step 2 :

$$\sigma_x = \sqrt{\frac{\Sigma\ x^2}{n} - \cancel{(\bar{x})}^2}$$

$$= \sqrt{\frac{2450}{5} - (22)^2}$$

$$= \sqrt{490 - 484}$$

$$= \sqrt{6}$$

= 2.45 [Applying log]

Step 3 :

$$\text{C.}V_A = \frac{\sigma_x}{x} \times 100$$

$$= \frac{2.45}{22} \times 100$$

= 11.136 %

Step 4 :

$$\bar{y} = \frac{\Sigma\ y}{n} = \frac{75}{5} = 15$$

Step 5 :

$$\sigma_y = \sqrt{\frac{\Sigma\ y^2}{n} - (\bar{y})^2}$$

$$= \sqrt{\frac{1193}{5} - (15)^2}$$

$$= \sqrt{238.6 - 225}$$

$$= \sqrt{13.6}$$

= 3.69 [Applying log]

Step 6 :

$$= \text{C.}V_B = \frac{\sigma_y}{\bar{y}} \times 100$$

$$= \frac{3.69}{15} \times 100$$

= 24.59 %

Step 7 : Since

$\text{C.}V_A < \text{C.}V_B$

Since the C.V. corresponding to city A is less than Corresponding to city B, the prices show more stability in city A

MOMENTS

4. The first four raw moments of a distribution are 2,20,40 and 50 respectively. Find the first four central moments of the distribution. (3 Marks)

Ans: The first four raw moments of the distribution are

$\mu_1' = 2$, $\mu_2' = 20$, $\mu_3' = 40$ and $\mu_4' = 50$

We want to find first four central moments μ_1, μ_2, μ_3 and μ_4.

We know that $\mu_1 = 0$.

Using relations between raw moments and central moments, We get

$\mu_2 = \mu_2' - \mu_1'^2$

= 20-4 = 16

$\mu_3 = \mu_3' - 3\mu_2'\,\mu_1' + 2\mu_1'^3$

= 40 − 3(20)(2) + 2(8)

= 40 − 120 + 16

= − 64

$\mu_4 = \mu_4' - 4\mu_3' \mu_1' + 6\mu_2' \mu_1^2 - 3\mu_1'^4$

$= 50 - 4(40)(2) + 6(20)(4) - 3(16)$

$= 50-320+480-48$

$= 162$

Thus, the first four central moments are $\mu_1 = 0$, $\mu_2 = 16$, $\mu_3 = -64$ and $\mu_4 = 162$

THEORY OF ATTRIBUTES

5. A report regarding 21km running race is given below. Total number of participants in the race was 1000, out of which 570 were men. 720 participants successfully completed the race. Number of successful men was 512. Find the number of successful women and unsuccessful women & unsuccessful men. (4 Marks)

Solution : Let α denote woman, β denote unsuccessful participant.

We are given that
N= 1000, (A) = 570, (B) = 720, (AB) = 512.
To determine $(\alpha\ B)$, $(\alpha\ \beta), (A\,\beta)$.
We prepare the following 2 × 2 table to find the remaining frequencies.

Table 2.5

	B	β	**Total**
A	(AB) = 512 $(\alpha\, B) = 208$	$(A\,\beta) = 58$ $(\alpha\ \beta) = 222$	(A) = 570 $(\alpha\ )$ = 430
Total	(B) = 720	(β) = 280	N = 1000

∴ Number of successful women = $(\alpha\, B) = 208$
Number of unsuccessful women = $(\alpha\ \beta) = 222$
Number of unsuccessful men = $(A\,\beta) = 58$

6. Out of 240 players, 105 play cricket, 126 play hockey, 75 play football, 24 play both cricket and hockey. 30 play both cricket and football,21 play hockey and football, 9 players play all the three games. Find the number of players playing. (3 Marks)

(i) only two games

(ii) at least two games

(iii) only one game.

Ans:

Solution: Number of players only two games = Number of players playing cricket and hockey but not football + Number of players paying hockey and football but not cricket + Number of players playing cricket and football but not hockey = $(A\,\beta\gamma) + (\alpha\, BC)$ + $(A\,\beta C)$

Now,
$(A\ BY) = (AB) - (ABC) = 24 - 9 = 15$
$(\alpha\, BC) = (BC) - (ABC) = 21 - 9 = 12$
$(A\,\beta C) = (AC) - (ABC) = 30 - 9 = 21$

∴ Number of players playing only two games is 15 + 12 + 21+=48

(ii) Number of players playing at least two game
= Number of Players playing only two games
+ Number of players playing all the three games

= $(A\,\beta Y) + (\alpha\, BC) + (A\,\beta C)$ + (ABC)
= 48 + 9 = 57

(iii) Number of players playing only one game = Total number of players – Number of players playing at least two games = 240 – 57= 183

TRIGONOMETRY

7. **If $2cos^2x$ + 7 sin x = 5, find the permissible values of sin x** **(3 Marks)**

Ans.

$$\text{If } 2cos^2x + 7\sin x = 5$$

$$2(1 - Sin^2x) + 7\sin x = 5$$

$$2 - 2sin^2x + 7\sin x = 5$$

$$2sin^2x - 7\sin x + 3 = 0$$

$$2sin^2x - 6\sin x - \sin x + 3 = 0$$

$$2\sin x\ (\sin x - 3) - 1\ (\sin x - 3) = 0$$

$$(\sin x - 3)\ (2\sin x - 1) = 0$$

Sin x = 3 or Sin x = $\frac{1}{2}$

Since $-1 \leq \text{Sin X} \leq 1$

$$\therefore \boxed{\text{Sin x} = \frac{1}{2}}$$

8. **Eliminateθ, if x = asec θ + b tan θ ; y =asec θ- b tanθ** **(2 Marks)**

Ans. x = a sec θ + b tan θ …………………(I)

Y = a sec θ - b tan θ ……………….(II)

On adding (I) and (II), on subtracting (II) from (I)

x + y = 2asec θ, x − y= 2b tan θ

$\therefore$ sec θ = $\frac{x+y}{2a}$; tan θ = $\frac{x-y}{2b}$

We have, $\sec^2\theta - \tan^2\theta = 1$

$\left(\frac{x+y}{2a}\right)^2$ - $\left(\frac{x-y}{2b}\right)^2$ = 1

$\frac{(x+y)^2}{4a^2}$ - $\frac{(x-y)^2}{4b^2}$ = 1.

LOCUS

9. **If sinθ = $\frac{-12}{13}$, $\pi^c < \theta < \frac{3\pi^c}{2}$, Find the value of sec θ − tan θ** **(4 Marks)**

Ans.

Sin θ = $-\frac{12}{13}$

$$cos^2 = 1 - sin^2\theta$$

$$= 1 - \left(\frac{-12}{13}\right)^2$$

$$= 1 - \frac{144}{169}$$

$$= \frac{25}{169}$$

cos θ = $\pm \frac{5}{13}$

Since $\pi < \theta < \frac{3\pi}{2}$

$\therefore$ **cos θ =** $\frac{-5}{13}$,

Sec θ = $-\frac{1}{\cos\theta}$

Sec θ = $-\frac{-13}{5}$

tan θ = $\frac{\text{Sin}\,\theta}{\cos\theta}$

$$= \frac{\frac{-12}{\cancel{13}}}{\frac{-5}{\cancel{13}}}$$

$$= \frac{12}{5}$$

consider

Sec θ - tan θ

$$= \frac{-13}{5} - \frac{12}{5}$$

$$= -\frac{25}{5}$$

$$= -5$$

ANGLE AND ITS MEASUREMENT

10. **One angle of a quadrilateral has measure $\frac{2\pi}{9}$ radians and the measures of the other three angles are in the ratio 3 :5 : 8, find the measures in radians.** **(3 Marks)**

Ans: The sum of angles of a quadrilateral is 360°.
One of the angles is given to be
$\frac{2\pi}{9} = \frac{2\pi}{9} \times \frac{180}{\pi} = 40°$

$\therefore$ Sum of the remaining three angles is 360° – 40° = 320°

Since these three angles are in the ratio 3 : 5 :8,

degree measures of these angles are $3k$, $5k$, $8k$, where k is constant.

$\therefore\ 3k + 5k + 8k = 320°$

$\therefore\ 16k = 320°$

$\therefore\ K = 20°$

$\therefore$ The measures of three angles are

$(3k)° = (3 \times 20)° = 60°$

$(5k)° = (5 \times 20)° = 100°$

and $(8k)° = (8 \times 20)° = 160°$

$\therefore$ Three angles are 30o, 100o , 160o.

These three angles in radians are

$60° = \left[60\,x\,\frac{\pi}{180}\right]^{c} = \frac{\pi^{c}}{3}$

$100° = \left[100\,x\,\frac{\pi}{180}\right]^{c} = \frac{5\pi^{c}}{9}$

$160° = \left[160\,x\,\frac{\pi}{180}\right]^{c} = \frac{8\pi^{c}}{9}$

LOGARITHM

11. **If** $\log\left(\frac{x-y}{5}\right) = \frac{1}{2}\log x + \frac{1}{2}\log y$, **Show that** $x^2 + y^2 = 27xy$ **(3 Marks)**

Ans.

$\log\left(\frac{x-y}{5}\right) = \frac{1}{2}\log x + \frac{1}{2}\log y$,

$\log\left(\frac{x-y}{5}\right) = \frac{1}{2}\ [\log x + \log y]$,

$\log\left(\frac{x-y}{5}\right) = \frac{1}{2}\log (xy)$,

$\log\left(\frac{x-y}{5}\right) = \log (xy)^{1/2}$,

$\log\left(\frac{x-y}{5}\right) = \log \sqrt{xy}$,

Dropping log :

$\frac{x-y}{5} = \sqrt{xy}$,

Squaring

$$\left(\frac{X-y}{5}\right)^2 = xy$$

$$\frac{(x-y)^2}{25} = xy$$

$$x^2 - 2xy + y^2 = 25xy$$

$$\boxed{x^2 + y^2 = 27xy}$$

COMPLEX NUMBER **(3 Marks)**

12. **If a and b are real and $(i^4 + 3i)$ a + $(i-1)$b + $5i^3$ = 0, Find a and b**

Ans. $(i^4 + 3i)\ a + (i-1)b + 5i^3 = 0 = 0 + 0\,i$

$\therefore\ (1 + 3i)\ a + (i-1)b + 5i^3 = 0 + 0\,i$

$\therefore\ a + 3a\,i + b\,i - b - 5\,i = 0 + 0\,i$

$(a\text{-}b) + (3a + b - 5)\,i = 0 + 0\,i$

Equating real and imaginary parts on both sides, we get,

a – b = 0 and 3a + b-5 = 0

$\therefore$ **a = b and 3a + b = 5**

$\therefore\ 3a + a = 5$

$4a = 5$

a = $\frac{5}{4}$

$\therefore$ **a = b =** $\frac{5}{4}$