J. K. SHAH CLASSES

FYJC - TEST 01- SOLUTION

BRANCH : **BORIVALI & VASAI**
CHAPTER : LOGARITHM & COMPLEX NUMBERS

BATCH : FYJC B1+B2+C
TIME : 1 hr 15 mins
MARKS : 30

SOLUTION

Section – 1

1) $a^2 + b^2 = 14ab$

$a^2 + 2ab + b^2 = 14ab + 2ab$

$(a+b)^2 = 16ab$

Inserting log
$\text{Log } (a + b)^2 = \log 16ab$
$2\log(a + b) = \log 16 + \log a + \log b$
$2\log(a+b) = \log 4^2 + \log a + \log b$
$2\log (a+b) = 2\log 4 + \log a+\log b$ ....PROVED

2) LHS =

$= \frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z}$

$= \frac{1}{1+log_a bc} + \frac{1}{1+log_b ac} + \frac{1}{1+log_c ab}$

$= \frac{1}{1+\frac{logbc}{loga}} + \frac{1}{\frac{logb+logac}{logb}} + \frac{1}{\frac{\log c+logab}{logc}}$

$= \frac{1}{\frac{loga+logbc}{loga}} + \frac{1}{\frac{logb+logac}{logb}} + \frac{1}{\frac{logc+logab}{logc}}$

$= \frac{loga}{loga+logbc} + \frac{logb}{logb+logac} + \frac{logc}{logc+logab}$

$= \frac{loga}{logabc} + \frac{logb}{logabc} + \frac{logc}{logabc}$

$= \log_{abc} a + \log_{abc} b + \log_{abc} c$

$= \log_{abc} abc$
$=1$ ....PROVED

Alternatively:

$=\frac{loga+logb+logc}{logabc}$

$= \frac{logabc}{logabc}$

= 1 ……….PROVED

3. $a^2 = b^3 = c^4 = d^5$

Inserting log

$Loga^2 = logb^3 = logc^4 = logd^5 = K$
2 log a = 3logb = 4logc = 5logd = K
Loga = $\frac{K}{2}$, logb = $\frac{K}{3}$,loga = $\frac{K}{4}$, logb = $\frac{K}{5}$
L.H.S.= log_a^{bcd}

$= \frac{logbcd}{loga}$

$= \frac{logb+logc+logd}{loga}$

$= \frac{\frac{K}{3}+\frac{K}{4}+\frac{K}{5}}{\frac{K}{2}}$

$= \frac{k\left(\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right)}{\frac{k}{2}}$

$= \frac{\left(\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right)}{\left(\frac{1}{2}\right)}$

$= \left(\frac{20+15+12}{\frac{60}{\left(\frac{1}{2}\right)}}\right)$

$=\frac{47}{30}$ = RHS …………PROVED

4. $log_2x + log_4x + log_6x = \frac{21}{4}$
$\frac{logx}{log2} + \frac{logx}{log4} + \frac{logx}{log16} = \frac{21}{4}$

$\frac{logx}{log2} + \frac{logx}{log2^2} + \frac{logx}{log2^4} = \frac{21}{4}$

$\frac{logx}{log2}$ + $\frac{logx}{2log2}$ + $\frac{logx}{4log2}$= $\frac{21}{4}$

$\frac{logx}{log2}$ $\left(\frac{1}{1} + \frac{1}{2} + \frac{1}{4}\right)$ = $\frac{21}{4}$

$$\frac{logx}{log2}\left(\frac{4+2+1}{4}\right) = \frac{21}{4}$$

$\frac{logx}{log2}$ $\left(\frac{7}{4}\right)$ = $\frac{21}{4}$

$\frac{logx}{log2}$ = 3

Logx = 3log2

Logx = log2^3

X = 8

5. log2 + log(x + 3) – log(3x – 5) = log 3
log [2(x + 3)] – log (3x – 5) = log3
log (2x + 6) – log(3x – 5) = log3
∴ log $\left(\frac{2x+6}{3x-5}\right)$ = log3
∴ $\frac{2x+6}{3x-5}$ = 3
$2x + 6 = 9x - 15$
∴ 7x = 21
$x = 3$

SECTION – 2

1. $\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{10}}\right)^{10}$ + $\left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)^{10}$

$= \left[\left(\frac{1+i}{\sqrt{2}}\right)^2\right]^5$ + $\left[\left(\frac{1-i}{\sqrt{2}}\right)^2\right]^5$

= $\left[\frac{1+2i+i^2}{2}\right]^5$ + $\left[\frac{1-2i+i^2}{2}\right]^5$

= $\left[\frac{1+2i+1}{2}\right]^5$ + $\left[\frac{1-2i-1}{2}\right]^5$

= $\left[\frac{2i}{2}\right]^5$ + $\left[\frac{-2i}{2}\right]^5$

= $(i)^5$ + $(-i)^5$

= $(i)^5$ - $(-i)^5$

= 0

2. $x = \frac{9}{2+\sqrt{5}\,i}$

$x = \frac{9}{2+\sqrt{5}i} \; \frac{2-\sqrt{5}\,i}{2-\sqrt{5}\,i}$

$x = \frac{9(2-\sqrt{5}\,i)}{4-5i^2}$

$x = \frac{9(2-\sqrt{5}\,i)}{4+5}$

$x = \frac{9(2-\sqrt{5}\,i)}{9}$

$x = 2-\sqrt{5}\,i$

$x - 2 = -\sqrt{5}\,i$

$(x-2)^2 = (-\sqrt{5}\,i)^2$

$x^2 - 4x + 4 = 5i^2$

$x^2 - 4x + 4 = -5$

$x^2 - 4x + 4 + 5 = 0$

$x^2 - 4x + 9 = 0$

$$
\begin{array}{r|l}
 & x + 3 \\
\hline
x^2 - 4x + 9 & x^3 - x^2 - 3x + 30 \\
 & x^3 - 4x^2 + 9x \\
 & - \quad + \quad - \\
\hline
 & 3x^2 - 12x + 30 \\
 & 3x^2 - 12x + 27 \\
 & - \quad + \quad - \\
\hline
 & 3
\end{array}
$$

$\therefore$ $x^3-x^2-3x+30$
$= (x^2-4x+9)(x+3)+3$
$= 0(x+3)+3$
$= 0+3$
$= 3$

3. Let $\sqrt{6+8i}$ = a+ib, a, b $\in$ R
On squaring
$6+8i = (a+ib)^2$
$6+8i = a^2 - b^2 + 2abi$
Equating real and imaginary parts,
$6 = a^2 - b^2$ …………(1)
$8 = 2ab$ …………(2)
$\therefore$ $a = \frac{4}{b}$

$6 = \left(\frac{4}{b}\right)^2 - b^2$

$6 = \frac{16}{b^2} - b^2$

$\therefore b^4 + 6b^2 - 16 = 0$

Put $b^2 = m$

$\therefore m^2 + 6m - 16 = 0$

$(m+8)(m-2) = 0$

$\therefore m = -8$ or $m = 2$

i.e. $b^2 = -8$ or $b^2 = 2$

but $b \in R$ $\therefore b^2 \neq -8$

$\therefore b^2 = 2$ $b = \pm\sqrt{2}$

when $b = \sqrt{2}$, $a = 2\sqrt{2}$

$\therefore$ Square root of 6+8i

$= 2\sqrt{2} + \sqrt{2}\,i = \sqrt{2}\,(2+i)$

when $b = -\sqrt{2}$, $a = -2\sqrt{2}$

$\therefore$ Square root of 6+8i

$= -2\sqrt{2} - \sqrt{2}i = -\sqrt{2}(2+i)$

$\sqrt{6+8i} = \pm\sqrt{2}\,(2+i)$