SOLUTION TO MATHEMATICAL LOGIC

Q1. Use appropriate symbols and connectives to express the following SYMBOLICALLY

01. Dhanashri is beautiful and she is intelligent $\equiv$ $p \wedge q$

where $p \equiv$ Dhanashri is beautiful

$q \equiv$ Dhanashri is intelligent

02 Either we play Kabaddi or go for cycling $\equiv$ $p \vee q$

where $p \equiv$ We play Kabaddi

$q \equiv$ We go cycling

03. Ashok failed or Nirmala passed and she is happy $\equiv$ $p \vee (q \wedge r)$

where $p \equiv$ Ashok failed

$q \equiv$ Nirmala passed

$r \equiv$ Nirmala is happy

04. Pinku never works hard yet she gets good marks $\equiv$ $\sim p \wedge q$

(Pinku does not work hard and still she gets good marks)

where $p \equiv$ Pinku works hard

$q \equiv$ Pinku gets good marks

05. Though God created man , Man created many $\equiv$ $p \wedge q$

(God created Man and Man created Many)

where $p \equiv$ God created Man

$q \equiv$ Man created Many

06. Eventhough it is not cloudy , it is still raining $\equiv$ $\sim p \wedge q$

(It is not cloudy and still it is raining)

where $p \equiv$ It is cloudy

$q \equiv$ It is still raining

07. The drug is effective though it has side effects $\equiv$ $p \wedge q$

(The drug is effective but (and) it has side effects)

where $p \equiv$ The drug is effective

$q \equiv$ The drug has side effects

08. Inspite of bad weather , India won the cricket match $\equiv$ $p \wedge q$

(the weather is bad and still India won the cric ket match)

where $p \equiv$ The weather is bad

$q \equiv$ India won the cricket match

09. Yuvraj neither plays cricket nor tennis $\equiv$ $\sim p \wedge \sim q$

(Yuvraj does not play cricket and he does not play tennis)

where $p \equiv$ Yuvraj plays cricket

$q \equiv$ Yuvraj plays tennis

10. It may or may not rain but sky is cloudy $\equiv$ $(p \vee \sim p) \wedge q$

where $p \equiv$ It may rain

$q \equiv$ The sky is cloudy

11. If Kutub – Minar is in Delhi then Hyderabad is in Andhra Pradesh $\equiv p \rightarrow q$

where $p \equiv$ Kutub – Minar is in Delhi

$q \equiv$ Hyderabad is in Andhra Pradesh

12. Two triangles have equal areas only if they are congruent $\equiv p \rightarrow q$

(Recall : Rhombus only if parallelogram means Rhombus $\rightarrow$ Parallelogram)

where $p \equiv$ Two triangles have equal areas

$q \equiv$ two triangles are congruent

13. It is not true that Subhash passed , then he is happy $\equiv$ $\sim (p \rightarrow q)$

where $p \equiv$ Subhash passed

$q \equiv$ Subhash is happy

14. If the question is not <u>e</u>asy then we not <u>f</u>ail $\equiv$ $\sim e \rightarrow \sim f$

where $e \equiv$ The question is easy

$f \equiv$ we will fail

15. if Pravin likes a radio programme and the programme is a sponsored programme then the programme is not in the evening before 7 O'clock $\equiv$ $(p \wedge q) \rightarrow \sim q$

where $p \equiv$ Pravin likes a radio programme

$q \equiv$ the programme is a sponsored programme

$r \equiv$ programme is in the evening before 7 O'clock

16. ABC is a triangle hence the points A, B & C are not collinear $\equiv$ $p \rightarrow \sim q$

(means : if ABC is a triangle then points A , B & C are not collinear)

where $p \equiv$ ABC is a triangle

$q \equiv$ points A , B & C are collinear

17. A person is successful only if he is a politician or he has good connections

$\equiv$ $p \rightarrow (q \vee r)$

(Recall : 'only if' stands for 'implies')

where $p \equiv$ A person is successful

$q \equiv$ A person is a politician

$q \equiv$ A person has good connections

18. As Ram is tall he can be a good basket ball player $\equiv$ $p \rightarrow q$

(means : if Ram is tall then he can be a good basket ball player)

where $p \equiv$ Ram is tall

$q \equiv$ Ram can be a good basket ball player

19. $\sqrt{25} = 4$ is necessary condition for the number 8 to be an even number

(Try to understand this : Parallelogram is necessary condition for rhombus and hence we say " if Rhombus then it is a parallelogram

$\therefore$ the above statement should be read as if 8 is even then condition for that is $\sqrt{25} = 4$)

$\equiv$ $p \rightarrow q$

where $p \equiv$ 8 is an even number

$q \equiv \sqrt{25} = 4$

20. Milk is white if and only if the sky is not blue $\equiv$ $p \leftrightarrow \sim q$

where $p \equiv$ Milk is white

$q \equiv$ Sky is blue

Q2. CONSTRUCT TRUTH TABLES

1. $(p \wedge q) \wedge \sim p$

p	q	~p	p ∧ q	(p ∧ q) ∧ ~p
T	T	F	T	F
T	F	F	F	F
F	T	T	F	F
F	F	T	F	F

2. $(\sim p \vee q) \wedge (\sim p \wedge \sim q)$

p	q	~ p	~ q	~p ∨ q	~p ∧ ~q	(~p ∨ q) ∧ (~p ∧ ~q)
T	T	F	F	T	F	F
T	F	F	T	F	F	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

3. $(p \wedge \sim q) \wedge \sim(q \wedge \sim p)$

p	q	~ p	~ q	p ∧ ~q	q ∧ ~p	~(q ∧ ~p)	(p ∧~q) ∧ ~(q ∧ ~p)
T	T	F	F	F	F	T	F
T	F	F	T	T	F	T	T
F	T	T	F	F	T	F	F
F	F	T	T	F	F	T	F

4. $(p \wedge q) \rightarrow (p \vee \sim q)$

p	q	~ q	p ∧ q	p ∨ ~q	(p ∧ q) → (p ∨ ~q)
T	T	F	T	T	T
T	F	T	F	T	T
F	T	F	F	F	T
F	F	T	F	T	T

5. $(p \to q) \leftrightarrow (\sim p \lor q)$

p	q	$\sim p$	$p \to q$	$\sim p \lor q$	$(p \to q) \leftrightarrow (\sim p \lor q)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

6. $(p \land \sim q) \leftrightarrow (q \to p)$

p	q	$\sim q$	$p \land \sim q$	$q \to p$	$(p \land \sim q) \leftrightarrow (q \to p)$
T	T	F	F	T	F
T	F	T	T	T	T
F	T	F	F	F	T
F	F	T	F	T	F

7. $(\sim p \lor \sim q) \leftrightarrow \sim(p \land q)$

p	q	$\sim p$	$\sim q$	$\sim p \lor \sim q$	$p \land q$	$\sim (p \land q)$	$(\sim p \lor \sim q) \leftrightarrow \sim(p \land q)$
T	T	F	F	F	T	F	T
T	F	F	T	T	F	T	T
F	T	T	F	T	F	T	T
F	F	T	T	T	F	T	T

Q3. Determine whether the following statement pattern is TAUTOLOGY or CONTRADICTION or neither

1. $[p \land (p \to q)] \to q$

p	q	$p \to q$	$p \land (p \to q)$	$[p \land (p \to q)] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Since all the truth values in the last column are 'T' , given statement is 'Tautology'

2. $(\sim p \vee q) \vee (q \rightarrow p)$

p	q	~ p	~p ∨ q	q → p	(~p ∨ q) ∨ (q → p)
T	T	F	T	T	T
T	F	F	F	T	T
F	T	T	T	F	T
F	F	T	T	T	T

Since all the truth values in the last column are 'T' , given statement is 'Tautology'

3. $(p \wedge \sim q) \leftrightarrow (p \rightarrow q)$

p	q	~ q	p ∧ ~q	p → q	(p ∧ ~q) ↔ (q → p)
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	F	T	F
F	F	T	F	T	F

Since all the values in the last column are 'F' , given statement is 'Contradiction'

4. $(p \wedge q) \wedge (p \rightarrow \sim q)$

p	q	~ q	p ∧ q	p → ~q	(p ∧ q) ∧ (p → ~q)
T	T	F	T	F	F
T	F	T	F	T	F
F	T	F	F	T	F
F	F	T	F	T	F

Since all the values in the last column are 'F' , given statement is 'Contradiction'

6. $(\sim p \wedge \sim q) \wedge (q \wedge r)$

p	q	r	~p	~q	~p ∧~q	q ∧ r	(~p ∧~q) ∧ (q ∧ r)
T	T	T	F	F	F	T	F
T	T	F	F	F	F	F	F
T	F	T	F	T	F	F	F
T	F	F	F	T	F	F	F
F	T	T	T	F	F	T	F
F	T	F	T	F	F	F	F
F	F	T	T	T	T	F	F
F	F	F	T	T	T	F	F

Since all the values in the last column are 'F' , given statement is 'Contradiction'

Q3. Prove that the following statements are LOGICALLY EQUIVALENT

2. $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

p	q	r	$q \vee r$	$p \wedge q$	$p \wedge r$	COL A $p \wedge (q \vee r)$	COL B $(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

Since truth values in col A and col B are identical $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

3. a) $\sim (p \vee q) \equiv \sim p \wedge \sim q$

p	q	$\sim p$	$\sim q$	$p \vee q$	COL A $\sim (p \vee q)$	COL B $\sim p \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

Since truth values in col A and col B are identical $\sim (p \vee q) \equiv \sim p \wedge \sim q$

4. $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

p	q	COL A $p \leftrightarrow q$	$p \rightarrow q$	$q \rightarrow p$	COL B $(p \rightarrow q) \wedge (q \rightarrow p)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

Since truth values in col A and col B are identical $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

5. $p \rightarrow q \equiv \sim q \rightarrow \sim p \equiv \sim p \vee q$

p	q	$\sim p$	$\sim q$	COL A $p \rightarrow q$	COL B $\sim q \rightarrow \sim p$	COL C $\sim p \vee q$
T	T	F	F	T	T	T
T	F	F	T	F	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

7. $(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

					COL A			COL B
p	q	r	$q \vee r$	$p \vee q$	$(p \vee q) \rightarrow r$	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	F	F	F	F
T	F	T	T	T	T	T	T	T
T	F	F	F	T	F	F	T	F
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	T	F	F
F	F	T	T	F	T	T	T	T
F	F	F	F	F	T	T	T	T

Since truth values in col A and col B are identical $(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

Q4. if p and q are true statements and r , s are false statements , find the truth values

1. $p \vee (q \wedge r)$

Replacing by their truth values

$\equiv T \vee (T \wedge F)$

(RECALL : AND MEIN EK JATHI AND DOOSRI NAHI JATHI ; ITS FALSE)

$\equiv T \vee F$

$\equiv T$

(RECALL : OR MEIN EK JATHI , DOOSRI NAHI JATHI ; ITS TRUE)

2. $(p \wedge \sim r) \wedge (\sim q \wedge s)$

Replacing by their truth values

$\equiv (T \wedge \sim F) \wedge (\sim T \wedge F)$

$\equiv (T \wedge T) \wedge (F \wedge F)$

RECALL AND MEIN DONO JATHI ITS TRUE ; DONO NAHI JATHI ITS FALSE

$\equiv T \wedge F$

$\equiv F$

(RECALL : AND MEIN EK JATHI AND DOOSRI NAHI JATHI ; ITS FALSE)

3. $\sim (p \wedge \sim r) \vee (\sim q \vee s)$

Replacing by their truth values

$\equiv \ \sim (T \wedge \sim F) \vee (\sim T \vee F)$

$\equiv \ \sim (T \wedge T) \vee (F \vee F)$

$\equiv \ \sim T \vee F$

$\equiv \ F \vee F$

$\equiv \ F$

OR MEIN DONO NAHI JATHI THEN IT IS FALSE

4. $(p \wedge \sim r) \rightarrow (q \wedge s)$

Replacing by their truth values

$\equiv \ (T \wedge \sim F) \rightarrow (T \wedge F)$

$\equiv \ (T \wedge T) \rightarrow (T \wedge F)$

$\equiv \ T \rightarrow F$

$\equiv \ F$

5. $\left[(p \rightarrow q) \rightarrow (q \rightarrow r)\right] \rightarrow (r \rightarrow s)$

Replacing by their truth values

$\equiv \ \left[(T \rightarrow T) \rightarrow (T \rightarrow F)\right] \rightarrow (F \rightarrow F)$

$\equiv \ \left[T \rightarrow F\right] \rightarrow T$

$\equiv \ F \rightarrow T$

$\equiv \ T$

Q6. from the following set of statements , identify the pairs of statements having same meaning

BASED ON $p \rightarrow q \equiv \sim q \rightarrow \sim p$

1. a) if man is rich then he is happy $\equiv$ $R \rightarrow H$

b) if man is not rich then he is not happy $\equiv$ $\sim R \rightarrow \sim H$

c) if man is unhappy then he is not rich $\equiv$ $\sim H \rightarrow \sim R$

d) if man is happy then he is rich $\equiv$ $H \rightarrow R$

SOLUTION : Since $P \rightarrow Q \equiv \sim Q \rightarrow \sim P$

$R \rightarrow H \equiv \sim H \rightarrow \sim R$; statement (a) & (c) have same meaning

$H \rightarrow R \equiv \sim R \rightarrow \sim H$; statement (b) & (d) have same meaning

2 a) if a man is rich then he buys a car ≡ $R \to C$

b) if a man is not rich then he does not buy a car ≡ $\sim R \to \sim C$

c) if a man buys a car , then he is rich ≡ $C \to R$

d) if man does nor buy a car then he is not rich ≡ $\sim C \to \sim R$

SOLUTION : Since $P \to Q \equiv \sim Q \to \sim P$

$R \to C \equiv \sim C \to \sim R$; statement (a) & (d) have same meaning

$C \to R \equiv \sim R \to \sim C$; statement (b) & (c) have same meaning

Q7. Rewrite the statement without using 'conditional'

a) If productivity increases then wages rise

≡ $P \to Q$

≡ $\sim P \vee Q$

≡ Productivity does not increase or the wages rise

b) If it is cold , Madhu wears a hat

≡ $P \to Q$

≡ $\sim P \vee Q$

≡ It is not cold or Madhu wears a hat

Q8. Rewrite the statement removing the connective 'if and only if' and using the connectives 'not' ; 'or' ; 'and'

a) Kiran is rich if and only if he is honest

≡ $P \leftrightarrow Q$

≡ $(P \to Q) \wedge (Q \to P)$

≡ $(\sim P \vee Q) \wedge (\sim Q \vee P)$

≡ Kiran is not rich or He is honest and Kiran is not honest or he is rich

b) The demand falls if and only if the price increases

≡ $P \leftrightarrow Q$

≡ $(P \rightarrow Q) \wedge (Q \rightarrow P)$

≡ $(\sim P \vee Q) \wedge (\sim Q \vee P)$

≡ Demand does not fall or price increases and Price does not increase or demand falls

Q9. Write CONVERSE – CONTRA-POSITIVE – INVERSE for the given conditional statements

1. If Ravi is good in Logic then Ravi is good in Mathematics

SOLUTION :

LET $P \rightarrow Q$ ≡ If Ravi is good in Logic then Ravi is good in Mathematics

CONVERSE : $Q \rightarrow P$

If Ravi is good in Mathematics then he is good in Logic

CONTRAPOSITIVE : $\sim Q \rightarrow \sim P$

If Ravi is not good in Mathematics then he is not good in Logic

INVERSE : $\sim P \rightarrow \sim Q$

If Ravi is not good in Logic then he is not good in Mathematics

2. If function is differentiable then it is continuous

SOLUTION :

LET $P \rightarrow Q$ ≡ If function is differentiable then it is continuous

CONVERSE : $Q \rightarrow P$

If the function is continuous then it is differentiable

CONTRAPOSITIVE : $\sim Q \rightarrow \sim P$

If the function is not continuous then it is not differentiable

INVERSE : $\sim P \rightarrow \sim Q$

If the function is not differentiable then it is continuous

3. if a man is a bachelor , then he is unhappy

SOLUTION :

LET $P \to Q \equiv$ if a man is a bachelor then he is unhappy

CONVERSE : $Q \to P$

If man is unhappy then he is a bachelor

CONTRAPOSITIVE : $\sim Q \to \sim P$

If man is happy then he is not a bachelor

INVERSE : $\sim P \to \sim Q$

If a man is not a bachelor then he is happy

4. The crop will be destroyed if there is a flood

SOLUTION :

LET $P \to Q \equiv$ if there is a flood then the crops will be destroyed

CONVERSE : $Q \to P$

If the crop will be destroyed then there is a flood

CONTRAPOSITIVE : $\sim Q \to \sim P$

If the crop will not be destroyed then there is no flood

INVERSE : $\sim P \to \sim Q$

If there is no flood then the crop will not be destroyed

5. A family becomes literate if the women in it are literate

SOLUTION :

LET $P \to Q \equiv$ if the women in the family are literate then the family becomes literate

CONVERSE : $Q \to P$

If the family becomes literate then the women in the family are literate

CONTRAPOSITIVE : $\sim Q \to \sim P$

If the family does not become literate then the women in the family are not literate

INVERSE : $\sim P \to \sim Q$

if the women in the family are not literate then the family does not becomes literate

6. **Quadrilateral is a rhombus only if it is a parallelogram**

SOLUTION :

LET $P \to Q$ ≡ if the quadrilateral is a rhombus then it is a parallelogram

CONVERSE : $Q \to P$

if the quadrilateral is a parallelogram then it is a rhombus

CONTRAPOSITIVE : $\sim Q \to \sim P$

if the quadrilateral is a not parallelogram then it is not a rhombus

INVERSE : $\sim P \to \sim Q$

if the quadrilateral is not a rhombus then it is not a parallelogram

Q10. Write the NEGATIONS of the following statements

De Morgan's Law

$$\sim (p \vee q) \equiv \sim p \wedge \sim q$$
$$\sim (p \wedge q) \equiv \sim p \vee \sim q$$

1. **Tajmahal is in India and Everest is in Nepal**

Using : $\sim(P \wedge Q) \equiv \sim P \vee \sim Q$

Negation : Tajmahal is not in India OR Everest is not in Nepal

2. **Madhu is fair and Mahesh is intelligent**

Using : $\sim(P \wedge Q) \equiv \sim P \vee \sim Q$

Negation : Madhu is not fair OR Mahesh is not intelligent

3. **policeman is honest and he is not rich**

Using : $\sim(P \wedge Q) \equiv \sim P \vee \sim Q$

Negation : Policeman is honest OR he is rich

4. **It is cold or it is raining**

Using : $\sim(P \vee Q) \equiv \sim P \wedge \sim Q$

Negation : It is not cold AND it is not raining

5. **I will have tea or coffee**

Using : $\sim(P \vee Q) \equiv \sim P \wedge \sim Q$

Negation : I will not have tea AND I will not have coffee

6. **5 is prime number or 20 is a composite number**

Using : $\sim(P \vee Q) \equiv \sim P \wedge \sim Q$

Negation : 5 is not a prime number AND 20 is not a composite number

7. **the question paper is easy or we shall pass**

Using : $\sim(P \vee Q) \equiv \sim P \wedge \sim Q$

Negation : the question paper is not easy AND we shall not pass

8. **Ram is intelligent but lazy**

Using : $\sim(P \wedge Q) \equiv \sim P \vee \sim Q$

Negation : Ram is not intelligent OR Ram is not lazy

09. **Kitchen is small but neat**

Using : $\sim(P \wedge Q) \equiv \sim P \vee \sim Q$

Negation : Kitchen is not small OR Kitchen is not neat

10. **the teacher must have both charisma and diplomacy**

Using : $\sim(P \wedge Q) \equiv \sim P \vee \sim Q$

Negation : the teacher must not have charisma OR she must not have diplomacy

11. **10 > 5 and 2 < 7**

Using : $\sim(P \wedge Q) \equiv \sim P \vee \sim Q$

Negation : 10 is not greater than 5 OR 2 is not less than 7

12. **Ashok reads daily news paper DNA or TOI**

Using : $\sim(P \vee Q) \equiv \sim P \wedge \sim Q$

Negation : Ashok does not read daily newspaper DNA AND Ashok does not read TOI

$\sim(P \to Q) \equiv P \wedge \sim Q$

Recall How To Remember : When is implies false ?

When Ali is going and Bob is not going

13. **If ABC is triangle then $\angle A + \angle B + \angle C = 180°$.**

Using : $\sim(P \to Q) \equiv P \wedge \sim Q$

Negation : ABC is a triangle and $\angle A + \angle B + \angle C \neq 180°$.

14. **if the diagonals of a parallelogram are perpendicular then it is a rhombus**

Using : $\sim(P \to Q) \equiv P \wedge \sim Q$

Negation : Diagonals of parallelogram are perpendicular and it is not a rhombus

15. **if question paper is easy then Pravin will pass**

Using : $\sim(P \to Q) \equiv P \wedge \sim Q$

Negation : Question paper is easy and Pravin will not pass

16. **if the lines are parallel then their slopes are equal**

Using : $\sim(P \to Q) \equiv P \wedge \sim Q$

Negation : lines are parallel and their slopes are not equal

17. **if monsoon is good then farmers are happy**

Using : $\sim(P \to Q) \equiv P \wedge \sim Q$

Negation : Monsoon is good and farmers are not happy

18. **if $2 + 5 = 10$ then $4 + 10 = 20$**

Using : $\sim(P \to Q) \equiv P \wedge \sim Q$

Negation : $2 + 5 = 10$ and $4 + 10 \neq 20$

19. **If it snows then Gajashri does not drive the car**

Using : $\sim(P \to Q) \equiv P \wedge \sim Q$

Negation : It snows and Gajashri does drive the car

$$\sim (p \leftrightarrow q) \equiv (\sim p \wedge q) \vee (p \wedge \sim q)$$

Recall How To Remember : Ali is going and Bob is not going

OR

Bob is going and Ali is not going

20. Price increases if and only if Demand falls

Using : $\sim(P \leftrightarrow Q) \equiv (P \wedge \sim Q) \vee (Q \wedge \sim P)$

Negation : Price increases and demand does not fall

OR

Demand falls and price does not increase

21. Tomorrow will be Monday if and only if today is Sunday

Using : $\sim(P \leftrightarrow Q) \equiv (P \wedge \sim Q) \vee (Q \wedge \sim P)$

Negation : Tomorrow will be Monday and today is not Sunday

OR

Today is Sunday and Tomorrow will not be Monday

22. Rajdhar is successful if and only if he is hardworking

Using : $\sim(P \leftrightarrow Q) \equiv (P \wedge \sim Q) \vee (Q \wedge \sim P)$

Negation : Rajdhar is successful and he is not hardworking

OR

Rajdhar is hardworking and he is not successful

23. A student will get a seat to M.B.A. if and only if he is rich

Using : $\sim(P \leftrightarrow Q) \equiv (P \wedge \sim Q) \vee (Q \wedge \sim P)$

Negation : A student will get a seat to MBA and he is not rich

OR

The student is rich and he will not get a seat to MBA

24. **$(a + b)^2 = a^2 + b^2$ if and only if $ab = 0$**

Using : $\sim(P \leftrightarrow Q) \equiv (P \wedge \sim Q) \vee (Q \wedge \sim P)$

Negation : $(a + b)^2 = a^2 + b^2$ and $ab \neq 0$

OR

$ab = 0$ and $(a + b)^2 \neq a^2 + b^2$

Q11. write NEGATIONS of the following

EVERYTHING KA SOMETHING

SOMETHING KA NOTHING

NOTHING KA SOMETHING

1. **every student from this class passed**

We change 'Everything' to 'Something'

Negation : Some students from this class did not pass

2. **all students are sincere**

We change 'Everything' to 'Something'

Negation : Some students are not sincere

3. **all politicians are corrupt**

We change 'Everything' to 'Something'

Negation : Some politicians are not correct

4. **All natural numbers are integers**

We change 'Everything' to 'Something'

Negation : Some natural numbers are not integers

5. **All parents care for their children**

We change 'Everything' to 'Something'

Negation : Some parents do not care for their children

6. **all girls are made of sugar or honey**

We change 'Everything' to 'Something'

Using : $\sim(P \vee Q) \equiv \sim P \wedge \sim Q$

Negation : Some girls are not made of sugar and honey

7. **some dogs are intelligent**

We change 'Something' to 'Nothing'

Negation : No dog is intelligent

8. **Some students of the standard XII are eighteen years old**

We change 'Something' to 'Nothing'

Negation : No student of standard XII is eighteen years old

9. **Some buildings in this area are multistoried**

We change 'Something' to 'Nothing'

Negation : No building in this area is multistoried

10. **No man is animal**

We change 'Nothing' to 'Something'

Negation : Some men are animals

11. **No policeman is polite**

We change 'Nothing' to 'Something'

Negation : Some policemen are polite

12. **some bosses are good**

We change 'Something' to 'Nothing'

Negation : No boss is good

13. **Some members of the Indian cricket team are not committed**

Recall here we decided that since there is a 'not' in the given statement we will not change 'Something' to 'Nothing' but will change it to 'Everything' so that we get chance to drop the 'not'

We change 'Something' to 'Everything'

Negation : Every member of the Indian cricket team is committed

14. **Some students have not paid the fees**

We change 'Something' to 'Everything'

Negation : Every student has paid the fees

15. **every integer is a rational and every rational is a real**

We change 'Everything' to 'Something'

Using : **$\sim(P \wedge Q) \equiv \sim P \vee \sim Q$**

Negation : Some integers are not rational OR Some rationals are not real

16. **All students have completed their homework and the teacher is present**

We change 'Everything' to 'Something'

Using : **$\sim(P \wedge Q) \equiv \sim P \vee \sim Q$**

Negation : Some students have not completed their homework

OR

Teacher is not present

17. **$\forall\, n \in N\, ,\, n + 7 > 8$**

Negation : $\exists\, n \in N$, such that $n + 7$ is not greater than 8

18. **$\forall\, x \in N$, $x^2 + x$ is an even number**

Negation : $\exists\, x \in N$, such that $x^2 + x$ is not an even number

19. **$\exists\, n \in N$, such that $n^2 = n$**

Negation : $\forall\, n \in N\, ,\, n^2 \neq n$

20. **$\exists\, x \in R$, such that $x^2 < x$**

Negation : $\forall\, x \in R\, ,\, x^2 \geq x$ **OR** $\forall\, x \in R$, x^2 is not less than x

21. **$\exists\, n \in N$, such that $n + 4 > 9$**

Negation : $\forall\, n \in N$, $n + 4$ is not greater than 9

Q12. Using rules of negation , write down the negation of the following statements

1. $(p \wedge \sim q) \wedge (\sim p \vee \sim q)$

Solution : $\sim \left[(p \wedge \sim q) \wedge (\sim p \vee \sim q) \right]$

$\equiv \sim (p \wedge \sim q) \vee \sim (\sim p \vee \sim q)$ ..... De Morgan's Law

$\equiv \left[\sim p \vee \sim (\sim q) \right] \vee \left[\sim (\sim p) \wedge \sim (\sim q) \right]$ ..... De Morgan's Law

$\equiv (\sim p \vee q) \vee (p \wedge q)$

2. $(\sim p \wedge \sim q) \vee (p \wedge \sim q)$

Solution : $\sim \left[(\sim p \wedge \sim q) \vee (p \wedge \sim q) \right]$

$\equiv \sim (\sim p \wedge \sim q) \wedge \sim (p \wedge \sim q)$ ..... De Morgan's Law

$\equiv \left[\sim (\sim p) \vee \sim (\sim q) \right] \wedge \left[\sim p \vee \sim (\sim q) \right]$ ..... De Morgan's Law

$\equiv (p \vee q) \wedge (\sim p \vee q)$

3. $(p \wedge q) \rightarrow (\sim p \vee r)$

Solution : $\sim \left[(p \wedge q) \rightarrow (\sim p \vee r) \right]$

$\equiv (p \wedge q) \wedge \sim (\sim p \vee r)$ ..... $\sim(P \rightarrow Q) \equiv P \wedge \sim Q$

$\equiv (p \wedge q) \wedge \left[\sim (\sim p) \wedge \sim r \right]$ ..... De Morgan's Law

$\equiv (p \wedge q) \wedge (p \wedge \sim r)$

4. $p \rightarrow (q \wedge r)$

Solution : $\sim \left[p \rightarrow (q \wedge r) \right]$

$\equiv p \wedge \sim (q \wedge r)$ ..... $\sim(P \rightarrow Q) \equiv P \wedge \sim Q$

$\equiv (p \wedge q) \wedge (\sim q \vee \sim r)$ ..... De Morgan's Law

5. $(p \rightarrow \sim q) \wedge (\sim q \rightarrow p)$

Solution : $\sim \left[(p \rightarrow \sim q) \wedge (\sim q \rightarrow p) \right]$

$\equiv \sim (p \rightarrow \sim q) \vee \sim (\sim q \rightarrow p)$ ..... De Morgan's Law

$\equiv \left[p \wedge \sim (\sim q) \right] \vee (\sim q \wedge \sim p)$ ..... $\sim(P \rightarrow Q) \equiv P \wedge \sim Q$

$\equiv (p \wedge q) \vee (\sim q \wedge \sim p)$

DUALITY

Two compound statements are said to be dual of each other if one can be obtained from the other by replcing $\vee$ by $\wedge$ and $\wedge$ by $\vee$ and c by t and t by c

Q13. Write DUALS of each of the following

1. $(p \vee q) \wedge r$

   DUAL : $(p \wedge q) \vee r$

2. $(p \wedge t) \vee (c \wedge \sim q)$

   DUAL : $(p \vee c) \wedge (t \vee \sim q)$

3. $t \vee (p \wedge q)$

   DUAL : $c \wedge (p \vee q)$

4. $[\sim (p \wedge q)] \vee [p \wedge \sim (q \vee \sim s)]$

   DUAL :

   $[\sim (p \vee q)] \wedge [p \vee \sim (q \wedge \sim s)]$

5. $(p \vee q) \vee r \equiv p \vee (q \vee r)$

   DUAL :

   $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

6. $\sim (p \vee q) \vee (\sim p \wedge q) \equiv \sim p$

   DUAL :

   $\sim (p \wedge q) \wedge (\sim p \vee q) \equiv \sim p$

7. $p \wedge (\sim q \vee c)$

   DUAL : $p \vee (\sim q \wedge c)$

8. $(p \to q) \vee (q \to p)$

   Solution

   $\equiv (\sim p \vee q) \vee (\sim q \vee p)$

   DUAL :

   $\equiv (\sim p \wedge q) \wedge (\sim q \wedge p)$

Q14. Write DUALS of each of the following

1. Parth likes milk or tea

   Dual : Parth Likes milk and tea

2. Dhanashree is a doctor and she is clever

   DUAL : Dhanashree is a doctor or she is clever

3. **Manjiri and Hitendra cannot read Urdu**

DUAL : Manjiri or Hitendra cannot read Urdu

4. **Yuvraj or Nirmala are going to Delhi**

DUAL : Yuvraj and Nirmala are going to Delhi

5. **If Santosh passes in Accountancy , then Kusum passes in Logic**

$\equiv$ $p \rightarrow q$

$\equiv$ $\sim p \vee q$

DUAL :

$\equiv$ $\sim p \wedge q$

$\equiv$ Santosh does not pass in Accountancy and Kusum passes in Logic

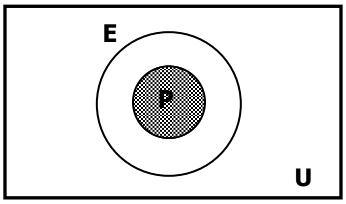
Q15. Express the truth of each of the following statement by VENN DIAGRAM

1. **all professors are educated**

P ≡ **set of all professors**

E ≡ **set of all educated people**

U ≡ **set of all human beings**



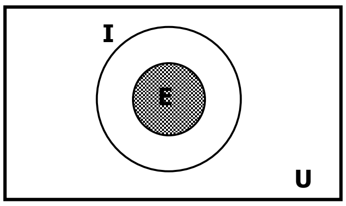
2. **Equilateral triangles are isosceles**

(It means : - All equilateral triangles are isosceles)

E ≡ **set of all equilateral triangles**

I ≡ **set of all isosceles triangles**

U ≡ **set of all triangles**

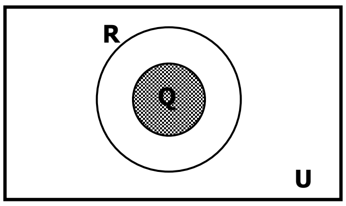


3. **All rational numbers are real numbers**

Q ≡ **set of all rational numbers**

R ≡ **set of all real numbers**

U ≡ **set of all numbers (complex)**



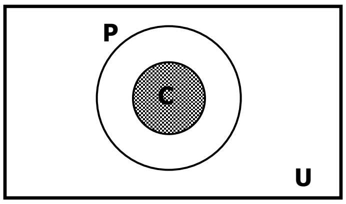
4. co-operative industry is a proprietary firm

(It means : - All co-operative industries are proprietary firm)

C ≡ set of all cooperative industries

P ≡ set of all proprietary firms

U ≡ set of all firms



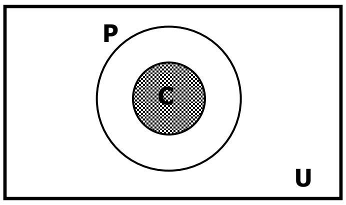
5. circle is a polygon

(It means : - All circle are polygons)

C ≡ set of all circles

P ≡ set of all polygons

U ≡ set of all geometrical figures



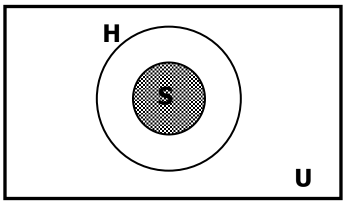
6. Sunday implies holiday

(It means : - All Sundays are Holidays)

S ≡ set of all Sundays

H ≡ set of all Holidays

U ≡ set of all days in a year



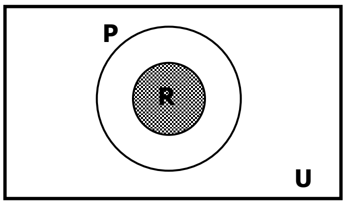
7. If a quadrilateral is a rhombus then its is a parallelogram

(It means : - All rhombus are parallelograms)

R ≡ set of all rhombus

P ≡ set of all parallelograms

U ≡ set of all quadrilaterals



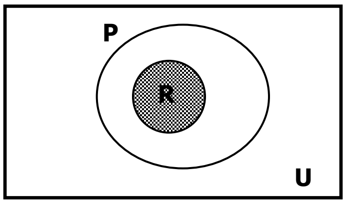
8. All natural numbers are real numbers and x is not a natural number

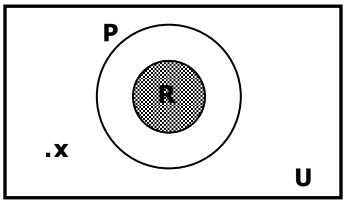
(It means : - All rhombus are parallelograms)

N ≡ set of all natural numbers

R ≡ set of all real numbers

U ≡ set of all numbers (complex)



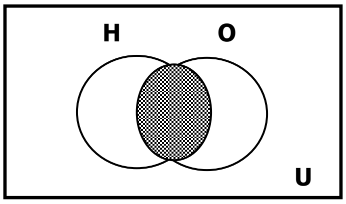


9. Some hardworking students are obedient

H ≡ set of all hard working students

O ≡ set of all obedient people

U ≡ set of all human beings

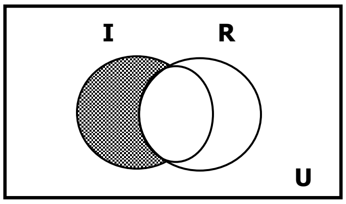


10. Some nonresident Indians are not rich

I ≡ set of all non resident Indians

R ≡ set of all rich people

U ≡ set of all human beings

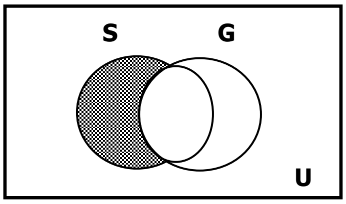


11. Many servants are not graduates

S ≡ set of all servants

G ≡ set of all graduates

U ≡ set of all human beings

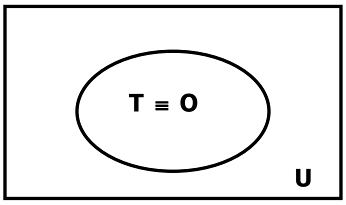


12. All teachers are scholars and scholars are teachers

T ≡ set of all teachers

O ≡ set of all scholars

U ≡ set of all human beings

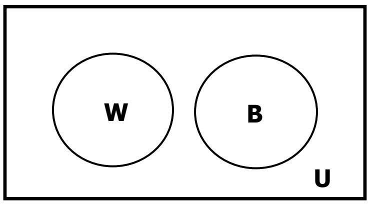


13. No wicketkeeper is bowler in a cricket team

W ≡ set of all wicket keepers

B ≡ set of all bowlers

U ≡ set of all players in a cricket team

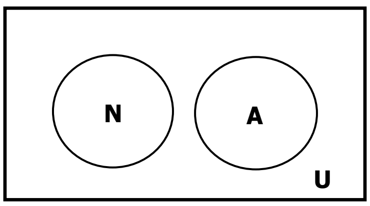


14. No naval person is an air force person

N ≡ set of all naval persons

A ≡ set of all air force persons

U ≡ set of all human beings

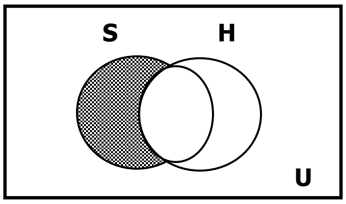


15 a. There are students who are not scholars

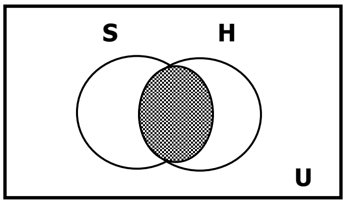
b. There are scholars who are students

c. There are persons who are scholars and students

S ≡ set of all students ; H ≡ set of all scholars ; U ≡ set of all human beings



Statement (a)



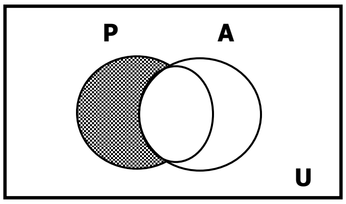
statement (b) & (c)

16. a. Some politicians are actors

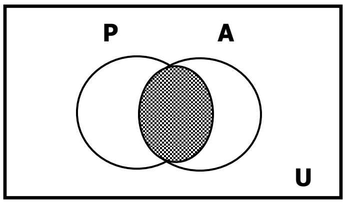
b. There are politicians who are actors

c. There are politicians who are not actors

P ≡ set of all politicians ; A ≡ set of all actors ; U ≡ set of all human beings



Statement (c)



statement (a) & (b)

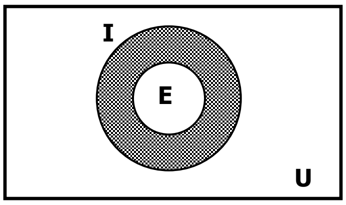
17. Some Isosceles triangles are not equilateral triangles

(Some Isosceles triangles are not equilateral triangles but all equilateral triangles are isosceles)

E ≡ set of all equilateral triangles

I ≡ set of all isosceles triangles

U ≡ set of all triangles



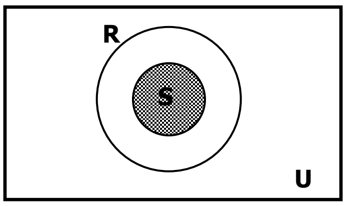
18. Some rectangles are squares

(Some rectangles are squares and all squares are rectangles)

R ≡ set of all rectangles

S ≡ set of all squares

U ≡ set of all quadrilaterals



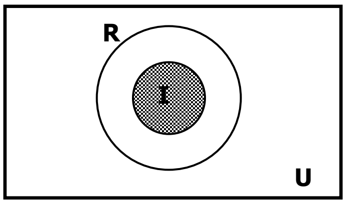
19. Some real numbers are integers

(Some real numbers are integers but all integers are real numbers)

R ≡ set of all real numbers

S ≡ set of all integers

U ≡ set of all numbers (complex)



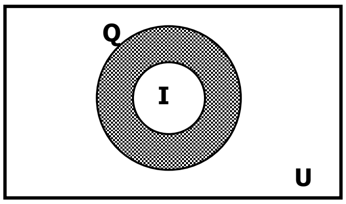
20. Some rational numbers are not integers

(Some rational numbers are not integers but all integers are rational numbers)

Q ≡ set of all rational numbers

I ≡ set of all integers

U ≡ set of all real numbers



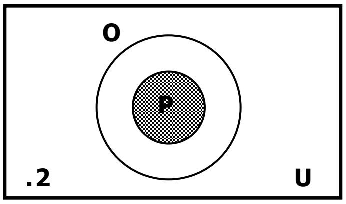
21. If n is a prime number and $n \neq 2$, then it is odd

(all prime numbers except 2 are odd)

P ≡ set of all prime numbers n , $n \neq 2$

O ≡ set of all odd numbers

U ≡ set of all real numbers



Q16. Using ALGEBRA OF STATEMENTS prove :

1. $p \vee (q \wedge \sim q) \equiv p$

Solution : $p \vee (q \wedge \sim q)$

$\equiv p \vee c$ .... Complement Law

$\equiv p$ .... Identity Law

2. $p \vee (\sim p \wedge q) \equiv p \vee q$

Solution $p \vee (\sim p \wedge q)$

$\equiv (p \vee \sim p) \wedge (p \vee q)$ .... Distributive Law

$\equiv t \wedge (p \vee q)$ .... Complement Law

$\equiv p \vee q$ .... Identity Law

. 3. $\sim(p \vee q) \vee (\sim p \wedge q) \equiv \sim p$

Solution $\sim(p \vee q) \vee (\sim p \wedge q)$

$\equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q)$ .... De Morgan's Law

$\equiv \sim p \wedge (\sim q \vee q)$ .... Distributive Law

$\equiv \sim p \wedge t$ .... Complement Law

$\equiv \sim p$ .... Identity Law

4. $p \wedge [(\sim p \vee q) \vee \sim q] \equiv p$

Solution $p \wedge [(\sim p \vee q) \vee \sim q]$

$\equiv p \wedge [\sim p \vee (q \vee \sim q)]$ .... Associative Law

$\equiv p \wedge (\sim p \vee t)$ .... Complement Law

$\equiv p \wedge t$ .... Identity Law

$\equiv p$ .... Identity Law

5. $[p \wedge (q \vee r)] \vee [(\sim r \wedge \sim q) \wedge p] \equiv p$

Solution $[p \wedge (q \vee r)] \vee [p \wedge (\sim q \wedge \sim r)]$ .... Commutative Law

$\equiv p \wedge [(q \vee r) \vee (\sim q \wedge \sim r)]$ .... Distributive Law

$\equiv p \wedge t$ .... Complement Law

$\equiv p$ .... Identity Law

6. $(p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \vee \sim q) \equiv t$

Solution $[(p \wedge q) \vee (p \wedge \sim q)] \vee (\sim p \vee \sim q)$

$\equiv [p \wedge (q \vee \sim q)] \vee (\sim p \vee \sim q)$ .... Distributive Law

$\equiv (p \wedge t) \vee (\sim p \vee \sim q)$ .... Complement Law

$\equiv p \vee (\sim p \vee \sim q)$ .... Identity Law

$\equiv (p \vee \sim p) \vee \sim q$ .... Associative Law

$\equiv t \vee \sim q$ .... Complement Law

$\equiv t$ .... Identity Law

7. $(p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q) \equiv p \vee \sim q$

Solution $\left[(p \wedge q) \vee (p \wedge \sim q) \right] \vee (\sim p \wedge \sim q)$

$\equiv \left[p \wedge (q \vee \sim q) \right] \vee (\sim p \wedge \sim q)$ .... Distributive Law

$\equiv (p \wedge t) \vee (\sim p \wedge \sim q)$ .... Complement Law

$\equiv p \vee (\sim p \wedge \sim q)$ .... Identity Law

$\equiv (p \vee \sim p) \wedge (p \vee \sim q)$ .... Distributive Law

$\equiv t \wedge (p \vee \sim q)$ .... Complement Law

$\equiv (p \vee \sim q)$ .... Identity Law

8. $(p \vee q) \wedge \sim p \rightarrow q$ is a tautology

Solution $\left[(p \vee q) \wedge \sim p \right] \rightarrow q$

$\equiv \left[(p \wedge \sim p) \vee (q \wedge \sim p) \right] \rightarrow q$ .... Distributive Law

$\equiv \left[c \vee (q \wedge \sim p) \right] \rightarrow q$ .... Complement Law

$\equiv (q \wedge \sim p) \rightarrow q$ .... Identity Law

$\equiv \sim (q \wedge \sim p) \vee q$ .... $P \rightarrow Q \equiv \sim P \vee Q$

$\equiv (\sim q \vee p) \vee q$ .... DeMorgan's Law

$\equiv q \vee (\sim q \vee p)$ .... Commutative Law

$\equiv (q \vee \sim q) \vee p$ .... Associative Law

$\equiv t \vee p$ .... Complement Law

$\equiv t$ .... Identity Law