J.K. SHAH CLASSES

MATHEMATICS & STATISTICS

SYJC TEST – 04 – SET 2

DURATION – 1 $^{1}/_{2}$ HR **MARKS – 40**

SECTION – I

Q1. Attempt any THREE of the following (2 marks each) **(6 marks)**

01. The total cost for production of Q items is given as $C = Q^3 - 600Q^2 + 1200Q$

Find the values of Q for which average cost is decreasing

SOLUTION

SOLUTION SET

$C = Q^3 - 600Q^2 + 1200Q$

AVERAGE COST

$$C_A = \frac{C}{Q}$$

$$= \frac{Q^3 - 600Q^2 + 1200Q}{Q}$$

$$= Q^2 - 600Q + 1200$$

For average cost decreasing ,

$$\frac{dC_A}{dQ} < 0$$

$$2Q - 600 < 0$$

$$2Q < 600$$

$$Q < 300$$ **ans : average cost is decreasing for Q < 300**

02. if the avg revenue R_A is 50 and elasticity of demand $\eta = 5$, find the marginal revenue

SOLUTION

$$R_m = R_A \left(1 - \frac{1}{\eta}\right)$$

$$= 50 \left(1 - \frac{1}{5}\right)$$

$$= 50 \times \frac{4}{5}$$

$$= 40$$

03. The expenditure of a person with income I is given by $E_C = 0.000035\ I^2 + 0.045\ I$

Find marginal propensity to consume & marginal propensity to save (APC) when I = 5000

SOLUTION

$$\mathbf{E_c = 0.000035I^2 + 0.045I}$$

$$\mathbf{MPC}\Big|_{I = 5000} = \frac{dEc}{dI}$$

$$= \frac{d\ 0.000035\ I^{2} + 0.045\ I}{dI}$$

$$= 0.000070\ I + 0.045$$

$$= 0.00007(5000) + 0.045$$

$$= 0.35 + 0.045$$

$$= \mathbf{0.395}$$

$$\mathbf{MPS}\Big|_{I = 8000} = 1 - MPC\Big|_{I = 8000}$$

$$= 1 - 0.395$$

$$= \mathbf{0.605}$$

Q2. Attempt any TWO of the following (3 marks each) **(6 marks)**

01. Cost of assembling x wallclocks is $\left(\frac{x^3}{3} - 40x^2\right)$ and labor charges are 500x .

Find no. of wallclocks to be manufactured for which the marginal cost is minimum

SOLUTION

STEP 1 : MARGINAL COST C_M

$$C = \frac{x^3}{3} - 40x^2 + 500x$$

$$C_M = \frac{dC}{dx}$$

$$= x^2 - 80x + 500$$

STEP 2 :

$$\frac{dC_M}{dx} = 2x - 80$$

$$\frac{d^2C_M}{dx^2} = 2$$

STEP 3 :

$$\frac{dC_M}{dx} = 0$$

$$2x - 80 = 0$$

$$x = 40$$

STEP 4 :

$$\left.\frac{d^2C_M}{dx^2}\right|_{x = 20} = 2 > 0$$

Marginal cost is minimum at x = 20

02. if the demand function is $D = 50 - 3p - p^2$. Find the elasticity of demand at $p = 5$ & comment

SOLUTION

STEP 1 : $D = 50 - 3p - p^2$.

$$\frac{dD}{dp} = -3 - 2p$$

STEP 2 : $\eta = \frac{-P}{D} \cdot \frac{dD}{dp}$

$$= - \frac{p}{50 - 3p - p^2} \cdot (-3 - 2p)$$

$$= \frac{3p + 2p^2}{50 - 3p - p^2}$$

STEP 3 : $\eta \Big|_{p = 5}$

$$= \frac{3(5) + 2(5)^2}{50 - 3(5) - (5)^2}$$

$$= \frac{15 + 2(25)}{50 - 15 - 25}$$

$$= \frac{65}{10}$$

$$= 6.5 > 1 \quad . \quad \text{Demand is relatively elastic}$$

03. find values of x for which $f(x) = x^3 - 3x^2 - 9x + 25$ is increasing

SOLUTION

For f(x) increasing ,

$f'(x) > 0$

$3x^2 - 6x - 9 > 0$

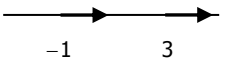
$x^2 - 2x - 3 > 0$

$(x - 3)(x + 1) > 0$

CASE 1 :

$x - 3 > 0$ & $x + 1 > 0$

$x > 3$ & $x > -1$



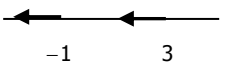
$x > 3$

$x \in (3, \infty)$

CASE 2 :

$x - 3 < 0$ & $x + 1 < 0$

$x < 3$ & $x < -1$



$x < -1$

$x \in (-\infty, -1)$

***f* is increasing for $x \in (3, \infty)$ & $x \in (-\infty, -1)$**

Q3. Attempt any TWO of the following (4 marks each) **(8 marks)**

01. Examine the function $f(x) = x + \frac{25}{x}$ for maxima and minima

STEP 1 :

$f(x) = x + \frac{25}{x}$

STEP 2 :

$f'(x) = 1 - \frac{25}{x^2} = 1 - 25x^{-2}$

$f''(x) = 0 + 50x^{-3}$

$= \frac{50}{x^3}$

STEP 3 :

$f'(x) = 0$

$1 - \frac{25}{x^2} = 0$

$1 = \frac{25}{x^2}$

$x^2 = 25$

$x = \pm 5$

STEP 4 :

$f''(x)\Big|_{x=5} = \frac{50}{5^3} > 0$

f is minimum at $x = 5$

$f''(x)\Big|_{x=-5} = \frac{50}{(-5)^3} < 0$

f is maximum at $x = -5$

STEP 5 :

Since f is minimum at $x = 5$ cont.

Minimum value of f

$= f(x)\Big|_{x=5}$

$= 5 + \frac{25}{5}$

$= 10$

Since f is maximum at $x = -5$

Maximum value of f

$= f(x)\Big|_{x=-5}$

$= -5 + \frac{25}{-5}$

$= -5 - 5$

$= -10$

02. Find how many lanterns (x) should be ordered so that the order is the most economical if the price for lantern is given as

$$p = 4x + \frac{64}{x^2} + \frac{17}{x}$$

SOLUTION

STEP 1 : COST

$$C = p.x$$

$$= \left(4x + \frac{64}{x^2} - \frac{17}{x}\right). x$$

$$= 4x^2 + \frac{64}{x} - 17$$

STEP 2 :

$$\frac{dC}{dx} = 8x - \frac{64}{x^2} = 8x - 64x^{-2}$$

$$\frac{d^2C}{dx^2} = 8 + 128x^{-3}$$

$$= 8 + \frac{128}{x^3}.$$

STEP 3 :

$$\frac{dC}{dx} = 0$$

$$8x - \frac{64}{x^2} = 0$$

$$8x = \frac{64}{x^2}$$

$$8x^3 = 64$$

$$x^3 = 8 \quad \therefore \quad x = 2$$

STEP 4 :

$$\left.\frac{d^2C}{dx^2}\right|_{x = 2} = 8 + \frac{128}{2^3} > 0$$

Cost is minimum at x = 2

Hence number of lanterns to be ordered = 2

03. Comment on elasticity of demand of a commodity for p = 200 , when demand function is $p = 400 - \frac{q^2}{2}$

SOLUTION

STEP 1 : $p = 400 - \frac{q^2}{2}$

$$\frac{q^2}{2} = 400 - p$$

$$q^2 = 800 - 2p$$

$$q = \sqrt{800 - 2p}$$

$$\frac{dq}{dp} = \frac{1}{2\sqrt{800 - 2p}} \cdot \frac{d}{dp}(800 - 2p)$$

$$\frac{dq}{dp} = \frac{1}{2\sqrt{800 - 2p}} \cdot -2$$

$$\frac{dq}{dp} = \frac{-1}{\sqrt{800 - 2p}}$$

STEP 2 : $\eta = \frac{-P}{D} \cdot \frac{dD}{dp}$

$= \frac{-p}{q} \cdot \frac{dq}{dp}$

IN THIS SUM DEMAND 'D' IS DENOTED AS 'q'

$$= \frac{-p}{\sqrt{800 - 2p}} \cdot \frac{-1}{\sqrt{800 - 2p}}$$

$$= \frac{p}{800 - 2p}$$

STEP 3 : $\eta \Big|_{p = 200} = \frac{200}{800 - 400}$

$$= \frac{200}{400}$$

$$= 0.5 < 1$$

Demand is relatively inelastic

SECTION - II

Q4. Attempt any THREE of the following (2 marks each) **(6 marks)**

01. If the present worth of a bill due six months hence is ₹ 2,500 at 10% p.a. , what is the sum due

SOLUTION

F.V. = P.W.+ INT ON P.W. FOR 6 MONTHS @10% p.a.

$$F.V. = 2500 + 2500 \times \frac{6}{12} \times \frac{10}{100}$$

$$F.V. = 2500 + 125$$

$$F.V. = ₹\ 2,625$$

02. Find the accumulated value after 1 year of annuity immediate in which ₹ 20,000 is invested every quarter at 16% p.a. compounded quarterly ($1.04^4 = 1.1699$)

SOLUTION :

$C = ₹\ 20,000 \quad ; \quad i = 4\% = 0.04 \quad ; \quad n = 4$

$$A = C\left[\frac{(1+i)^n - 1}{I}\right]$$

$$= 20000\left[\frac{(1+0.04)^4 - 1}{0.04}\right]$$

$$= 20000\left[\frac{1.04^4 - 1}{0.04}\right]$$

$$= 20000\left[\frac{1.1699 - 1}{0.04}\right]$$

$$= 20000\ \frac{0.1699}{0.04}$$

$$= 20000\ \frac{16.99}{4}$$

$$= 5000 \times 16.99$$

$$= ₹\ 84,950$$

03. a shop is valued at ₹ 2,40,000 for 75% of its value . If the rate of premium is 90 paise percent , find the premium paid by the owner of the shop . If the agent gets commission at 15% of the premium , find the agents commission

Solution

Property value = ₹ 2,40,000

Insured value = $\frac{75}{100}$ x 2,40,000

= ₹ 1,80,000

Rate of premium = 90 paise percent

= 0.90%

Premium = $\frac{0.9}{100}$ x 1,80,000

= $\frac{9}{1000}$ x 1,80,000

= ₹ 1,620

Agents commission

= $\frac{15}{100}$ x 1620

= ₹ 243

04. Find the present value of an immediate annuity of ₹ 40,000 per annum for 3 years with interest compounded at 8% p.a. ($1.08^{-3} = 0.7938$)

SOLUTION :

C = ₹ 40,000 ; i = 8% = 0.08 ; n = 3

$$P = C\left(\frac{1-(1+i)^{-n}}{i}\right)$$

$$= 40000\left(\frac{1-(1+0.08)^{-3}}{0.08}\right)$$

$$= 40000\left(\frac{1-1.08^{-3}}{0.08}\right)$$

$$= 40000\left(\frac{1-0.7938}{0.06}\right)$$

$$= 40000\,\frac{0.2062}{0.08}$$

$= 40000 \frac{20.62}{8}$

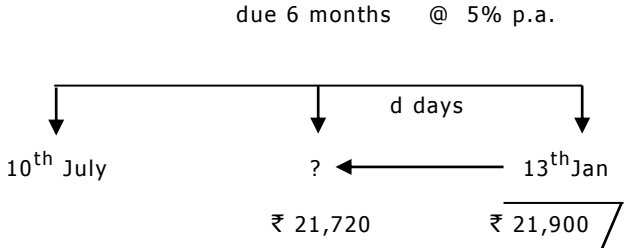
$= 5000 \times 20.62$

$= ₹\ 1,03,100$

Q5. Attempt any TWO of the following (3 marks each) **(6 marks)**

01. A bill of ₹ 21,900 drawn on July 10 for 6 months was discounted for ₹ 21,720 at 5% p.a. On which day the bill was discounted

SOLUTION



STEP 1 :

Date of drawing = 10 / 07

Add period of bill + 6 months

Nominal due date = 10 / 01

Add Grace days + 3 days

Legal due date = 13 / 01 …….

13th January

STEP 2 :

Let Unexpired period = d days

STEP 3 :

B.D. = F.V. – C.V.

= 21,900 – 21,720

= ₹ 180

STEP 4 :

B.D. = Interest on F.V. for 'd' days @ 5% p.a.

$$180 = \frac{\cancel{21900} \times d}{\cancel{365}} \times \frac{\cancel{5}}{\cancel{100}}$$

$\cancel{73}$

d = 60

STEP 5 :

Dt. of Discount

= 13th Jan – 60 days

Jan Dec Nov

= 13 + 31 + 16

= 14th November

02. Mr. Rana plans to save for his son's higher studies . He wants to accumulate a sum of ₹ 2,00,000 at the end of 4 years . How much should he invest at the end of each year from now , if he can get interest compounded at 10% p.a. ($1.1^4 = 1.4641$)

SOLUTION :

A = ₹ 2,00,000 ; i = 10% = 0.1 ; n = 4

$$A = C\left[\frac{(1+i)^n - 1}{i}\right]$$

$$2,00,000 = C\left[\frac{(1+0.1)^4 - 1}{0.1}\right]$$

$$2,00,000 = C\left[\frac{(1.1)^4 - 1}{0.1}\right]$$

$$2,00,000 = C\left[\frac{1.4641 - 1}{0.1}\right]$$

$$2,00,000 = C\left[\frac{0.4641}{0.1}\right]$$

$$2,00,000 = C\left[\frac{4.641}{1}\right]$$

$$C = \frac{2,00,000}{4.641} \longrightarrow$$

= ₹ 43,090

LOG CALC
5.3010
− 0.6666
AL 4.6344
43090

03. A person takes a policy for ₹ 80,000 for a period of 20 years . He pays premium of 10 years during which bonus was declared at the average rate of ₹ 20 per year per thousand . Find the paid up value of policy if he discontinues paying premium after 10 years

SOLUTION

Policy value = ₹ 80,000 for 20 years

Person pays premium for 10 years and discontinues

STEP – 1 : TOTAL BONUS ACCRUED

Rate of Bonus

= ₹ 20 per thousand per annum

Bonus = $\frac{20}{1000}$ x 80,000

= ₹ 1600 p.a

Bonus accrued

In 10 years = 1600 x 10

= ₹ 16,000

STEP – 2 : PAID UP VALUE

Paid Up Value

$$= \frac{\text{No. of Premiums paid}}{\text{total no. of Premiums originally stipulated in the policy}} \times \text{Policy value}$$

+

bonus accrued if any

= $\frac{10}{20}$ x 80,000 + 16,000

= 40,000 + 16,000

= 56,000

Q6. Attempt any TWO of the following (4 marks each) **(8 marks)**

01. a car valued at ₹ 4,00,000 is insured for ₹ 2,50,000 . The rate of premium is 5% less 20% . How much loss does the owner bear including premium , if the value of the car is reduced to 60% of its original value

Solution

Value of car = ₹ 4,00,000

Insured value = ₹ 2,50,000

Rate of premium = 5 % less 20%.

Premium = $\frac{5}{100}$ x 2,50,000

= ₹ 12,500

less 20% disc – 2,500

Net Premium = ₹ 10,000

Since the value of the car is reduced to 60% of its original value , the loss on the car is 40%

Loss = $\frac{40}{100}$ x 4,00,000

= ₹ 1,60,000

Claim = $\frac{\text{insured val.}}{\text{Property val.}}$ x loss

= $\frac{2,50,000}{4,00,000}$ x 1,60,000

= ₹ 1,00,000

Loss	=	1,60,000
Less claim	–	1,00,000
Net loss	=	60,000
Add premium	+	10,000
Net loss Incl. premium	= ₹	70,000

02. Find the true discount , banker's discount and banker's gain on a bill of ₹ 36,600 due 4 months hence discounted at 5% p.a

SOLUTION

STEP 1 :

FV = PW + Int on PW for 4 months @ 5% p.a.

$$36600 = PW + PW \times \frac{4}{12} \times \frac{5}{100}$$

$$36600 = PW + \frac{PW}{60}$$

$$36600 = \frac{61\ PW}{60}$$

$$PW = \frac{36600 \times 60}{61}$$

$$= ₹\ 36,000$$

STEP 2 :

TD = Int on PW for 4 months @ 5% p.a.

$$= 36000 \times \frac{4}{12} \times \frac{5}{100}$$

$$= ₹\ 600$$

STEP 3 :

BD = Int on FV for 4 months @ 5% p.a.

$$= 36600 \times \frac{4}{12} \times \frac{5}{100}$$

$$= ₹\ 610$$

STEP 4 :

BG = BD – TD

= 610 – 600

= ₹ 10

03. a merchant takes out fire insurance policy to cover 80% of the book value of his stock . A fire broke out and stock worth ₹ 80,000 was completely destroyed while the rest was damaged and reduced to 20% of the book value . If he receives a sum of ₹ 67,200 as proportional of the book compensation under the policy , find the book value of the stock

SOLUTION

Value of stock = ₹ x

Insured value = 80% of the stock

Loss

stock worth ₹ 80,000 was completely destroyed while the rest was damaged and reduced to 20% of the book value

Loss

$$= 80{,}000 + \frac{80}{100}(x - 80{,}000)$$

$$= 80{,}000 + \frac{80x}{100} - 64{,}000$$

$$= 16{,}000 + \frac{4x}{5}$$

Claim = ₹ 67,200

Since stock was insured for 80% of its value

Claim = 80% of loss

$$67{,}200 = \frac{80}{100} \times \left(16{,}000 + \frac{4x}{5}\right)$$

$$\frac{67200 \times 100}{80} = 16{,}000 + \frac{4x}{5}$$

$$84{,}000 = 16{,}000 + \frac{4x}{5}$$

$$68{,}000 = \frac{4x}{5}$$

$$x = \frac{68{,}000 \times 5}{4}$$

$$x = ₹\ 85{,}000$$