J.K. SHAH CLASSES

SOLUTION SET

MATHEMATICS & STATISTICS

SYJC PRELIUM – 02 – SET B

DURATION – 3 HR MARKS – 80

SECTION - I

Q1. (A)Attempt any six of the following (12)

01. Find X and Y if $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$; $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

SOLUTION

$$X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \qquad X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$$

$$X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \qquad X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$2X = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} \qquad 2Y = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} \qquad Y = \frac{1}{2}\begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} \qquad Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

02. find $\frac{dy}{dx}$ if $y = \sin^{-1}\sqrt{1 - x^2}$

SOLUTION

Put $x = \cos\theta$

$$y = \sin^{-1}\sqrt{1 - \cos^2\theta}$$

$$y = \sin^{-1}\sqrt{\sin^2\theta}$$

$$y = \sin^{-1}(\sin\theta)$$

$$y = \theta$$

$$y = \cos^{-1} x$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1 - x^2}}$$

03. Find the value of k if the function

$$f(x) = \frac{\sin 9x}{2x} \quad ; \quad x \neq 0$$

$$= k \quad ; \quad x = 0 \qquad \text{is continuous at } x = 0$$

SOLUTION

Step 1

$\lim_{x \to 0} f(x)$

$$= \lim_{x \to 0} \frac{\sin 9x}{2x}$$

$$= \lim_{x \to 0} \frac{9}{2} \frac{\sin 9x}{9x}$$

$$= \frac{9}{2}(1)$$

$$= \frac{9}{2}$$

Step 2 :

$f(0) = k$ ……… given

Step 3 :

Since f is continuous at $x = 0$

$$f(0) = \lim_{x \to 0} f(x)$$

$$k = 9/2$$

04. Write negations of the following statements

1. $\forall x \in N$, $x^2 + x$ is an even number

   Negation : $\exists x \in N$, such that $x^2 + x$ is not an even number

2. if triangles are congruent then their areas are equal

   Using : $\sim(P \rightarrow Q) \equiv P \wedge \sim Q$

   Negation : triangles are congruent and their areas are not equal

05. find elasticity of demand if the marginal revenue is Rs 50 and the price is Rs 75

SOLUTION

$$R_m = R_A\left(1 - \frac{1}{\eta}\right)$$

$$50 = 75\left(1 - \frac{1}{\eta}\right)$$

$$\frac{50}{75} = 1 - \frac{1}{\eta}$$

$$\frac{2}{3} = 1 - \frac{1}{\eta}$$

$$\frac{1}{\eta} = 1 - \frac{2}{3}$$

$$\frac{1}{\eta} = \frac{1}{3} \qquad \eta = 3$$

06. State which of the following sentences are statements . In case of statement , write down the truth value

a) Every quadratic equation has only real roots

ans : the given sentence is a logical statement . Truth value : F

b) $\sqrt{-4}$ is a rational number

ans : the given sentence is a logical statement . Truth value : F

07. Evaluate : $\displaystyle\int \frac{\sec x.\tan x}{\sec^2 x + 4}\,dx$

SOLUTION PUT sec x = t

Secx.tanx . dx = dt

THE SUM IS

$$= \int \frac{1}{t^2 + 4}\,dt$$

$$= \int \frac{1}{t^2 + 2^2}\,dt$$

$$= \frac{1}{a}\tan^{-1}\frac{t}{a} + c$$

$$= \frac{1}{2}\tan^{-1}\frac{t}{2} + c$$

Resubs.

$$= \frac{1}{2}\tan^{-1}\left(\frac{\sec x}{2}\right) + c$$

08. if $A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$; $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then find $|AB|$

SOLUTION

AB

$$= \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1+3 & 2+4 \\ 2+6 & 4+8 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 6 \\ 8 & 12 \end{bmatrix}$$

$$|AB| = 4(12) - 8(6) = 48 - 48 = 0$$

Q2. (A)Attempt any TWO of the following **(06)**

01. $f(x) = \dfrac{3 - \sqrt{2x+7}}{x-1}$; $x \neq 1$

$= -1/3$; $x = 1$ Discuss continuity at $x = 1$

SOLUTION

STEP 1

$\lim\limits_{x \to 1} f(x)$

$$= \lim_{x \to 1} \frac{3 - \sqrt{2x+7}}{x-1}$$

$$= \lim_{x \to 1} \frac{3 - \sqrt{2x+7}}{x-1} \cdot \frac{3 + \sqrt{2x+7}}{3 + \sqrt{2x+7}}$$

$$= \lim_{x \to 1} \frac{9 - (2x+7)}{x-1} \cdot \frac{1}{3 + \sqrt{2x+7}}$$

$$= \lim_{x \to 1} \frac{9 - 2x - 7}{x-1} \cdot \frac{1}{3 + \sqrt{2x+7}}$$

$$= \lim_{x \to 1} \frac{2 - 2x}{x-1} \cdot \frac{1}{3 + \sqrt{2x+7}}$$

$$= \lim_{x \to 1} \frac{2(1-x)}{x-1} \cdot \frac{1}{3 + \sqrt{2x+7}}$$

$$= \lim_{x \to 1} \frac{-2\cancel{(x-1)}}{\cancel{x-1}} \cdot \frac{1}{3 + \sqrt{2x+7}} \qquad x - 1 \neq 0$$

$$= \frac{-2}{3 + \sqrt{2+7}}$$

$$= \frac{-2}{3+3}$$

$$= \frac{-1}{3}$$

STEP 2 :

$f(1) = -1/3$ ……… given

STEP 3 :

$f(1) = \lim_{x \to 1} f(x)$; f is continuous at x = 1

02. Write the converse , inverse and the contrapositive of the statement

"The crops will be destroyed if there is a flood "

SOLUTION :

LET $P \rightarrow Q$ $\equiv$ if there is a flood then the crops will be destroyed

CONVERSE : $Q \rightarrow P$

If the crops will be destroyed then there will be a flood

CONTRAPOSITIVE : $\sim Q \rightarrow \sim P$

If the crops will not be destroyed then there will be no flood

INVERSE : $\sim P \rightarrow \sim Q$

If there is no flood then the crops will not be destroyed

03. Find $\frac{dy}{dx}$ if $y = \tan^{-1}\left(\frac{5x}{1-6x^2}\right)$

SOLUTION

$$y = \tan^{-1}\left(\frac{3x+2x}{1-3x.2x}\right)$$

$$y = \tan^{-1} 3x + \tan^{-1} 2x$$

$$\frac{dy}{dx} = \frac{1}{1+9x^2} . \frac{d}{dx}(3x) + \frac{1}{1+4x^2} \frac{d}{dx}(2x($$

$$\frac{dy}{dx} = \frac{3}{1+9x^2} + \frac{2}{1+4x^2}$$

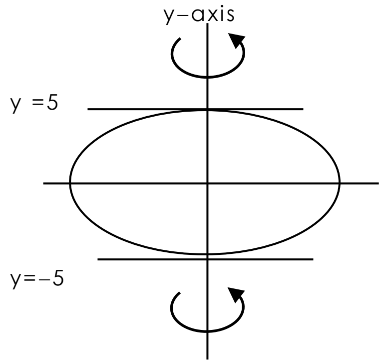
(B) Attempt any TWO of the following **(08)**

01. Find the volume of a solid obtained by the complete revolution of the ellipse

$$\frac{x^2}{25} + \frac{y^2}{36} = 1 \qquad \text{about Y – axis}$$

SOLUTION

STEP 1 :



$$\frac{x^2}{25} + \frac{y^2}{36} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$a^2 = 25$; $a = 5$

$b^2 = 36$, $b = 6$

STEP 2 :

$$\frac{x^2}{25} + \frac{y^2}{36} = 1$$

$$\frac{x^2}{25} = 1 - \frac{y^2}{36}$$

$$\frac{x^2}{25} = \frac{36 - y^2}{36}$$

$$x^2 = \frac{25}{36}(36 - y^2)$$

STEP 3 :

$$V = \pi \int_{-6}^{6} x^2 . dy$$

About y – axis

$$= \pi \int_{-6}^{6} \frac{25}{36}(36 - y^2).dy$$

$$= \frac{25\pi}{36} \int_{-6}^{6} (36 - y^2).dy$$

$$= \frac{25\pi}{36}\left[36y - \frac{y^3}{3}\right]_{-6}^{6}$$

$$= \frac{25\pi}{36}\left\{\left(216 - \frac{216}{3}\right) - \left(-216 + \frac{216}{3}\right)\right\}$$

$$= \frac{25\pi}{36}\left\{(216 - 72) - (-216 + 72)\right\}$$

$$= \frac{25\pi}{36}(144 + 144)$$

$$= \frac{25\pi}{36}(288)$$

$= 200\,\pi$ cubic units

02. Evaluate : $\int \log(1 + x^2)\, dx$

$$= \int \log(1 + x^2).\, 1dx$$

$$= \log(1 + x^2) \int 1\, dx - \int \left(\frac{d}{dx} \log(1 + x^2) \int 1\, dx\right) dx$$

$$= \log(1 + x^2) . x - \int \frac{2x}{1 + x^2.} . x \quad dx$$

$$= x. \log(1 + x^2) - 2\int \frac{x^2}{1 + x^2.} \quad dx$$

$$= x. \log(1 + x^2) - 2\int \frac{1 + x^2 - 1}{1 + x^2.} . dx$$

$$= x. \log(1 + x^2) - 2\int 1 - \frac{1}{1 + x^2.} \quad dx$$

$$= x. \log(1 + x^2) - 2\left(x - \tan^{-1}x\right) + c$$

$$= x. \log(1 + x^2) - 2x + 2\tan^{-1}x + c$$

03. Find how many lanterns (x) should be ordered so that the order is the most economical if the price for lantern is given as

$$p = 4x + \frac{64}{x^2} + \frac{7}{x}$$

SOLUTION

STEP 1 : COST

$$C = p.x$$

$$= \left(4x + \frac{64}{x^2} + \frac{7}{x}\right) . x$$

$$= 4x^2 + \frac{64}{x} + 7$$

STEP 2 :

$$\frac{dC}{dx} = 8x - \frac{64}{x^2} = 8x - 64x^{-2}$$

$$\frac{d^2C}{dx^2} = 8 + 128x^{-3}$$

$$= 8 + \frac{128}{x^3}.$$

STEP 3 :

$$\frac{dC}{dx} = 0$$

$$8x - \frac{64}{x^2} = 0$$

$$8x = \frac{64}{x^2}$$

$$8x^3 = 64$$

$$x^3 = 8 \qquad \therefore \quad x = 2$$

STEP 4 :

$$\left.\frac{d^2C}{dx^2}\right|_{x = 2} = 8 + \frac{128}{2^3} > 0$$

Cost is minimum at $x = 2$

No of lanterns to be ordered $= 2$

Q3. (A)Attempt any TWO of the following **(06)**

01. Using ALGEBRA OF STATEMENTS , prove

$$p \wedge \left[(\sim p \vee q) \vee \sim q \right] \equiv p$$

Solution

$p \wedge \left[(\sim p \vee q) \vee \sim q \right]$

$\equiv p \wedge \left[\sim p \vee (q \vee \sim q) \right]$ ……… Associative Law

$\equiv p \wedge (\sim p \vee \mathbf{t})$ ……… Complement Law

$\equiv p \wedge \mathbf{t}$ ……… Identity Law

$\equiv p$ ……… Identity Law

02. $f(x) = \dfrac{(e^{3x} - 1)^2}{x.\log(1 + 3x)}$; $x \neq 0$

$= 10$; $x = 0$ Discuss the continuity at x = 0

Solution :

Step 1

$$\lim_{x \to 0} f(x)$$

$$= \lim_{x \to 0} \frac{(e^{3x} - 1)^2}{x.\log(1 + 3x)}$$

Dividing Numerator & Denominator by x^2

$x \to 0$, $x \neq 0$, $x^2 \neq 0$

$$= \lim_{x \to 0} \frac{\dfrac{(e^{3x} - 1)^2}{x^2}}{\dfrac{x.\log(1 + 3x)}{x^2}}$$

$$= \lim_{x \to 0} \frac{\left(\dfrac{e^{3x} - 1}{x}\right)^2}{\dfrac{\log(1 + 3x)}{x}}$$

$$= \lim_{x \to 0} \frac{\left(3\, \dfrac{e^{3x} - 1}{3x}\right)^2}{\log(1 + 3x)^{\frac{1}{x}}}$$

$$= \lim_{x \to 0} \frac{\left(3 \cdot \frac{e^{3x} - 1}{3x}\right)^2}{\log\left((1 + 3x)^{\frac{1}{3x}}\right)^3}$$

$$= \frac{(3.\log e)^2}{\log e^3}$$

$$= \frac{9}{3.\log e} = 3$$

Step 2 :

$f(0) = 10$ ……… given

Step 3 :

$$f(0) \neq \lim_{x \to 0} f(x)$$

$\therefore f$ is discontinuous at $x = 0$

Step 4 :

Removable Discontinuity

f can be made continuous at $x = 0$ by redefining it as

$$f(x) = \frac{(e^{3x} - 1)^2}{x.\log(1 + 3x)} \quad ; \quad x \neq 0$$

$$= 3 \quad ; \quad x = 0$$

03. if $\sin y = x.\sin(5 + y)$; prove that $\frac{dy}{dx} = \frac{\sin^2(5 + y)}{\sin a}$

SOLUTION

$$\sin y = x.\sin(5 + y)$$

$$x = \frac{\sin y}{\sin(5 + y)}$$

Differentiating wrt y

$$\frac{dx}{dy} = \frac{\sin(5 + y)\frac{d}{dy}\sin y - \sin y\frac{d}{dy}\sin(5 + y)}{\sin^2(5 + y)}$$

$$\frac{dx}{dy} = \frac{\sin(5 + y) . \cos y - \sin y . \cos(5 + y)\frac{d}{dy}(5 + y)}{\sin^2(5 + y)}$$

$$\frac{dx}{dy} = \frac{\sin(5 + y).\cos y - \cos(5 + y) . \sin y}{\sin^2(5 + y)}$$

$$\frac{dx}{dy} = \frac{\sin(5 + y - y)}{\sin^2(5 + y)}$$

$$\frac{dx}{dy} = \frac{\sin 5}{\sin^2(5 + y)}$$

Now $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$

$\therefore$ $\frac{dy}{dx} = \frac{\sin^2(5+ y)}{\sin 5}$

(B) Attempt any TWO of the following **(08)**

01. $$\int_4^7 \frac{(11-x)^2}{x^2 + (11-x)^2}\,dx \qquad \ldots\ldots \quad \textbf{(1)}$$

$$\textbf{USING} \int_a^b f(x)dx = \int_b^b f(a+b-x)\,dx$$

$$I = \int_4^7 \frac{\left[11-(4+7-x)\right]^2}{(4+7-x)^2 + \left[11-(4+7-x)\right]^2}\,dx$$

$$I = \int_4^7 \frac{\left[11-(11-x)\right]^2}{(11-x)^2 + \left[11-(11-x)\right]^2}\,dx$$

$$I = \int_4^7 \frac{(11-11+x)^2}{(11-x)^2 + (11-11+x)^2}\,dx$$

$$I = \int_4^7 \frac{x^2}{(11-x)^2 + x^2}\,dx \qquad \ldots\ldots \quad \textbf{(2)}$$

(1) + (2)

$$2I = \int_4^7 \frac{(11-x)^2 + x^2}{(11-x)^2 + x^2}\,dx$$

$$2I = \int_4^7 1\,dx$$

$$2I = \Big[x\Big]_4^7$$

$$2I = 7-4$$

$$2I = 3$$

$$I = 3/2$$

02. $$\int \frac{x^2}{x^4 + 5x^2 + 6}\,dx$$

$$\int \frac{x^2}{(x^2 + 2)(x^2 + 3)}\,dx$$

SOLUTION

$$\frac{x^2}{(x^2 + 2)(x^2 + 3)} = \frac{A}{x^2 + 2} + \frac{B}{x^2 + 3}$$

x^2 = t (say)

$$\frac{t}{(t + 2)(t + 3)} = \frac{A}{t + 2} + \frac{B}{t + 3}$$

$$t = A(t + 3) + B(t + 2)$$

Put $t = -3$

$$-3 = B(-3 + 2)$$

$$-3 = B(-1) \qquad \therefore\ B = 3$$

Put $t = -2$

$$-2 = A(-2 + 3)$$

$$-2 = A(1) \qquad \therefore\ A = -2$$

THEREFORE

$$\frac{t}{(t + 2)(t + 3)} = \frac{-2}{t + 2} + \frac{3}{t + 3}$$

HENCE

$$\frac{x^2}{(x^2 + 2)(x^2 + 3)} = \frac{-2}{x^2 + 2} + \frac{3}{x^2 + 3}$$

BACK IN THE SUM

$$= \int \frac{-2}{x^2 + 2} + \frac{3}{x^2 + 3}\,dx$$

$$= \int \frac{-2}{x^2 + \sqrt{2}^2} + \frac{3}{x^2 + \sqrt{3}^2}\,dx$$

$$= -2 \cdot \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + 3\ \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c$$

$$= -\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \sqrt{3} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c$$

03. $A = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{pmatrix}$ Verify : $A.(\text{adj } A) = (\text{adj } A).A = |A|.I$

COFACTOR'S

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 0 & -2 \\ 0 & 3 \end{vmatrix} = 1(0-0) = 0$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} = -1(9+2) = -11$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} = 1(0-0) = 0$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 2 \\ 0 & 3 \end{vmatrix} = -1(-3-0) = 3$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = 1(3-2) = 1$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = -1(0+1) = -1$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 2 \\ 0 & -2 \end{vmatrix} = 1(2-0) = 2$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} = -1(-2-6) = 8$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -1 \\ 3 & 0 \end{vmatrix} = 1(0+3) = 3$$

COFACTOR MATRIX OF A

$$\begin{pmatrix} 0 & -11 & 0 \\ 3 & 1 & -1 \\ 2 & 8 & 3 \end{pmatrix}$$

ADJ A **=** TRANSPOSE OF THE COFACTOR MATRIX

$$= \begin{pmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{pmatrix}$$

|A|

$$= 1(0 + 0) + 1(9 + 2) + 2(0 - 0)$$

$$= 11$$

LHS 1

= A.(adj A)

$$= \begin{pmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{pmatrix} \begin{pmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 0 + 11 + 0 & 3 - 1 - 2 & 2 - 8 + 6 \\ 0 - 0 - 0 & 9 + 0 + 2 & 6 + 0 - 6 \\ 0 - 0 + 0 & 3 + 0 - 3 & 2 + 0 + 9 \end{pmatrix}$$

$$= \begin{pmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{pmatrix}$$

LHS 2

= (adj A) . A

$$= \begin{pmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{pmatrix} \begin{pmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 0 + 9 + 2 & 0 + 0 + 0 & 0 - 6 + 6 \\ -11 + 3 + 8 & 11 + 0 + 0 & -22 - 2 + 24 \\ 0 - 3 + 3 & 0 - 0 + 0 & 0 + 2 + 9 \end{pmatrix}$$

$$= \begin{pmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{pmatrix}$$

RHS

= |A|.I

$$= 11 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{pmatrix}$$

HENCE A.(adj A) = (adj A).A = |A|.I

SECTION – II

Q4. (A)Attempt any six of the following (12)

01. Find correlation coefficient between x and y for the following data

$n = 100$, $\overline{x} = 62$, $\overline{y} = 53$, $\sigma x = 10$, $\sigma y = 12$, $\Sigma(x - \overline{x})(y - \overline{y}) = 8000$

SOLUTION $r = \dfrac{cov(x,y)}{\sigma x \,.\, \sigma y}$

$$= \frac{\dfrac{\Sigma(x - \overline{x})(y - \overline{y})}{n}}{\sigma x \,.\, \sigma y}$$

$$= \frac{\dfrac{8000}{100}}{10.12}$$

$$= \frac{80}{10.12}$$

$$= \frac{2}{3}$$

02. a car is insured for 80% of its value . An accident took place and the car was damaged to the extent of 40% of its value . How much can be claimed under the policy if the annual premium at 75 paise percent amounts to ₹ 360 . Also state the value of the car

SOLUTION

Value of car = ₹ x

Insured value $= \dfrac{80x}{100} = \dfrac{4x}{5}$

Rate of premium = 77 paise percent

= 0.75%

Premium = ₹ 360

$360 = \dfrac{0.75}{100} \times \dfrac{4x}{5}$

$360 = \dfrac{75}{10000} \times \dfrac{4x}{5}$

$360 = \dfrac{6x}{1000}$

$x = 60,000$

value of car = ₹ 60,000

Loss = $\frac{40}{100}$ x 60,000

= ₹ 24,000

Claim = 80% of loss

= $\frac{80}{100}$ x 24,000

= ₹ 19,200

03. The coefficient of rank correlation for a certain group of data is 0.5 . If $\Sigma d^2 = 42$, assuming no ranks are repeated ; find the no. of pairs of observation

SOLUTION

$R = 0.5$; $\Sigma d^2 = 42$

$$R = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

$$0.5 = 1 - \frac{6(42)}{n(n^2 - 1)}$$

$$\frac{6(42)}{n(n^2 - 1)} = 1 - 0.5$$

$$\frac{6(42)}{n(n^2 - 1)} = 0.5$$

$$\frac{6(42)}{n(n^2 - 1)} = \frac{1}{2}$$

$$n(n^2 - 1) = 6 \times 42 \times 2$$

$$n(n^2 - 1) = 3 \times 2 \times 3 \times 2 \times 7 \times 2$$

$$(n - 1).n.(n + 1) = 7 \times 8 \times 9$$

On comparing , n = 8

04. Ameena started a business by investing certain capital . After 3 months she admitted Yasmin as a partner who invested the same amount . Finding the need for more capital they invited Shabana into the partnership after 4 months with same capital as they had . At the end of the year profit was ₹ 23,400 . How much was the share of each in the profit .

SOLUTION

STEP 1 :

Profits will be shared in the ratio of

'PERIOD OF INVESTMENT'

	AMEENA		YASMIN		SHABANA
=	12	:	9	:	5

TOTAL = 26

STEP 2 :

PROFIT = ₹ 23,400

Ameena's share of profit= $\frac{12}{\cancel{26}} \times \overset{900}{\cancel{23,400}}$ = ₹ 10,800

Yasmin's share of profit = $\frac{9}{\cancel{26}} \times \overset{900}{\cancel{23,400}}$ = ₹ 8,100

Shabana's share of profit= $\frac{5}{\cancel{26}} \times \overset{900}{\cancel{23,400}}$ = ₹ 4,500

05. Calculate CDR for district A and B and compare

Age Group (Years)	**DISTRICT A**		**DISTRICT B**	
	NO. OF PERSONS **P**	NO. OF DEATHS **D**	NO. OF PERSONS **P**	NO. OF DEATHS **D**
0 – 10	1000	18	3000	70
10 – 55	3000	32	7000	50
Above 55	2000	41	1000	24
	ΣP = 6000	**ΣD = 91**	**ΣP = 11000**	**ΣD = 144**

CDR(A) $= \frac{\Sigma D}{\Sigma P} \times 1000$

$= \frac{91}{6000} \times 1000$

= 15.17

(DEATHS PER THOUSAND)

CDR(B) $= \frac{\Sigma D}{\Sigma P} \times 1000$

$= \frac{144}{11000} \times 1000$

= 13.09

(DEATHS PER THOUSAND)

COMMENT : CDR(B) < CDR(A) . HENCE DISTRICT B IS HEALTHIER THAN DISTRICT A

06. the probability of defective bolts in a workshop is 40% . Find the mean and variance of defective bolts out os 10 bolts

SOLUTION $n = 10$,

r,v,x = no of defective bolts

p = probability of defective bolt $= \frac{40}{100} = \frac{2}{5}$

$q = 1 - p = \frac{3}{5}$

$X \sim B(10, {}^{2}/_{5})$

Mean $= np = 10 \times \frac{2}{5} = 5$

Variance $= npq = 10 \times \frac{2}{5} \times \frac{3}{5} = 2.4$

07. The ratio of prices of two cycles was 16:23 . Two years later when the price of first cycle has increased by 10% and that of second by ₹ 477 ; the ratio of prices becomes 11 : 20 . Find the original prices of two cycles

SOLUTION

Let price of cycle 1 = 16x

Price of cycle 2 = 23x

As per the given condition

$$\frac{16x + \frac{10}{100}(16x)}{23x + 477} = \frac{11}{20}$$

$$16x + \frac{8x}{5} = \frac{11}{20}(23x + 477)$$

$$\frac{88x}{5} = \frac{11}{20}(23x + 477)$$

$$32x = 23x + 477$$

$$9x = 477 \qquad x = 53$$

$\therefore$

price of cycle 1 = 16(53) = ₹ 848

price of cycle 2 = 23(53) = ₹ 1219

08. for an immediate annuity paid for 3 years with interest compounded at 10% p.a. its present value is ₹ 10,000 . What is the accumulated value after 3 years ($1.1^3 = 1.331$)

SOLUTION $A = P(1 + i)^n$

$= 10000(1 + 0.1)^3$

$= 10000(1.1)^3$

$= 10000(1.331)$

= ₹ 13,310

Q5. (A)Attempt any Two of the following **(06)**

01. The probability that a student from an evening college will be a graduate is 0.4 . Determine the probability that out of 5 students

(i) one will be graduate (ii) at least one will be graduate

SOLUTION

5 students , n = 5

For a trial Success – student is a graduate

p – probability of success = 4/10 = 2/5

q – probability of failure = 3/5

r.v. X – no of successes = 0 , 1 , 2 ,.... , 5 **X ~ B (5 , 2/5)**

a) P(one will be graduate)

$$= P(X = 1)$$

$$= P(1)$$

$$= {}^5C_1 \cdot p^1 \cdot q^4$$

$$= {}^5C_1 \left(\frac{2}{5}\right)^1 \left(\frac{3}{5}\right)^4$$

$$= \frac{5 \cdot 2 \cdot 81}{3125}$$

$$= \frac{162}{625}$$

b) P(at least one will be graduate)

$$= P(X \geq 1)$$

$$= 1 - P(0)$$

$$= 1 - {}^5C_0 \left(\frac{2}{5}\right)^0 \left(\frac{3}{5}\right)^5$$

$$= 1 - \frac{1 \cdot 1 \cdot 243}{3125}$$

$$= \frac{3125 - 243}{3125}$$

$$= \frac{2882}{3125}$$

02. Compute rank correlation coefficient for the following data

Rx :	1	2	3	4	5	6
Ry :	6	3	2	1	4	5

SOLUTION

x	y	d = |x − y|	d^2
1	6	5	25
2	3	1	1
3	2	1	1
4	1	3	9
5	4	1	1
6	5	1	1
			$\Sigma d^2 = 38$

$$R = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

$$= 1 - \frac{6(38)}{6(36 - 1)}$$

$$= 1 - \frac{38}{35}$$

$$= \frac{-3}{35}$$

$$= -0.086$$

03. the income of the agent remains unchanged though the rate of commission is increased from 6% to 7.5% . Find the percentage reduction in the value of the business

SOLUTION

Let initial sales = ₹ 100

Rate of commission = 6%

∴ Commission = □ 6

Let the new sales = ₹ x

Rate of commission = 7.5%

∴ Commission = $\frac{7.5x}{100}$

Since the income of the broker remains unchanged

$$\frac{7.5\,x}{100} = 6$$

$$x = \frac{6 \times 1000}{75}$$

$$x = 80$$

∴ new sales = ₹ 80

Hence percentage reduction in the value of the business = 20%

(B) Attempt any Two of the following **(08)**

01. the value of godown of ₹ 40,000 contains stock worth ₹ 2,40,000 . They were insured for ₹ 25,000 and 80% of the stock respectively . Due to fire , stock worth ₹ 30,000 was completely destroyed while remaining was reduced to 60% of its value . The damage to the godown was ₹ 20,000 . What sum can be claimed under the policy

SOLUTION

GODOWN

Property value = ₹ 40,000

Insured value = ₹ 25,000

Loss = ₹ 20,000

Claim = $\frac{\text{insured val.}}{\text{Property val.}} \times \text{loss}$

$= \frac{25,000}{40,000} \times 20,000$

= ₹ 12,500

STOCK IN GODOWN

Value of stock = ₹ 2,40,000

Insured value = 80% of the stock

Loss

stock worth ₹ 30,000 was completely destroyed while remaining was reduced to 60% of its value

$= 30,000 + \frac{40}{100}(2,40,000 - 30,000)$

$= 30,000 + \frac{40}{100}(2,10,000)$

= 30, 000 + 84,000

= ₹ 1,14,000

Since 80% of the stock was insured

Claim = 80% of loss

$= \frac{80}{100} \times 1,14,000$

= ₹ 91,200

Hence

Total claim = 12, 500 + 91,200 = ₹ 1,03,700

02. X : 6 2 10 4 8

Y : 9 11 ? 8 7

Arithmetic means of X and Y series are 6 and 8 respectively . Calculate correlation coefficient

SOLUTION :

$$\bar{y} = \frac{\Sigma y}{n} \qquad 8 = \frac{9 + 11 + b + 8 + 7}{5}$$

$$40 = 35 + b \qquad b = 5$$

x	y	$x-\bar{x}$	$y-\bar{y}$	$(x-\bar{x})^2$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
6	9	0	1	0	1	0
2	11	–4	3	16	9	–12
10	5	4	–3	16	9	–12
4	8	–2	0	4	0	0
8	7	2	–1	4	1	–2
30 Σx	40 Σy	0 $\Sigma(x-\bar{x})$	0 $\Sigma(y-\bar{y})$	40 $\Sigma(x-\bar{x})^2$	20 $\Sigma(y-\bar{y})^2$	–26 $\Sigma(x-\bar{x})(y-\bar{y})$
$\bar{x} = 6$ $\bar{y} = 8$						

$$r = \frac{\Sigma(x-\bar{x}).(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}.}$$

$$r = \frac{-26}{\sqrt{40} \times \sqrt{20}}$$

$$r = \frac{-26}{\sqrt{40 \times 20}}$$

$$r' = \frac{26}{\sqrt{40 \times 20}}$$

taking log on both sides

$$\log r' = \log 26 - \frac{1}{2}\left(\log 40 + \log 20\right)$$

$$\log r' = 1.4150 - \frac{1}{2}[\,1.6021 + 1.3010\,]$$

$$\log r' = 1.4150 - \frac{1}{2}(2.9031)$$

$$\log r' = 1.4150 - 1.4516$$

$$\log r' = \bar{1}.9634$$

$$r' = AL(\bar{1}.9634) = 0.9191$$

$$r = -0.9191$$

03. A bill of □ 21,900 drawn on July 10 for 6 months was discounted for ₹ 21,720 at 5% p.a. On which day the bill was discounted

SOLUTION

due 6 months @ 5% p.a.

d days

10^{th} July ? ← 13^{th}Jan

₹ 21,720 ₹ 21,900

STEP 1:

Date of drawing	=	10 / 07
Add period of bill	+	6 months
Nominal due date	=	10 / 01
Add Grace days	+	3 days
Legal due date	=	13 / 01 ……. 13th January

STEP 2:

Let Unexpired period = d days

STEP 3 :

B.D. = F.V. – C.V.

= 21,900 – 21,720

= ₹ 180

STEP 4 :

B.D. = Interest on F.V. for 'd' days @ 5% p.a.

$$180 = \overset{3}{\cancel{21900}} \times \frac{d}{\underset{\cancel{73}}{\cancel{365}}} \times \frac{\cancel{5}}{\cancel{100}}$$

d = 60

STEP 5:

Dt. of Discount

= 13^{th} Jan – 60 days

Jan Dec Nov

= 13 + 31 + 16

= 14^{th} November

Q6. (A) Attempt any Two of the following (06)

01. 100 misprints are distributed randomly throughout the 100 pages of a book . Assuming the distribution of the number of misprints to be Poisson , find the probability that a page at random will contain at least three misprints ($e^{-1} = 0.368$)

SOLUTION

m = average number of misprints per page = 100/100 = 1

r.v $X \sim P(1)$

P(a page at random will contain at least three misprints)

$= P(x \geq 3)$

$= P(3) + P(4) + \ldots\ldots$

$= 1 - \left(P(0) + P(1) + P(2) \right)$

$= 1 - \left(\frac{e^{-1}.1^0}{0!} + \frac{e^{-1}.1^1}{1!} + \frac{e^{-1}.1^2}{2!} \right)$ Using $P(x) = \frac{e^{-m}.m^x}{x!}$

$= 1 - e^{-1}.\left(\frac{1}{1} + \frac{1}{1} + \frac{1}{2} \right)$

$= 1 - 0.368(1 + 1 + 0.5)$

$= 1 - 0.368(2.5)$

$= 1 - 0.92$

$= 0.08$

02. Suppose X is a random variable with pdf

$$f(x) = \frac{c}{x} \quad ; \quad 1 < x < 3 \quad ; \quad c > 0$$

Find c & E(X)

i) $$\int_1^3 \frac{c}{x} \, dx = 1$$

$$c \int_1^3 \frac{1}{x} \, dx = 1$$

$$c \Big[\log x\Big]_1^3 = 1$$

$$c \Big[\log 3 - \log 1\Big] = 1$$

$$c \log 3 = 1$$

$$c = \frac{1}{\log 3}$$

Hence X is a r.v. with pdf

$$f(x) = \frac{1}{x.\log 3} \quad ; \quad 1 < x < 3$$

ii) $$E(x) = \int_1^3 x.f(x) \, dx$$

$$= \int_1^3 x. \frac{1}{x.\log 3} \, dx$$

$$= \int_1^3 \frac{1}{\log 3} \, dx$$

$$= \left[\frac{x}{\log 3}\right]_1^3$$

$$= \left(\frac{3}{\log 3}\right) - \left(\frac{1}{\log 3}\right) = \frac{2}{\log 3}$$

03. In a factory there are six jobs to be performed , each of which should go through machines A and B in the order A – B . Determine the sequence for performing the jobs that would minimize the total elapsed time T . Find T and the idle time on the two machines

Job	J_1	J_2	J_3	J_4	J_5	J_6
M_A	1	3	8	5	6	3
M_B	5	6	3	2	2	10

Step 1 : Finding the optimal sequence

Min time = 1 on job J_1 on machine M_1 . Place the job at the start of the sequence

J_1					

Next min time= 2 on jobs J_4 & J_5 on machine M_B . Place the jobs at the end of the sequence randomly

J_1				**J_4**	**J_5**

Placed Randomly

Next min time = 3 on jobs J_2 & J_6 on machine M_A and on job J_3 on machine M_B respectively . Place J_2 & J_6 at the start next to J_1 randomly and J_3 at the end next to J_4

J_1	**J_2**	**J_6**	**J_3**	**J_4**	**J_5**

Placed Randomly

OPTIMAL SEQUENCE

J_1	**J_2**	**J_6**	**J_3**	**J_4**	**J_5**

Step 2 : Work table

According to the optimal sequence

Job	**J_1**	**J_2**	**J_6**	**J_3**	**J_4**	**J_5**	**total process time**	
M_A	1	3	3	8	5	6	=	26 hrs
M_B	5	6	10	3	2	2	=	28 hrs

WORK TABLE

	MACHINES				Idle time
	M_A		M_B		
JOBS	IN	OUT	IN	OUT	on M_B
J_1	0	1	1	6	1
J_2	1	4	6	12	
J_6	4	7	12	22	
J_3	7	15	22	25	
J_4	15	20	25	27	
J_5	20	26	27	29	

Step 3 :

Total elapsed time T = 29 hrs

Idle time on M_A = T – (sum of processing time of all 6 jobs on M1)

= 29 – 26

= 3 hrs

Idle time on M_B = T – (sum of processing time of all 6 jobs on M2)

= 29 – 28

= 1 hr

Step 4 : All possible optimal sequences :

$$J_1 - J_2 - J_6 - J_3 - J_4 - J_5$$

OR

$$J_1 - J_6 - J_2 - J_3 - J_4 - J_5$$

OR

$$J_1 - J_2 - J_6 - J_3 - J_5 - J_4$$

OR

$$J_1 - J_6 - J_2 - J_3 - J_5 - J_4$$

(B) Attempt any Two of the following (08)

01. a pharmaceutical company has four branches , one at each city A , B , C and D . A branch manager is to be appointed one at each city , out of four candidates P , Q , R and S . The monthly business depends upon the city and effectiveness of the branch manager in that city

		CITY A	B	C	D	
	P	11	11	9	9	
BRANCH	Q	13	16	11	10	**MONTHLY BUSINESS (IN LACS)**
MANAGER	R	12	17	13	8	
	S	16	14	16	12	

Which manager should be appointed at which city so as to get maximum total monthly business .

$$\begin{matrix} 6 & 6 & 8 & 8 \\ 4 & 1 & 6 & 7 \\ 5 & 0 & 4 & 9 \\ 1 & 3 & 1 & 5 \end{matrix}$$

Subtracting all the elements in the matrix from its maximum

The matrix can now be solved for 'MINIMUM'

$$\begin{matrix} 0 & 0 & 2 & 2 \\ 3 & 0 & 5 & 6 \\ 5 & 0 & 4 & 9 \\ 0 & 2 & 0 & 4 \end{matrix}$$

Reducing the matrix using 'ROW MINIMUM'

$$\begin{matrix} 0 & 0 & 2 & 0 \\ 3 & 0 & 5 & 4 \\ 5 & 0 & 4 & 7 \\ 0 & 2 & 0 & 2 \end{matrix}$$

Reducing the matrix using 'COLUMN MINIMUM'

$$\begin{matrix} \boxed{0} & \cancel{0} & 2 & \cancel{0} \\ 3 & \boxed{0} & 5 & 4 \\ 5 & \cancel{0} & 4 & 7 \\ \cancel{0} & 2 & \boxed{0} & 2 \end{matrix}$$

– Allocation using 'SINGLE ZERO ROW-COLUMN METHOD'

– Allocation incomplete (3^{rd} row unassigned)

$$\begin{matrix} & \boxed{0} & \cancel{0} & 2 & \cancel{0} \\ \surd & 3 & \boxed{0} & 5 & 4 \\ \surd & 5 & \cancel{0} & 4 & 7 \\ & \cancel{0} & 2 & \boxed{0} & 2 \\ & & \surd & & \end{matrix}$$

– Drawing min. no. of lines to cover all '0's

0	3	2	0	Revise the matrix
0	0	2	1	Reducing all the uncovered elements by its minimum and adding the same at the intersection
2	0	1	4	
0	4	0	2	

$\cancel{0}$	3	2	$\boxed{0}$	– Reallocation using 'SINGLE ZERO ROW-COLUMN METHOD'
$\boxed{0}$	$\cancel{0}$	2	1	– Since all rows contain an 'assigned zero' , the assignment problem is complete
2	$\boxed{0}$	1	4	
$\cancel{0}$	4	$\boxed{0}$	2	

Optimal Assignment : P – D ; Q – A; R – B ; S – C

Maximum business = 9 + 13 + 17 + 16 = 55(lacs)

02. the management of a large furniture store would like to determine if there is any relationship between the number of people entering the store on a given day (X) and sales (in thousands of ☐) (Y) . The records of ten days is given

$\Sigma x = 580$; $\Sigma y = 370$; $\Sigma x^2 = 41658$; $\Sigma y^2 = 17206$; $\Sigma xy = 11494$

Obtain regression line X on Y

SOLUTION

$$\bar{x} = \frac{\Sigma x}{n} = \frac{580}{10} = 58 \quad ; \quad \bar{y} = \frac{\Sigma y}{10} = \frac{370}{10} = 37$$

$$bxy = \frac{n\Sigma xy - \Sigma x.\Sigma y}{n\Sigma y^2 - (\Sigma y)^2}$$

$$= \frac{10(11494) - (580)(370)}{10(17206) - (370)^2}$$

$$= \frac{114940 - 214600}{172060 - 136900}$$

$$= \frac{-99660}{35160}$$

$$= -\frac{9966}{3516}$$

$$= -2.835$$

LOG CALC
3.9986
– 3.5460
AL 0.4526
2.835

Equation

$$x - \bar{x} = bxy\,(y - \bar{y})$$

$$x - 58 = -2.835\,(y - 37)$$

$$x - 58 = -2.835\,y + 104.895$$

$$x = -2.835\,y + 104.895 + 58$$

$$x = -2.835\,y + 162.895$$

03. Minimize $z = 30x + 20y$, subject to

$x + y \leq 8$, $6x + 4y \geq 12$, $5x + 8y \geq 20$, $x, y \geq 0$

STEP 1

$x + y \leq 8$	$x + y = 8$ cuts x – axis at (8,0) cuts y – axis at (0,8)	Put (0,0) in $x + y \leq 8$ $0 \leq 8$ SS : ORIGIN SIDE
$6x + 4y \geq 12$	$6x + 4y = 12$ cuts x – axis at (2,0) cuts y – axis at (0,3)	Put (0,0) in $6x + 4y \geq 12$ $0 \geq 12$ (NOT SATISFIED) SS : NON-ORIGIN SIDE
$5x + 8y \geq 20$	$5x + 8y = 20$ cuts x – axis at (4,0) cuts y – axis at (0,2.5)	Put (0,0) in $5x + 8y \geq 20$ $0 \geq 20$ (NOT SATISFIED) SS : NON-ORIGIN SIDE
$x, y \geq 0$		SS : I QUADRANT

FOR C 2 x

$$6x + 4y = 12 \quad \ldots\ldots (1)$$
$$5x + 8y = 20 \quad \ldots\ldots (2)$$

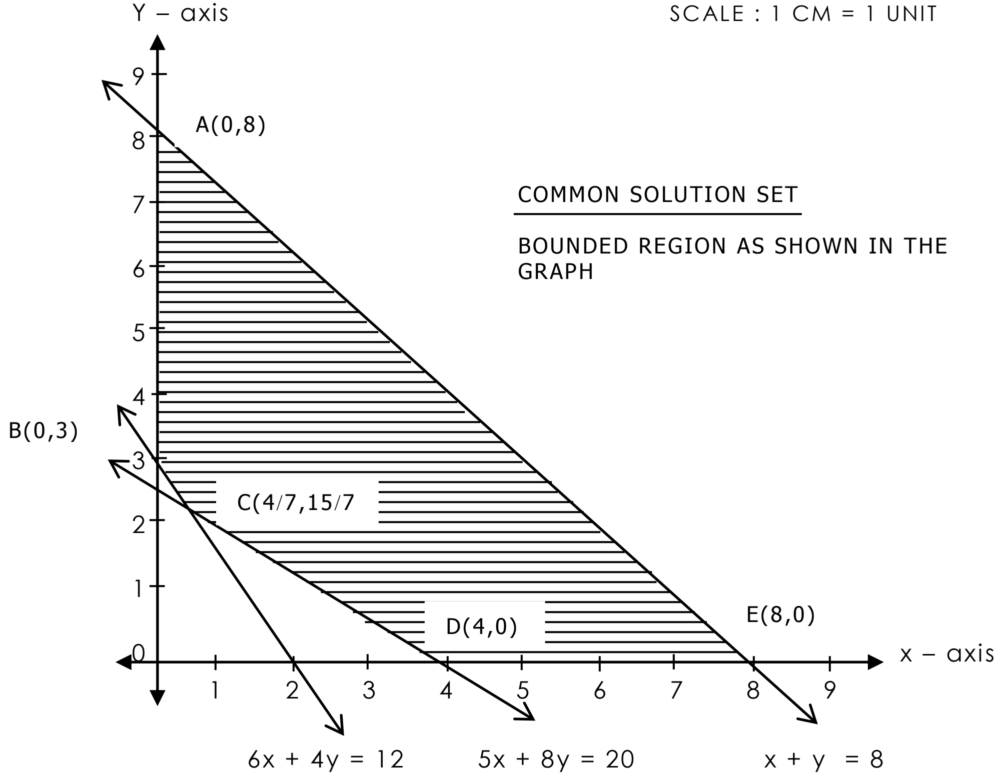
$$12x + 8y = 24$$
$$5x + 8y = 20$$
$$7x = 4$$
$$x = 4/7$$

subs in (1)

$$y = 15/7$$

$$C \equiv \left({}^{4}/_{7}, {}^{15}/_{7}\right)$$

STEP 2



STEP 3 :

CORNERS	Z = 30x + 20y	
A(0,2)	Z = 30(0) + 20(8)	= 160
B(0.3)	Z = 30(0) + 20(3)	= 60
C(4/7,15/7)	Z = $\frac{120}{7} + \frac{300}{7}$	= $\frac{420}{7}$ = 60
D(4.0)	Z = 30(4) + 20(0)	= 120
E(8,0)	Z = 30(8) + 20(0)	= 240

STEP 4

Zmin = 60 at all points on seg BC (INFINITE OPTIMAL SOLUTIONS)