J.K. SHAH CLASSES

MATHEMATICS & STATISTICS

SYJC PRELIUM - 01 - SET B **SET - B**

SOLUTION SET

DURATION - 3 HR MARKS - 80

SECTION - I

Q1. (A) Attempt any six of the following **(12)**

Q1

01. $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$, find $A^2 - 3A$

SOLUTION :

$$A^2 - 3A$$

$$= \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} - 3\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2 & 2+6 \\ 1+3 & 2+9 \end{bmatrix} - \begin{bmatrix} 3 & 6 \\ 3 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 8 & - & 3 & 6 \\ 4 & 11 & & 3 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 \\ 1 & 2 \end{bmatrix}$$

02. Differentiate $\log(1 + x^2)$ wrt $\tan^{-1}x$

SOLUTION :

$$u = \log(1 + x^2)$$

$$\frac{du}{dx} = \frac{1}{1 + x^2} \frac{d}{dx}(1 + x^2)$$

$$= \frac{2x}{1 + x^2}$$

$$v = \tan^{-1}x$$

$$\frac{dv}{dx} = \frac{1}{1 + x^2}$$

$$\frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{2x / (1+x^2)}{1/(1+ x^2)}$$

$$= 2x$$

03. if f(x) is continuous at x = 2 , then find f(2) where

$$f(x) = \frac{x^5 - 32}{x - 2}, \quad x \neq 2$$

SOLUTION : STEP 1 $\lim_{x \to 2} f(x)$

$$= \lim_{x \to 2} \frac{x^5 - 32}{x - 2}$$

$$= \lim_{x \to 2} \frac{x^5 - 2^5}{x - 2}$$

$$= 5(2)^{5-1}$$

$$= 5(2)^4 = 80$$

STEP 2 : Since function is continuous at x = 2

$$f(2) = \lim_{x \to 2} f(x) = 80$$

04. the total revenue $R = 10800x - 4x^3$ where x is number of units sold . Find x for which total revenue R is increasing

SOLUTION : R = 10800 – 4x3

For Revenue Increasing ,

$$\frac{dR}{dx} > 0$$

$$10800 - 12x^2 > 0$$

$$10800 > 12x^2$$

$$900 > x2$$

$$x^2 > 900$$

$$x > 30$$

05. if p : It is raining

q : It is humid

Write the following statements in the symbolic form

a) if it is raining then it is humid $\equiv p \rightarrow q$

b) It is raining but not humid $\equiv p \wedge \sim q$

06. Write negations of the following statements

a) Radha likes tea or coffee

Using $\sim(p \vee q) \equiv \sim p \wedge \sim q$

Negation : Radha does not like tea and does not like coffee

b) $\exists\, x \in R$ such that $x + 3 \geq 10$

Negation : $\forall\, x \in R\ ,\ x + 3 < 10$

07. Evaluate : $\displaystyle\int \frac{x+1}{x(x+\log x)}\, dx$

SOLUTION : Put $x + \log x = t$

$$1 + \frac{1}{x} \,.\, dx = dt$$

$$\frac{x+1}{x} \,.\, dx = dt$$

$$= \int \frac{1}{t}\, dt$$

$$= \log |\, t\, | + c$$

$$= \log |x + \log x| + c$$

08. Solve the equations $x + y = 4$ & $2x - y = 5$ using method of REDUCTION

SOLUTION :

$$\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

R2 + R1

$$\begin{bmatrix} 1 & 1 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} x + y \\ 3x \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \end{bmatrix}$$

By Equatlity of Matrices , $3x = 9 \quad \therefore\ x = 3$

$x + y = 4$

subs $x = 3 \quad \therefore\ y = 1$

Q2. (A) Attempt any TWO of the following **(06)**

Q2A

01. Solve the following equations by the inversion method

$2x + 3y = -5$ and $3x + y = 3$

STEP 1 :

$$\begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$$

$AX = B$

$A^{-1}AX = A^{-1}B$

$IX = A^{-1}B$

$X = A^{-1}B$

STEP 2 :

$AA^{-1} = I$

$$\begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$R_1 - R_2$

$$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$R_2 - 2R_1$

$$\begin{bmatrix} 1 & -2 \\ 0 & 7 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$$

$R_2/7$

$$\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -1 \\ -\frac{2}{7} & \frac{3}{7} \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} A^{-1} = \frac{1}{7}\begin{bmatrix} 7 & -7 \\ -2 & 3 \end{bmatrix}$$

$R_1 + 2R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \frac{1}{7}\begin{bmatrix} 3 & -1 \\ -2 & 3 \end{bmatrix}$$

$$I.A^{-1} = \frac{1}{7}\begin{bmatrix} 3 & -1 \\ -2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{7}\begin{bmatrix} 3 & -1 \\ -2 & 3 \end{bmatrix}$$

STEP 3 :

$X = A^{-1}B$

$$= \frac{1}{7}\begin{bmatrix} 3 & -1 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ -5 \end{bmatrix}$$

$$= \frac{1}{7}\begin{bmatrix} 9 + 5 \\ -6 - 15 \end{bmatrix}$$

$$= \frac{1}{7}\begin{bmatrix} 14 \\ -21 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

BY EQUALITY OF MATRICES

$x = 2$ & $y = -3$

02. Using the truth table , examine whether the statement pattern

$p \vee \sim (p \wedge q)$

is a tautology , a contradiction or a contingency

SOLUTION :

p	q	$p \wedge q$	$\sim(p \wedge q)$	$p \vee \sim(p \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Conclusion : Since all the values in the last column are 'T' , the given statement is TAUTULOGY

03. if the demand function is $D = 50 - 3p - p^2$. Find the elasticity of demand at p = 5

Also comment on the result

SOLUTION

STEP 1 : $D = 50 - 3p - p^2$.

$$\frac{dD}{dp} = -3 - 2p$$

STEP 2 : $\eta = \frac{-P}{D} . \frac{dD}{dp}$

$$= -\frac{p}{50 - 3p - p^2} . (-3 - 2p)$$

$$= \frac{3p + 2p^2}{50 - 3p - p^2}$$

STEP 3 : $\eta \Big|_{p = 5}$

$$= \frac{3(5) + 2(5)^2}{50 - 3(5) - (5)^2}$$

$$= \frac{15 + 2(25)}{50 - 15 - 25}$$

$$= \frac{65}{10}$$

$$= 6.5 > 1 .$$

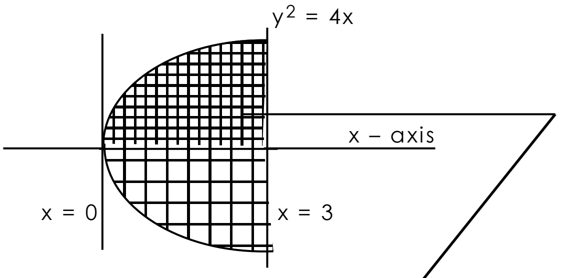
Demand is relatively elastic

(B) Attempt any TWO of the following **(08)**

Q2B

01. Find the area of the region bounded by curve $y^2 = 4x$ and line $x = 3$

SOLUTION



$$A = 2\int_0^3 y\,dx \quad \text{.... (BY SYMMETRY)}$$

$$= 2\int_0^3 \sqrt{4x}\,dx$$

$$= 2\int_0^3 2\sqrt{x}\,dx$$

$$= 4\int_0^3 x^{1/2}\,dx$$

$$= 4\left[\frac{x^{3/2}}{\frac{3}{2}}\right]_0^3$$

$$= \frac{8}{3}\left[x^{3/2}\right]_0^3$$

$$= \frac{8}{3}\left[3^{3/2} - 0^{3/2}\right]$$

$$= \frac{8}{3}\left[3\sqrt{3}\right]$$

$$= 8\sqrt{3} \text{ sq. units}$$

02. Evaluate : $\int \log(1 + x^2)\,dx$

SOLUTION

$$= \int \log(1 + x^2).\,1dx$$

$$= \log(1 + x^2)\int 1\,dx - \int\left(\frac{d}{dx}\log(1 + x^2)\int 1\,dx\right)dx$$

$$= \log(1 + x^2)\,.\,x - \int \frac{2x}{1 + x^2}.\,x\ dx$$

$$= x.\log(1 + x^2) - 2\int \frac{x^2}{1 + x^2}\,dx$$

$$= x.\log(1 + x^2) - 2\int \frac{1 + x^2 - 1}{1 + x^2}.\,dx$$

$$= x.\log(1+x^2) - 2\int \left(1 - \frac{1}{1+x^2}\right) dx$$

$$= x.\log(1+x^2) - 2\left(x - \tan^{-1}x\right) + c$$

$$= x.\log(1+x^2) - 2x + 2\tan^{-1}x + c$$

03. The total cost of producing x units is □ $(x^2 + 60x + 50)$ and the price per unit is □ $(180 - x)$. For what units the profit is maximum

SOLUTION

STEP 1 :

$R = p.x$

$= (180 - x).x$

$= 180x - x^2$

PROFIT

$\pi = R - C$

$\pi = 180x - x^2 - (x^2 + 60x + 50)$

$\pi = 180x - x^2 - x^2 - 60x - 50$

$\pi = 120x - 2x^2 - 50$

STEP 2 :

$$\frac{d\pi}{dx} = 120 - 4x$$

$$\frac{d^2\pi}{dx^2} = -4$$

STEP 3 :

$$\frac{d\pi}{dx} = 0$$

$120 - 4x = 0$

$120 = 4x$

$x = 30$

STEP 4 :

$$\left.\frac{d^2\pi}{dx^2}\right|_{x = 30} = -4 < 0$$

Profit is maximum at $x = 30$

Q3. (A) Attempt any TWO of the following **(06)**

Q3A

01. Write Converse – Contrapositive & Inverse statements for the given conditional statement

if the triangles are not congruent then their areas are not equal

SOLUTION :

LET $P \to Q$ ≡ if the triangles are not congruent then their areas are not equal

CONVERSE : $Q \to P$

If the areas of triangles are not equal then they are not congruent

CONTRAPOSITIVE : $\sim Q \to \sim P$

If the areas of the triangles are equal then they are congruent

INVERSE : $\sim P \to \sim Q$

If the two triangles are congruent then their areas are equal

02. find a & b if f(x) is continuous at x = 0 & f(1) = 2 where ;

$$f(x) = x^3 + a + b \quad ; \quad x \geq 0$$

$$= 2\sqrt{x^3 + 1} + a \quad ; \quad x < 0$$

SOLUTION :

STEP 1

$$\lim_{x \to 0+} f(x)$$

$$= \lim_{x \to 0} x^3 + a + b$$

$$= 0^2 + a + b = a + b$$

STEP 2

$$\lim_{x \to 0-} f(x)$$

$$= \lim_{x \to 0} 2\sqrt{x^3 + 1} + b$$

$$= 2\sqrt{0^3 + 1} + b$$

$$= 2 + b$$

STEP 3

$$f(0) = 0^2 + a + b$$

$$= a + b$$

STEP 4

Since f is continuous at x = 0

$$\lim_{x \to 0-} f(x) = \lim_{x \to 0+} f(x) = f(0)$$

$$2 + b = a + b = a + b$$

$$2 + b = a + b$$

$$a = 2$$

STEP 5

$$f(1) = 2$$

$$1^2 + a + b = 2$$

$$a + b = 1$$

Sub a = 2

$$2 + b = 1$$

$$b = -1$$

03. if $y = 5^x + x^x$; find $\frac{dy}{dx}$

SOLUTION :

$y = u + v$

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dv}$ .... (1)

Now

$u = x^x$

Taking log on both sides

$\log u = x . \log x$

$\frac{1}{u}\frac{du}{dx} = x \frac{d}{dx}\log x + \log x \frac{d}{dx} x$

$\frac{1}{u}\frac{du}{dx} = x . \frac{1}{x} + \log x . 1$

$\frac{1}{u}\frac{du}{dx} = 1 + \log x$

$\frac{du}{dx} = u (1 + \log x)$

$v = 5^x$

Differentiating wrt x

$\frac{dv}{dx} = 5^x.\log 5$

Hence

$\frac{dy}{dx} = x^x (1 + \log x) + 5^x.\log 5$

(B) Attempt any TWO of the following **(08)**

Q3B

01. $$\int_{\pi/6}^{\pi/3} \frac{\cot x}{1 + \cot x}\,dx$$

SOLUTION :

$$I = \int_{\pi/6}^{\pi/3} \frac{\cos x}{\cos x + \sin x}\,dx \quad \textbf{... (1)}$$

USING $$\int_a^b f(x)\,dx = \int_b^b f(a + b - x)\,dx$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\cos(\pi/2 - x)}{\cos(\pi/2 - x) + \sin(\pi/2 - x)}\,dx$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sin x}{\sin x + \cos x}\,dx \quad \textbf{... (2)}$$

(1) + (2)

$$2I = \int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sin x + \cos x}\,dx$$

$$2I = \int_{\pi/6}^{\pi/3} 1\,dx$$

$$2I = \Big[\, x \,\Big]_{\pi/6}^{\pi/3}$$

$$2I = \frac{\pi}{3} - \frac{\pi}{6}$$

$$2I = \frac{2\pi - \pi}{6}$$

$$2I = \frac{\pi}{6}$$

$$I = \frac{\pi}{12}$$

02. $$\int_0^1 \frac{x(\sin^{-1}x)^2 . dx}{\sqrt{1-x^2}}$$

SOLUTION :

$\sin^{-1}x = t \qquad x = \sin t \qquad$ when $x = 0 \,,\; t = \sin^{-1}0 = 0$

$\frac{1}{\sqrt{1-x^2}} dx = dt \qquad\qquad x = 1 \,,\; t = \sin^{-1}1 = \pi/2$

$$= \int_0^{\pi/2} t^2.\sin t \quad dx$$

$$= \left\{ t^2 \int \sin t \; dt - \int \left(\frac{d}{dt} t^2 \int \sin t \, dt \right) dt \right\}_0^{\pi/2}$$

$$= \left\{ t^2 . -\cos t - \int 2t . -\cos t \quad dt \right\}_0^{\pi/2}$$

$$= \left\{ -t^2 . \cos t + 2 \int t . \cos t \; dt \right\}_0^{\pi/2}$$

$$= \left\{ -t^2 . \cos t + 2 \left(t \int \cos t \; dt - \int \left(\frac{d}{dt} t \int \cos t \; dt \right) dt \right) \right\}_0^{\pi/2}$$

$$= \left\{ -t^2 . \cos t + 2 \left(t . \sin t - \int 1. \sin t \; dt \right) \right\}_0^{\pi/2}$$

$$= \left\{ -t^2 . \cos t + 2 \left(t . \sin t - \int \sin t \; dt \right) \right\}_0^{\pi/2}$$

$$= \left\{ -t^2 . \cos t + 2 \left[t . \sin t + \cos t \right] \right\}_0^{\pi/2}$$

$$= \left\{ -t^2 . \cos t + 2t.\sin t + 2\cos t \right\}_0^{\pi/2}$$

$$= \left(\frac{-\pi^2}{4} . \cos \frac{\pi}{2} + 2 \frac{\pi}{2} . \sin \frac{\pi}{2} + 2 \cos \frac{\pi}{2} \right) - \left(-0 . \cos 0 + 2(0).\sin 0 + 2 \cos 0 \right)$$

$$= \left(0 + \pi(1) + 0 \right) - \left(0 + 0 + 2(1) \right)$$

$$= \pi - 2$$

03. Find the inverse of the matrix A by using ADJOINT METHOD , $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$

SOLUTION :

COFACTOR'S

$A_{11} = (-1)^{1+1} \begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} = 1(2-6) = -4$

$A_{12} = (-1)^{1+2} \begin{vmatrix} 0 & 3 \\ 1 & 1 \end{vmatrix} = -1(0-3) = 3$

$A_{13} = (-1)^{1+3} \begin{vmatrix} 0 & 2 \\ 1 & 2 \end{vmatrix} = 1(0-2) = -2$

$A_{21} = (-1)^{2+1} \begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} = -1(0-2) = 2$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1(1-1) = 0$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix} = -1(2-0) = -2$

$A_{31} = (-1)^{3+1} \begin{vmatrix} 0 & 1 \\ 2 & 3 \end{vmatrix} = 1(0-2) = -2$

$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} = -1(3-0) = -3$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 1(2-0) = 2$

COFACTOR MATRIX OF A

$$= \begin{pmatrix} -4 & 3 & -2 \\ 2 & 0 & -2 \\ -2 & -3 & 2 \end{pmatrix}$$

ADJ A

= TRANSPOSE OF THE COFACTOR MATRIX

$$= \begin{pmatrix} -4 & 2 & -2 \\ 3 & 0 & -3 \\ -2 & -2 & 2 \end{pmatrix}$$

| A |

$= 1(2-6) - 0(0-3) + 1(0-2)$

$= 1(-4) - 0(-3) + 1(-2)$

$= -4 + 0 - 2$

$= -6$

$$A^{-1} = \frac{1}{|A|} \cdot \text{adj } A$$

$$= \frac{1}{-6} \begin{pmatrix} -4 & 2 & -2 \\ 3 & 0 & -3 \\ -2 & -2 & 2 \end{pmatrix}$$

$$= \frac{1}{6} \begin{pmatrix} 4 & -2 & 2 \\ -3 & 0 & 3 \\ 2 & 2 & -2 \end{pmatrix}$$

SECTION - II

Q4. (A)Attempt any six of the following **(12)**

Q4

01. the pdf of continuous random variable X is given by

$$f(x) = \frac{x}{4} \quad ; \quad 0 < x < 2$$

$$= 0 \quad ; \quad \text{otherwise} \quad \text{Find} \quad P(X \leq 1)$$

SOLUTION :

$$P(x \leq 1) = \int_0^1 \frac{x}{4}\, dx$$

$$= \left[\frac{x^2}{8}\right]_0^1$$

$$= \left(\frac{1}{8}\right) - \left(\frac{0}{8}\right)$$

$$= \frac{1}{8}$$

02. What is the sum due of Rs 5,000 due 4 months hence at 12.5% p.a. simple interest

SOLUTION :

F.V. = P.W.+ INT ON P.W. FOR 4 MONTHS @12.5% p.a.

$$F.V. = 5000 + 5000 \times \frac{4}{12} \times \frac{12.5}{100}$$

$$F.V. = 5000 + 5000 \times \frac{1}{3} \times \frac{125}{1000}$$

F.V. = 5000 + 208.33

F.V. = □ 5208.33

03. Values of two regression coefficients between the variables X and Y are byx = –0.4 and bxy = – 2.025 respectively . Obtain the value of correlation coefficient

SOLUTION

r^2 = byx x bxy

$r^2 = -0.4 \times -2.025$

$$r^2 = \frac{4}{10} \times \frac{2025}{1000}$$

$$r^2 = \frac{8100}{10000}$$

$$r^2 = \frac{81}{100}$$

$$r = \pm \frac{9}{10}$$

$$r = -\frac{9}{10} \quad \text{(byx \& bxy are –ve)}$$

04. Raghu , Madhu and Ramu started a business in partnership by investing ₹ 60,000 , ₹ 40,000 and ₹ 75,000 respectively . At the end of the year they found that they have incurred a loss of ₹ 24,500 . Find how much loss MADHU had to bear

SOLUTION

STEP 1 :

Loss will be shared in the **'RATIO OF THE INVESTMENT'**

	RAHGU		MADHU		RAMU	
=	60,000	:	40,000	:	75,000	
=	60	:	40	:	75	
=	12	:	8	:	15	TOTAL = 35

STEP 2 :

LOSS = ☐ 24,500

Raghu's share of loss = $\frac{12}{35}$ x 24,500 = ₹ 8,400

Madhu's share of loss = $\frac{8}{35}$ x 24,500 = ₹ 5,600

Ramu's share of loss = $\frac{15}{35}$ x 24,500 = ₹ 10,500

05. Compute the age specific death rate for the following

Age Group	Population	No. of deaths	SDR = $\frac{D}{P}$ X 1000	
0 – 10	11,000	220	$\frac{220}{11000}$ x 1000 = 20	
10 – 20	12,000	240	$\frac{240}{12000}$ x 1000 = 20	
20 – 60	9000	180	$\frac{180}{9000}$ x 1000 = 20	
60&above	2,000	90	$\frac{90}{2000}$ x 1000 = 45	DEATHS PER 000

06.

X = x	−1	0	1
P(x)	−0.2	1	0.2

Verify whether the above function can be regarded as p.m.f.

$p(-1) = -0.2$ Since $p(x) \geq 0 \ \forall x$, the above function is NOT a pmf

07. MARGINAL FREQUENCY DISTRIBUTION OF AGE OF HUSBANDS

CI	20 – 30	30 – 40	40 – 50	50 – 60	TOTAL
F	5	20	44	24	93

CONDITIONALMARGINAL FREQUENCY DISTRIBUTION OF AGE OF HUSBANDS WHEN AGE OF WIVES LIES IN 25 – 35

CI	20 – 30	30 – 40	40 – 50	50 – 60	TOTAL
F	0	10	25	2	37

08. from the regression equations : $2x - y - 15 = 0$ & $3x - 4y + 25$.

find $\bar{x}$ and $\bar{y}$

$\bar{x} = 17$, $\bar{y} = 19$

Q5. (A) Attempt any Two of the following (06)

01. a lot of 100 pens contain 10 defective pens . 5 pens are selected at random from the lot and sent to the retail store . What is the probability that the store will receive at least one defective pen

Q5A

SOLUTION

5 pens are selected at random , n = 5

For a trial Success – a defective pen

p – probability of success = 10/100 = 1/10

q – probability of failure = 1 – 1/10 = 9/10

r.v. X – no of successes = 0 , 1 , 2 , 3 , 4 , 5 **X ~ B (5 , 1/10)**

P(at least 1 defective pen)

$= P(X \geq 1)$

$= P(1) + P(2) + \ldots\ldots + P(5)$

$= 1 - P(0)$

$= 1 - {}^5C_0 \cdot p^0 \cdot q^5$

$= 1 - {}^5C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^5$

$= 1 - \frac{59049}{100000}$

$= 1 - 0.59049 = 0.40951$

02. in a town,10 accidents take place in a span of 50 days. Assuming that the number of accidents follow Poisson Distribution , find the probability that there will be one or more accidents per day ($e^{-0.2} = 0.8187$)

SOLUTION

m = average number of accidents per day = 10/50 = 0.2

r.v X = number of accidents in a day , X ~ P(0.2)

P(one or more accidents per day)

$= P(x \geq 1)$

$= 1 - P(0)$

$= 1 - \frac{e^{-0.2} \cdot 0.2^0}{0!}$ Using $P(x) = \frac{e^{-m} \cdot m^x}{x!}$

$= 1 - e^{-0.2} \cdot (1)$

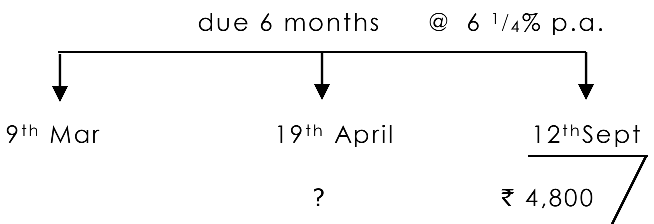
$= 1 - 0.8187$

$= 0.1813$

03. A bill of ₹ 4,800 was drawn on 9th March 2006 , at 6 months and was discounted on 19th April 2006 at 6 ¹/₄ % p.a. How much does the banker charge and how much does the holder receive.

SOLUTION

STEP 1 :



Date of drawing	=	9 / 03
Add period of bill	+	6 months
Nominal due date	=	9 / 09
Add Grace days	+	3 days
Legal due date	=	12 / 09

STEP 2 :

Unexpired period

= 19th April – 12th September

APR MAY JUN JUL AUG SEP

= 11 + 31 + 30 + 31 + 31 + 12

= 146 days

STEP 3 :

B.D. = Int. on F.V. for 146 days @ 25/4% p.a.

$$= 4800 \times \frac{146}{365} \times \frac{25}{400}$$

5

= ₹ 120

STEP 4 :

C.V. = F.V. – B.D.

= 4800 – 120

= ₹ 4,680

(B) Attempt any Two of the following **(08)**

01. Property valued at ₹ 7 lakh is insured to the extent of ₹ 5,60,000 at 5/8 % less 20%. How much loss does the owner bear including premium if the property is damaged to the extent of 40% of its value

Q5B

Solution

Property value = ₹ 7,00,000

Insured value = ₹ 5,60,000

Rate of premium = 5/8 % less 20%.

Premium = $\frac{5}{800} \times 5,60,000$

= ₹ 3,500

less 20% disc − 700

Net Premium = ₹ 2,800

Loss = $\frac{40}{100} \times 7,00,000$

= ₹ 2,80,000

Claim = $\frac{\text{insured val.}}{\text{Property val.}} \times \text{loss}$

= $\frac{5,60,000}{7,00,000} \times 2,80,000$

= ₹ 2,24,000

Loss	=	2,80,000
Less claim	−	2,24,000
Net loss	=	56,000
Add premium	+	2,800
Net loss Incl. premium	= ₹	58,800

02. given the following table which relates to the number of animals of a certain species at age x . Complete the life table

AGE x	lx	dx = lx − lx+1	qx = $\frac{dx}{lx}$	px = 1 − qx	Lx = $\frac{lx + lx+1}{2}$	Tx	$e_x^0 = \frac{Tx}{lx}$
0	1000	1000 − 940= 60	$\frac{60}{1000}$ = 0.06	1 − 0.06 = 0.94	940 + 30 = 970	2835	$\frac{2835}{1000}$ = 2.835
1	940	940 − 780 = 160	$\frac{160}{940}$ = 0.17	1 − 0.17 = 0.83	780 + 80 = 860	1865	$\frac{1865}{940}$ = 1.985
2	780	780 − 590 = 190	$\frac{190}{780}$ = 0.24	1 − 0.24 = 0.76	590 + 95 = 685	1005	$\frac{1005}{780}$ = 1.288
3	590	590 − 25 = 565	$\frac{565}{590}$ = 0.96	1 − 0.96 = 0.04	25+ 282.5= 307.5	320	$\frac{320}{590}$ = 0.5423
4	25	25 − 0 = 25	$\frac{25}{25}$ = 1	1 − 1 = 0	0 + 12. 5 = 12.5	12.5	$\frac{12.5}{25}$ = 0.5
5	0	----	---- ----	----	---- ----		

LOG CALCULATIONS FOR 'qx'

LOG 160 − LOG 940

2.2041
− 2.9731
AL $\bar{1}.2310$
0.1702

LOG 190 − LOG 780

2.2788
− 2.8921
AL $\bar{1}.3867$
0.2436

LOG 565 − LOG 590

2.7520
− 2.7709
AL $\bar{1}.9811$
0.9574

LOG CALCULATIONS FOR 'e_x^0'

LOG 1865 − LOG 940

3.2707
− 2.9731
AL 0.2976
1.985

LOG 1005 − LOG 780

3.0021
− 2.8921
AL 0.1100
1.288

LOG 320 − LOG 590

2.5051
− 2.7709
AL $\bar{1}.7342$
0.5423

03.

A production unit makes special type of metal chips by combining copper and brass . The standard weight of the chip must be at least 5 gm. The basic ingredients copper and brass cost ₹ 8 and ₹ 5 per gm . The durability considerations dictate that the metal chip must not contain more than 4 gm of brass and should contain minimum 2 gm of copper .Find the minimum cost of the metal chip satisfying the above conditions

Let copper input = x gm

Brass input = y gm

CONSTRAINT

1. since standard weight of the chip must be at least 5 gm ;

   $x + y \geq 5$

2. since metal chip must not contain more than 4 gm of brass and should contain minimum 2 gm of copper

   $x \geq 2$; $y \leq 4$

3. since x & y are inputs in gm , cannot be – ve ; $x , y \geq 0$

OBJECTIVE FUNCTION

copper and brass cost □ 8 and □ 5 per gm

total cost = 8x + 5y (in □)

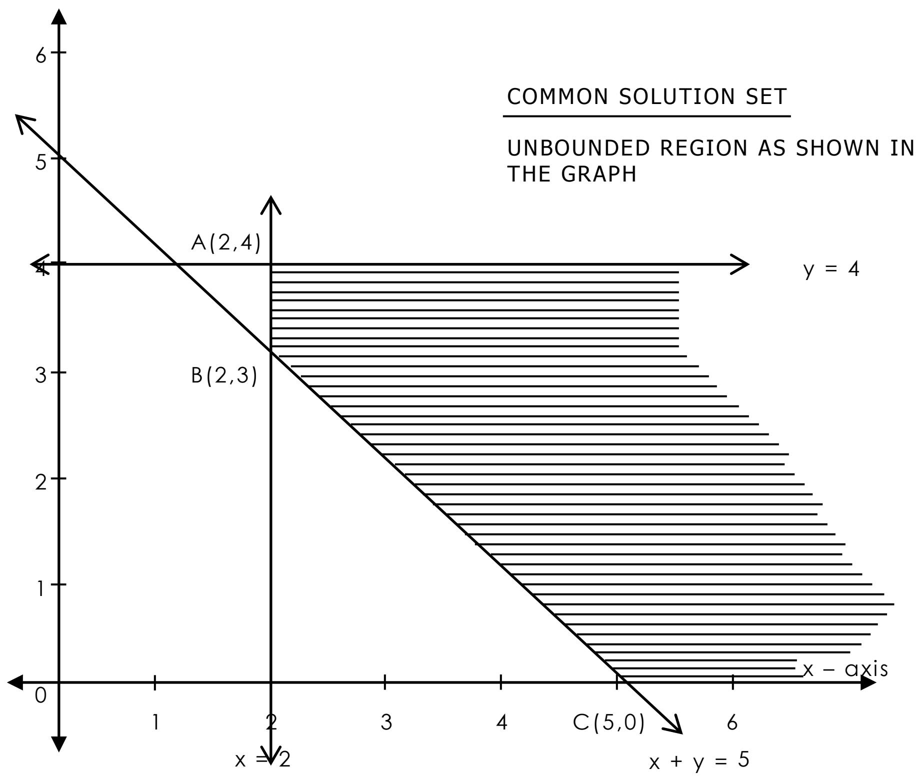
∴ Minimize $z = 8x + 5y$

LPP MODEL

Minimize $z = 8x + 5y$, Subject to

$x + y \geq 5$; $x \geq 2$; $y \leq 4$; $x , y \geq 0$

SCALE : 2CM = 1 UNIT



CORNERS	Z = 8x + 5y				
A(2,4)	Z = 8(2) + 5(4)	=	16 + 20	=	36
B(2,3)	Z = 8(2) + 5(3)	=	16 + 15	=	31
C(5,0)	Z = 8(5) + 5(0)	=	40 + 0	=	40

OPTIMAL SOLUTION : metal chip should contain 2 gm of copper and 3 gm of brass to keep the cost minimum to ₹ 31

Q6. (A) Attempt any Two of the following (06)

01. P and Q started a business with capitals in the ratio 4 : 3 . After 9 months P withdrew 25% of his capital and Q put in an equal amount in addition to his earlier capital . If at the end of the year P's share in the profit was ₹ 15,450 , find the total profit and Q's share of profit

SOLUTION

Q6A

PARTNER's NAME	CAPITAL INVESTED	PERIOD OF INVESTMENT
P	₹ 4k	9 MONTHS
	– 25%	
	₹ 3k	3 MONTHS
Q	₹ 3k	9 MONTHS
	+ k	
	₹ 4k	3 MONTHS

STEP 1 :

Profits will be shared in the

'RATIO OF PRODUCT OF CAPITAL INVESTED & PERIOD OF INVESTMENT'

	P		Q	
=	4k x 9 + 3k x 3	:	3k x 9 + 4k x 3	
=	36k + 9k	:	27k + 12k	
=	45k	:	39k	
=	15	:	13	TOTAL = 28

STEP 2 :

P share of profit = ₹ 15,450

P's share of profit = $\frac{15}{28}$ x Total Profit

15,450 = $\frac{15}{28}$ x Total Profit

Total Profit = $\frac{\cancel{15450}^{1030} \times 28}{\cancel{15}}$

= ₹ 28,840

Q's share of profit = $\frac{13}{\cancel{28}}$ x $\cancel{28,840}^{1030}$

= ₹ 13,390

02. find k if following is a pdf of r.v. X

$$f(x) = kx(1-x) \quad ; \quad 0 < x < 1$$
$$= 0 \quad ; \quad \text{otherwise}$$

Also P(1/4 < X < 1/2)

SOLUTION

$$\int_0^1 kx(1-x)\,dx = 1$$

$$k\int_0^1 (x - x^2)dx = 1$$

$$k\left(\frac{x^2}{2} - \frac{x^3}{3}\right)_0^1 = 1$$

$$k\left(\frac{1}{2} - \frac{1}{3}\right) = 1$$

$$k\left(\frac{1}{6}\right) = 1$$

$$k = 6$$

Hence , pdf of the r.v.x is given as

$$f(x) = 6x(1-x) \quad ; \quad 0 < x < 1$$
$$= 0 \quad ; \quad \text{otherwise}$$

$P(^1/_4 < x < {}^1/_2)$

$$= \int_{1/4}^{1/2} 6x(1-x)\,dx$$

$$= \int_{1/4}^{1/2} (6x - 6x^2)dx$$

$$= \left(\frac{6x^2}{2} - \frac{6x^3}{3}\right)_{1/4}^{1/2}$$

$$= \left(3x^2 - 2x^3\right)_{1/4}^{1/2}$$

$$= \left(\frac{3}{4} - \frac{2}{8}\right) - \left(\frac{3}{16} - \frac{2}{64}\right)$$

$$= \left(\frac{3}{4} - \frac{1}{4}\right) - \left(\frac{3}{16} - \frac{1}{32}\right)$$

$$= \frac{2}{4} - \frac{6-1}{32}$$

$$= \frac{1}{2} - \frac{5}{32}$$

$$= \frac{16-5}{32}$$

$$= \frac{11}{32}$$

03. $\Sigma x = 56$, $\Sigma y = 56$, $\Sigma x^2 = 476$, $\Sigma y^2 = 476$, $\Sigma xy = 469$ and $n = 7$ find

a) Regression equation of y on x b) estimate y when x = 12

SOLUTION

$$\bar{x} = \frac{\Sigma x}{n} = \frac{56}{7} = 8 \quad , \quad \bar{y} = \frac{\Sigma y}{n} = \frac{56}{7} = 8$$

$$b_{yx} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{7(469) - 56(56)}{7(476) - (56)^2}$$

$$= \frac{3283 - 3136}{3332 - 3136}$$

$$= \frac{147}{196}$$

$$= \frac{21}{28}$$

$$= 0.75$$

EQUATION

$$y - \bar{y} = byx\,(x - \bar{x})$$

$$y - 8 = 0.75(x - 8)$$

$$y - 8 = 0.75x - 6$$

$$y = 0.75x + 2$$

Put x = 12

$$y = 0.75(12) + 2$$

$$y = 9 + 2$$

$$y = 11$$

(B) Attempt any Two of the following **(08)**

Q6B

01. A courier agency employs 4 persons in 4 zones of a city . The number of letters delivered in a day in each zone by each person is given

PERSONS	ZONE	E	W	N	S
	P_1	18	25	22	26
	P_2	26	29	26	27
	P_3	28	31	36	30
	P_4	26	38	27	25

What is the optimal assignment so that maximum number of letters can be delivered and how many max no. of letters will be delivered by the agency

SOLUTION

20	13	16	8
12	9	12	11
10	7	2	8
12	0	11	13

Subtracting all the elements in the matrix by its maximum (38) .
this matrix can now be solved for
'MINIMAL ASSIGNMENT PROBLEM'

12	5	8	0
3	0	3	2
8	5	0	6
12	0	11	13

Reducing the matrix using 'ROW MINIMUM'

9	5	8	0
0	0	3	2
5	5	0	6
9	0	11	13

Reducing the matrix using 'COLUMN MINIMUM'

9	5	8	[0]
[0]	~~0~~	3	2
5	5	[0]	6
9	[0]	11	13

Allocation using 'SINGLE ZERO ROW COLUMN METHOD'

since each row contains an assigned zero , the assignment problem is solved

OPTIMAL ASSIGNMENT

P_1 – S , P_2 – E , P_3 – N , P_4 – W ,

Max letters delivered = 26 + 26 + 36 + 38

= 126

02. you are given below the following information about Profit rate (X) and growth rate (Y)

	X	Y
Mean	20	25
Variance	9	25

correlation coeff. = 0.9 .

a) Obtain the regression line of X on Y

b) Estimate the growth rate when the profit rate is 16%

SOLUTION

Y on X

$$byx = r \cdot \frac{\sigma y}{\sigma x}$$

$$= 0.9 \frac{5}{3}$$

$$= 1.5$$

Equation

$$y - \overline{y} = byx\,(x - \overline{x})$$

$$y - 25 = 1.5(x - 20)$$

Put x = 16

$$y - 25 = 1.5(16 - 20)$$

$$y - 25 = -6$$

$$y = 19$$

Growth rate = 19% when profit Rate is 16%

X on Y

$$bxy = r \frac{\sigma x}{\sigma y}$$

$$= 0.9 \frac{3}{5}$$

$$= 0.54$$

Equation

$$x - \overline{x} = bxy\,(y - \overline{y})$$

$$x - 20 = 0.54(y - 25)$$

$$x - 20 = 0.54y - 13.5$$

$$x = 0.54y - 13.5 + 20$$

$$x = 0.54y + 6.5$$

03. Compute correlation coefficient for the following data

x :	6	2	10	4	8
y :	9	11	5	8	7

SOLUTION

x	y	$x-\overline{x}$	$y-\overline{y}$	$(x-\overline{x})^2$	$(y-\overline{y})^2$	$(x-\overline{x})(y-\overline{y})$
6	9	0	1	0	1	0
2	11	−4	3	16	9	−12
10	5	4	−3	16	9	−12
4	8	−2	0	4	0	0
8	7	2	−1	4	1	−2
30 Σx	40 Σy	0 $\Sigma(x-\overline{x})$	0 $\Sigma(y-\overline{y})$	40 $\Sigma(x-\overline{x})^2$	20 $\Sigma(y-\overline{y})^2$	−26 $\Sigma(x-\overline{x})(y-\overline{y})$
$\overline{x} = 6$ $\overline{y} = 8$						

$$r = \frac{\Sigma(x-\overline{x}).(y-\overline{y})}{\sqrt{\Sigma(x-\overline{x})^2}\sqrt{\Sigma(y-\overline{y})^2}}.$$

$$r = \frac{-26}{\sqrt{40} \times \sqrt{20}}$$

$$r = \frac{-26}{\sqrt{40 \times 20}}$$

let

$$r' = \frac{26}{\sqrt{40 \times 20}} \quad \text{(Since log a , a > 0)}$$

taking log on both sides

$$\log r' = \log 26 - \frac{1}{2}\left(\log 40 + \log 20\right)$$

$$\log r' = 1.4150 - \frac{1}{2}[\,1.6021 + 1.3010\,]$$

$$\log r' = 1.4150 - \frac{1}{2}(2.9031)$$

$$\log r' = 1.4150 - 1.4516$$

$$\log r' = \overline{1}\,.\,9634$$

$$r' = AL(\overline{1}\,.\,9634)$$

$$r' = 0.9191$$

$$r = -0.9191$$