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# SUGGESTED SOLUTION

## SYJC

**SUBJECT- Mathematics & Statistics**

## Test Code – SYJ 6116

**BRANCH - () (Date :)**

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# SET A

## ANSWER : 1

**(i)** The dual of $(p \vee q) \vee r \equiv p \vee (q \vee r)$ is $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

**(2)**

**(ii)** $\begin{bmatrix}1\\2\\3\end{bmatrix}\begin{bmatrix}1 & 2 & 3\end{bmatrix} = \begin{bmatrix}1 & 2 & 3\\2 & 4 & 6\\3 & 6 & 9\end{bmatrix}$

**(2)**

**(iii)** $\int \frac{1}{x.\log\ x.\log(\log x)}\,dx$

(Hint : Put log (log x) = t]

$\log|\log(\log x)| + c$

**(2)**

**(iv)** $\int e^x \left[tan^{-1}x + \frac{1}{1+x^2}\right] dx$

Let I = $\int e^x \left(tan^{-1}x + \frac{1}{1+x^2}\right) dx$

Put f(x) = $tan^{-1}x$ $\therefore$ f′(x) = $\frac{1}{1+x^2}$

$\therefore$ I = $\int e^x\ [f(x) + f'(x)]\ dx$

= $e^x . f(x) + c = e^x . tan^{-1}\ x + c$

**(2)**

**(v)** x + 3y = 2, 3x + 5y = 4

The given equations can be written in the matrix form as :

$\begin{bmatrix}1 & 3\\3 & 5\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}2\\4\end{bmatrix}$

By $R_2 - 3R_1$

$\begin{bmatrix}1 & 3\\0 & -4\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}2\\-2\end{bmatrix}$

$\therefore \begin{bmatrix}x+3y\\0-4y\end{bmatrix} = \begin{bmatrix}2\\-2\end{bmatrix}$

By equality of matrices,

x + 3y = 2 .....(1)

– 4y = – 2 .....(2)

From (2), y = $\frac{1}{2}$

Substituting y = $\frac{1}{2}$ in (1), we get,

x + $\frac{3}{2}$ = 2

$\therefore$ x = 2 – $\frac{3}{2} = \frac{1}{2}$

Hence, x = $\frac{1}{2}$, y = $\frac{1}{2}$ is the required solution.

**(2)**

**(vi)** $f(x) = \frac{\sin 5x}{x}$, for $x \neq 0$

$= 1$, for x = 0, at x = 0.

$f(0) = 1$ (Given) ……(1)

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\sin 5x}{x}$$

$$= \lim_{x\to 0} \frac{\sin 5x}{5x} \times 5$$

$$= 5\lim_{x\to 0} \frac{\sin 5x}{5x}$$

$= 5 \times 1$ ……. [$x \to 0$, $5x \to 0$ and $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$]

$= 5$ …….(2)

From (1) and (2)

$$\lim_{x\to 0} f(x) \neq f(0)$$

$\therefore$ *f(x)* is **discontinuous** at x = 0.

**(2)**

**(vii)** $x = 4 + 25t^2$, $y = 16t^3$

Differentiating x and y w.r.t. *t*, we get,

$$\frac{dx}{dt} = \frac{d}{dt}(4 + 25t^2)$$

$$= 0 + 25 \times 2t = 50t$$

and $\frac{dy}{dt} = \frac{d}{dt}(16t^3) = 16 \times 3t^2 = 48t^2$

$$\therefore \frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)} = \frac{48t^2}{50t} = \frac{24}{25} \cdot t.$$

**(2)**

**(viii)** $f(x) = x^3 - 9x^2 + 27x + 7$

$\therefore f'(x) = \frac{d}{dx}(x^3 - 9x^2 + 27x + 7)$

$= 3x^2 - 9 \times 2x + 27 \times 1 + 0$

$= 3x^2 - 18x + 27$

$= 3(x^2 - 6x + 9) = 3(x - 3)^2 > 0$ for all $x \in R$, $x \neq 3$

$\therefore f'(x) > 0$ for all $x \in R - \{3\}$

$\therefore$ f (x) is increasing for all $x \in R - \{3\}$.

**(2)**

**ANSWER : 2(A)**

**(i)** For x < 1, f(x) = $\frac{\sin \pi x}{x-1} + a$

$\therefore \lim_{x \to 1^-} f(x) = \lim_{x \to 1} \left( \frac{\sin \pi x}{x-1} + a \right)$

Put x = 1 + h. Then x – 1 = h and as x $\to$ 1, h $\to$ 0.

$\therefore \lim_{x \to 1^-} f(x) = \lim_{h \to 0} \left[ \frac{\sin \pi(1+h)}{h} + a \right]$

$= \lim_{h \to 0} \left[ \frac{\sin(\pi + \pi h)}{h} + a \right]$

$= \lim_{h \to 0} \left[ \frac{-\sin \pi h}{h} + a \right]$

$= \lim_{h \to 0} \left[ -\frac{\sin \pi h}{\pi h} \times \pi + a \right]$

$= -\pi \lim_{h \to 0} \frac{\sin \pi h}{\pi h} + a$

$= -\pi \times 1 + a$ ….. [h $\to$ 0, πh $\to$ 0 and $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$]

$= -\pi + a$

Also, f(1) = 2π …… (Given)

Now, $f$(x) is continuous at x = 1.

$\therefore \quad \lim_{x \to 1^-} f(x) = f(1)$

$\therefore \quad -\pi + a = 2\pi$

$\therefore \quad a = 3\pi$

For x > 1, f(x) = $\frac{1+\cos \pi x}{\pi(1-x)^2} + b$

$\therefore \lim_{x \to 1^+} f(x) = \lim_{x \to 1} \left[ \frac{1+\cos \pi x}{\pi(1-x)^2} + b \right]$

Put x = 1 + h. Then 1 – x = – h and as x $\to$ 1, h $\to$ 0.

Also, 1 + cos πx = 1 + cos π (1 + h)

= 1 + cos (π + πh)

= 1 – cos πh

$$\therefore \lim_{x\to 1^+} f(x) = \lim_{h\to 0}\left[\frac{1-\cos\pi h}{\pi(-h)^2}+b\right]$$

$$= \lim_{h\to 0}\left[\frac{1-\cos\pi h}{\pi h^2}\cdot\frac{1+\cos\pi h}{1+\cos\pi h}+b\right]$$

$$= \lim_{h\to 0}\left[\frac{1-\cos^2\pi h}{\pi h^2(1+\cos\pi h)}+b\right]$$

$$= \lim_{h\to 0}\left[\frac{\sin^2\pi h}{\pi h^2\left(1+\cos\pi h\right)}+b\right]$$

$$= \lim_{h\to 0}\left[\left(\frac{\sin\pi h}{\pi h}\right)^2\times\frac{\pi}{1+\cos\pi h}+b\right]$$

$$= \pi\left(\lim_{h\to 0}\frac{\sin\pi h}{\pi h}\right)^2\times\frac{1}{\lim\limits_{h\to 0}(1+\cos\pi h)}+b$$

$= \pi\,(1)^2 \times \frac{1}{1+1} + b$..... [h $\to 0$, $\pi$h $\to$ 0 and $\lim\limits_{\theta\to 0}\frac{\sin\theta}{\theta}=1$]

$= \frac{\pi}{2} + b$

Since f is continuous at x = 1,

$\lim\limits_{x\to 1^+} f(x) = f(1)$

$\therefore \frac{\pi}{2} + b = 2\pi$

$\therefore$ b = $2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$

Hence, $a = 3\pi$ and $b = \frac{3\pi}{2}$.

**(3)**

**(ii)**

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| p | q | r | $\sim q$ | $\sim r$ | $\sim q \vee r$ | $p \wedge (\sim q \vee r)$ | $q \wedge \sim r$ | $p \to (q \wedge \sim r)$ | $\sim [p \to (q \wedge \sim r)]$ |
| T | T | T | F | F | T | T | F | F | T |
| T | T | F | F | T | F | F | T | T | F |
| T | F | T | T | F | T | T | F | F | T |
| T | F | F | T | T | T | T | F | F | T |
| F | T | T | F | F | T | F | F | T | F |
| F | T | F | F | T | F | F | T | T | F |
| F | F | T | T | F | T | F | F | T | F |
| F | F | F | T | T | T | F | F | T | F |

The entries in columns 7 and 10 are identical.

$\therefore p \wedge (\sim q \vee r) \equiv \sim [p \to (q \wedge \sim r)]$. **(3)**

**(iii)** sin y = x sin (5 + y)

$\therefore \quad x = \frac{\sin y}{\sin(5+y)}$

Differentiate both sides with respect to x

$\therefore\ 1 = \left(\frac{\sin(5+y)\cos y - \sin y \cos(5+y)}{sin^2(5+y)}\right)\frac{dy}{dx}$

$\therefore 1 = \left(\frac{\sin(5+y-y)}{sin^2(5+y)}\right)\frac{dy}{dx}$

$\therefore \qquad 1 = \frac{\sin 5}{sin^2(5+y)}\cdot\frac{dy}{dx}$

$\therefore\ \frac{dy}{dx} = \frac{sin^2(5+y)}{\sin 5}$

Note : Though some expressions appear to be in implicit form, by using proper transformations, they can be converted into explicit form.

**(3)**

**ANSWER : 2(B)**

**(i)** $A^2 = A \cdot A = \begin{bmatrix}1 & 2\\1 & 3\end{bmatrix}\begin{bmatrix}1 & 2\\1 & 3\end{bmatrix}$

$= \begin{bmatrix}1+2 & 2+6\\1+3 & 2+9\end{bmatrix} = \begin{bmatrix}3 & 8\\4 & 11\end{bmatrix}$

$\therefore\ A^2 - 4A + I$

$= \begin{bmatrix}3 & 8\\4 & 11\end{bmatrix} - 4\begin{bmatrix}1 & 2\\1 & 3\end{bmatrix} + \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$= \begin{bmatrix}3 & 8\\4 & 11\end{bmatrix} - \begin{bmatrix}4 & 8\\4 & 12\end{bmatrix} + \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$= \begin{bmatrix}3-4+1 & 8-8+0\\4-4+0 & 11-12+1\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

$\therefore \qquad A^2 - 4A + I = 0 \qquad ......(1)$

$\because \qquad |A| = \begin{vmatrix}1 & 2\\1 & 3\end{vmatrix}$

$= 3 - 2 = 1 \neq 0$

$\therefore \qquad A^{-1}$ exists.

Pre multiply (1) by $A^{-1}$, we get,

$A^{-1}(A^2 - 4A + I) = A^{-1} \cdot 0$

$\therefore A^{-1}(A \cdot A) - 4A^{-1} \cdot A + A^{-1} I = 0$

$\therefore\ (A^{-1} A) A - 4I + A^{-1} = 0$

$\therefore\ IA - 4I + A^{-1} = 0$

$\therefore$ A – 4I + $A^{-1}$ = 0

$\therefore$ $A^{-1}$ = 4I – A

$= 4\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} - \begin{bmatrix}1 & 2\\1 & 3\end{bmatrix}$

$= \begin{bmatrix}4 & 0\\0 & 4\end{bmatrix} - \begin{bmatrix}1 & 2\\1 & 3\end{bmatrix}$

$\therefore A^{-1} = \begin{bmatrix}3 & -2\\-1 & 1\end{bmatrix}$.

**(4)**

**(ii)** The demand function is

$D = \frac{2p+3}{3p-1}$

$\therefore \frac{dD}{dp} = \frac{d}{dp}\left(\frac{2p+3}{3p-1}\right)$

$= \frac{(3p-1)\frac{d}{dp}(2p+3)-(2p+3)\frac{d}{dp}(3p-1)}{(3p-1)^2}$

$= \frac{(3p-1)(2\ \times 1+0)-(2p+3)\ (3\times 1-0)}{(3p-1)^2}$

$= \frac{6p-2-6p-9}{(3p-1)^2} = \frac{-11}{(3p-1)^2}$

Elasticity of demand is given by

$\eta = -\frac{p}{D}\cdot\frac{dD}{dp}$

$= \frac{-p}{\left(\frac{2p+3}{3p-1}\right)} \times \frac{-11}{(3p-1)^2}$

$= \frac{11p}{(2p+3)(3p-1)} = \frac{11p}{6p^2+7p-3}$

If $\eta = \frac{11}{14}$, then

$\frac{11}{14} = \frac{11p}{6p^2+7p-3}$

$\therefore$ $66p^2 + 77p - 33 = 154p$

$\therefore$ $66p^2 - 77p - 33 = 0$

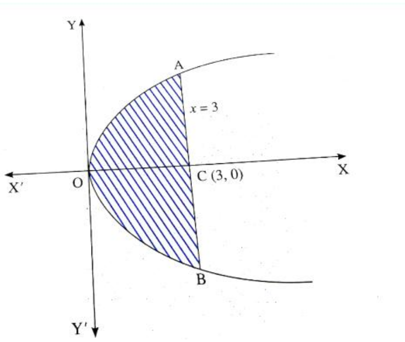
$\therefore$ $6p^2 - 7p - 3 = 0$

$\therefore$ $(2p - 3)(3p + 1) = 0$

$\therefore$ $2p - 3 = 0$ .....[$\because$ $p \geq 0$]

$\therefore$ $p = \frac{3}{2}$. **(4)**

**(iii)**




Required area = area of the region OABO

= 2(area of the region OACO)

$= 2\int_0^3 y\,dx$, where $y^2 = 4x$, i.e., $y = 2\sqrt{x}$

$= 2\int_0^3 2\sqrt{x}\,dx$

$= 4\int_0^3 x^{\frac{1}{2}}\,dx$

$= 4\cdot\left[\dfrac{x^{\frac{3}{2}}}{3/2}\right]_0^3$

$= \dfrac{8}{3}\left[x^{\frac{3}{2}}\right]_0^3$

$= \dfrac{\mathbf{8}}{\mathbf{3}}(\mathbf{3\sqrt{3} - 0})$

$= 8\sqrt{\mathbf{3}}$ sq. units.

**(4)**

**ANSWER : 3(A)**

**(i)** If $f$ is continuous at x = 0 and

$f(x) = 2\sqrt{x^3 + 1} + a$, for $x < 0$

$= x^3 + a + b$, for $x \geq 0$

and $f(1) = 2$

$f(x) = x^3 + a + b$, for $x \geq 0$

$\therefore f(1) = 1^3 + a + b = 1 + a + b$

But f(1) = 2 $\quad\therefore$ 1 + $a$ + b = 2

$\therefore$ $a$ + b = 1 ...... (1)

$\lim\limits_{x\to 0^+}$ f(x) = $\lim\limits_{x\to 0}$ ($x^3$ + $a$ + b)

= 0 + $a$ + b = $a$ + b

Also, $\lim\limits_{x\to 0^-}$ f(x) = $\lim\limits_{x\to 0}$ $(2\sqrt{x^3+1}+a)$

= $2\lim\limits_{x\to 0}\sqrt{x^3+1}+a$

= $2\sqrt{0+1}+a$ = 2 + $a$

Now, f(x) is continuous at x = 0

$\therefore$ $\lim\limits_{x\to 0^+}$ f(x) = $\lim\limits_{x\to 0}$ f(x)

$\therefore$ $a$ + b = 2 + $a$

$\therefore$ b = 2

Substituting b = 2 in (1), we get,

$a$ + 2 = 1 $\quad\therefore$ $a$ = – 1

Hence, $a$ = – 1 and $b$ = 2.

**(3)**

**(ii)** Consider

$[(p \vee q) \wedge \sim p] \to q$

$\equiv [(p \wedge \sim p) \vee (q \wedge \sim p)] \to q$ ...... By Distributive law

$\equiv [c \vee (q \wedge \sim p)] \to q$ ..... By complement law

$= (q \wedge \sim p) \to q$ ...... By Identity law

$\equiv (\sim p \wedge q) \to q$ ..... By Commutative law

$\equiv \sim(\sim p \wedge q) \vee q$ .... By $\sim\sim (p \to q)$

$\equiv \sim(p \wedge \sim q) \equiv \sim p \vee q$

$\equiv (p \vee \sim q) \vee q$ .... By De Morgan's law

$\equiv p \vee (\sim q \vee q)$ ..... By Associative law

$\equiv p \vee t$ ..... By Complement law

$\equiv t$ .... By Identity law

**(3)**

**(iii)** $y = (\sin x)^x + x \cdot \sin x$

Put $u = (\sin x)^x$

$\therefore \log u = \log (\sin x)^x = x \cdot \log \sin x$

Differentiating both sides w.r.t. x, we get,

$\frac{1}{u} \cdot \frac{du}{dx} = \frac{d}{dx}(x \cdot \log \sin x)$

$= x \cdot \frac{d}{dx}(\log \sin x) + (\log \sin x) \cdot \frac{d}{dx}(x)$

$= x \times \frac{1}{\sin x} \cdot \frac{d}{dx}(\sin x) + (\log \sin x) \times 1$

$= x \times \frac{1}{\sin x} \cdot \cos x + \log \sin x$

$\therefore \frac{du}{dx} = u\,[x \cot x + \log \sin x]$

$\therefore \frac{du}{dx} = (\sin x)^x\,[\log \sin x + x \cot x]$ ……(1)

Now, $y = u + x \cdot \sin x$

$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{d}{dx}(x \cdot \sin x)$

$= (\sin x)^x\,[\log \sin x + x \cot x] + x \cdot \frac{d}{dx}(\sin x) + (\sin x) \cdot \frac{d}{dx}(x)$ ….. [By (1)]

$= (\sin x)^x\,[\log \sin x + x \cot x] + x \cdot \cos x + (\sin x) \times 1$

$\therefore \frac{dy}{dx} = (\sin x)^x\,[\log \sin x + x \cot x] + (\sin x + x \cos x)$.

**(3)**

**ANSWER : 3 (B)**

**(i)** $\int \frac{2x^2-1}{(x^2+4)(x^2+5)}\,dx$

[Here numerator and denominator are function of $x^2$. Hence to convert the rational function into partial fractions we put $x^2 = t$.

After converting the rational function into partial functions we resubstitute $t = x^2$ and then the integration is performed.]

Let $I = \int \frac{2x^2-1}{(x^2+4)(x^2+5)}\,dx$

Consider, $\frac{2x^2-1}{(x^2+4)(x^2+5)}$

For finding partial fractions only, put $x^2 = t$.

$\therefore \frac{2x^2-1}{(x^2+4)(x^2+5)} = \frac{2t-1}{(t+4)(t+5)}$

$= \frac{A}{t+4} + \frac{B}{t+5}$ ……(Say)

$\therefore 2t - 1 = A(t + 5) + B(t + 4)$

Put t + 4 = 0, i.e., t = – 4, we get,

$2(-4) - 1 = A(1) + B(0)$ $\therefore$ A = – 9

Put t + 5 = 0, i.e., t = – 5, we get,

$2(-5) - 1 = A(0) + B(-1)$ $\therefore$ B = 11

$\therefore \frac{2t-1}{(t+4)(t+5)} = \frac{-9}{t+4} + \frac{11}{t+5}$

$\therefore \frac{2x^2-1}{(x^2+4)(x^2+5)} = \frac{-9}{(x^2+4)} + \frac{11}{(x^2+5)}$

$\therefore I = \int \left(\frac{-9}{x^2+4} + \frac{11}{x^2+5}\right) dx$

$= -9 \int \frac{1}{x^2+2^2} dx + 11 \int \frac{1}{x^2+(\sqrt{5})^2} dx$

$\therefore I = -\frac{9}{2}\tan^{-1}\left(\frac{x}{2}\right) + \frac{11}{\sqrt{5}}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + c.$

**(4)**

**(ii)** The cost function is given as

$C = \frac{x^2}{9} + 17x + 100$

$\therefore$ average cost = $C_A = \frac{C}{x}$

$= \left(\frac{x^2}{9} + 17x + 100\right)\frac{1}{x}$

$\therefore C_A = \frac{x}{9} + 17 + \frac{100}{x}$

$\therefore \frac{dC_A}{dx} = \frac{d}{dx}\left(\frac{x}{9} + 17 + \frac{100}{x}\right)$

$= \frac{1}{9} \times 1 + 0 + (100)(-1)x^{-2}$

$= \frac{1}{9} - \frac{100}{x^2}$

and $\frac{d^2C_A}{dx^2} = \frac{d}{dx}\left(\frac{1}{9} - \frac{100}{x^2}\right)$

$= 0 - (100)(-2)x^{-3} = \frac{200}{x^3}$

$\frac{dC_A}{dx} = 0$ gives $\frac{1}{9} - \frac{100}{x^2} = 0$

$\therefore x^2 - 900 = 0$ $\therefore x^2 = 900$

$\therefore$ x = 30 …….[$\because$ x > 0] and $\left[\frac{d^2C_A}{dx^2}\right]_{at\ x=30} = \frac{200}{(30)^3} > 0$

$\therefore C_A$ is minimum when x = 30 and the average cost at x = 30 is

$\left(\frac{x}{9} + 17 + \frac{100}{x}\right)_{at\ x=30}$

= $\frac{30}{9} + 17 + \frac{100}{30} = \frac{10}{3} + \frac{10}{3} + 17$ = $\frac{71}{3}$.

Marginal cost =$C_M$ = $\frac{dC}{dx} = \frac{d}{dx}\left(\frac{x^2}{9} + 17x + 100\right)$

= $\frac{1}{9} \times 2x + 17 \times 1 + 0$

= $\frac{2x}{9} + 17$

$\therefore$ $(C_M)_{at\ x\ =\ 30}$ = $\frac{2(30)}{9} + 17$

= $\frac{20}{3} + 17 = \frac{71}{3}$ and

Hence, the minimum average cost = $\frac{71}{3}$ and Marginal cost = $\frac{71}{3}$. **(4)**

**(iii)** $\int_{\pi/6}^{\pi/3} \frac{\cot x}{\cot x + 1} dx$

Let I = $\int_{\pi/6}^{\pi/3} \frac{\cot x}{\cot x + 1} dx$

= $\int_{\pi/6}^{\pi/3} \frac{\left(\frac{\cos x}{\sin x}\right)}{\left(\frac{\cos x}{\sin x}\right) + 1} dx$

$\therefore$ I = $\int_{\pi/6}^{\pi/3} \frac{\cos x}{\cos x + \sin x} dx$ .............(1)

We use the property, $\int_a^b f(x)dx$ = $\int_a^b f(a+b-x)dx$.

Hence, in I, we change x by $\frac{\pi}{6} + \frac{\pi}{3} - x$.

$\therefore$ I = $\int_{\pi/6}^{\pi/3} \frac{\cos\left(\frac{\pi}{6} + \frac{\pi}{3} - x\right)}{\cos\left(\frac{\pi}{6} + \frac{\pi}{3} - x\right) + \sin\left(\frac{\pi}{6} + \frac{\pi}{3} - x\right)} dx$

= $$\int_{\pi/6}^{\pi/3} \frac{\cos\left(\frac{\pi}{2}-x\right)}{\cos\left(\frac{\pi}{2}-x\right)+\sin\left(\frac{\pi}{2}-x\right)}dx$$

= $$\int_{\pi/6}^{\pi/3} \frac{\sin x}{\sin x+\cos x}dx \qquad \text{...... (2)}$$

Adding (1) and (2), we get

2I= $$\int_{\pi/6}^{\pi/3} \frac{\cos x}{\cos x+\sin x}dx + \int_{\pi/6}^{\pi/3} \frac{\sin x}{\sin x+\cos x}dx$$

= $$\int_{\pi/6}^{\pi/3} \frac{\cos x+\sin x}{\cos x+\sin x}dx$$

= $$\int_{\pi/6}^{\pi/3} 1dx = [x]_{\pi/6}^{\pi/3}$$

= $\frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$

$\therefore$ I = $\frac{\pi}{12}$ **(4)**

## ANSWER : 4

**(i)** Let the initial value of the business = Rs. 100

$\therefore$ original income of the broker at the rate of 6% is = Rs. 6 (1/2 mark)

Now, let the new value of the business = Rs. x.

$\therefore$ new income of the broker at the rate of 7.5%.

= $x \times \frac{7.5}{100} = x \times \frac{75}{1000} = \frac{3x}{40}$ (1/2 mark)

But the income of the broker remains unchanged.

$\therefore$ 6 = $\frac{3x}{40}$

$\therefore$ 240 = 3x

$\therefore$ x = $\frac{240}{3}$ = 80

$\therefore$ new value of the business = Rs. 80 (1/2 mark)

Hence, the percentage reduction in the value of the business is 20%. (1/2 mark)

**(2)**

**(ii)**

| Age group (years) | Population (in '000) $_nP_x$ | No. of deaths $_nD_x$ | Age – SDR |
|---|---|---|---|
| | | | $\frac{_nD_x}{_nP_x} \times 1000$ (1/2 mark) |
| Below 5 | 15 | 360 | $= \frac{360}{15,000} \times 1000 = 24$ (1/2 mark) |
| 5 – 30 | 20 | 400 | $= \frac{400}{20,000} \times 1000 = 20$ (1/2 mark) |
| Above 30 | 10 | 280 | $= \frac{280}{10,000} \times 1000 = 28$ (1/2 mark) |

**(2)**

**(iii)** Given : $b_{yx} = -0.6$, $b_{xy} = -0.3$

$r = \pm\sqrt{b_{yx} \cdot b_{xy}}$ (1/2 mark)

$= \pm\sqrt{(-0.6)(-0.3)}$

$= \pm\sqrt{0.18}$

$\therefore\ r = -0.4243$ (1/2 mark)

**Comment :**

Here, $b_{yx}$ and $b_{xy}$ both are negative. Hence r is also negative and hence X and Y are negatively correlated.

(1 mark)

**(2)**

**(iv)** In order that, given function is p.m.f., each must satisfy

(a) $P(X = x) \geq 0, \forall x$ and

(b) $\Sigma P(X = x) = 1$

(1/2 mark)

$P(X = 5.5) = \frac{5.5-5}{4} = \frac{0.5}{4} = \frac{1}{8}$,

$P(X = 6.5) = \frac{6.5-5}{4} = \frac{1.5}{4} = \frac{3}{8}$,

$P(X = 7.5) = \frac{7.5-5}{4} = \frac{2.5}{4} = \frac{5}{8}$

(1/2 mark)

Thus, $P(X = x) > 0, \forall x$ and

$\Sigma P(X = x) = \frac{1}{8} + \frac{3}{8} + \frac{5}{8} = \frac{9}{8} \neq 1$ (1/2 mark)

$\therefore$ the function is not a p.m.f. (1/2 mark)

**(2)**

**(v)** Given, Mean = 3.5 & Variance = 2.625

In the Binomial distribution,

Mean = n × p

i.e. 3.5 = n × p ...... (1)

and variance = n × p × q (1/2 mark)

2.625 = n × p × q .......(2)

Divide equation (i) by (ii)

$\frac{3.5}{2.625} = \frac{n \times p}{n \times p \times q}$

$1.33 = \frac{1}{q}$

$\therefore q = \frac{1}{1.33}$

q = 0.75 OR = $\frac{3}{4}$

OR

Divide equation (ii) by (i)

$\frac{2.625}{3.5} = \frac{n \times p \times q}{n \times p}$ (1/2 mark)

0.75 = q

$\therefore$ q = 0.75 or = $\frac{3}{4}$

we have p + q = 1

i.e. p = 1 – q

= 1 – 0.75 (1/2 mark)

= 0.25 OR = $\frac{1}{4}$

$\therefore$ substitute p = $\frac{1}{4}$ in equation (i)

$3.5 = n \times \frac{1}{4}$

3.5 × 4 = n (1/2 mark)

$\therefore$ n = 14

**(2)**

**(vi)** Given : $\bar{x}$ = 53, $\bar{y}$ = 28, $b_{yx}$ = – 1.5, $b_{xy}$ = – 0.2

Using regression equation of Y on X, we estimate Y when X = 50.

Now, $y - \bar{y} = b_{yx}(x - \bar{x})$

$\therefore$ y – 28 = – 1.5 (x – 53)

$\therefore$ y = – 1.5x + 79.5 + 28 (1 mark)

$\therefore$ y = –1.5x + 107.5

**Estimate of Y when X = 50 :**

Put x = 50 in y = – 1.5x + 107.5

$\therefore$ y = – 1.5 × 50 + 107.5

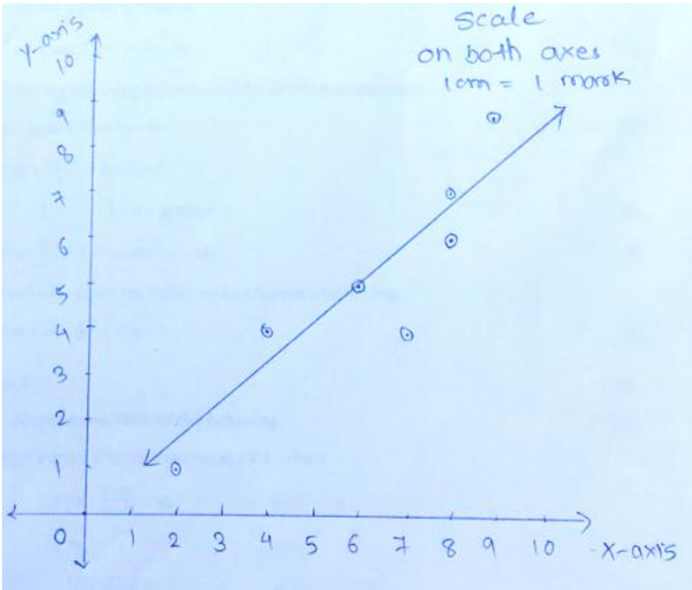
$\therefore$ y = – 75 + 107.5

$\therefore$ y = 32.5

(1 mark)

**(2)**

**(vii)** We take marks in Physics on X – axis and marks in Mathematics on Y – axis and plot the points as below.




We get a band of points rising from left to right. This indicates the positive correlation between marks in physics and marks in Mathematics. **(1 mark for graph and 1 mark for conclusion)**

**(viii)** Given : r = 0.3, Cov (x, y) = 12, $\sigma_x^2$ = 9 $\therefore$ $\sigma_x$ = 3. (1/2 mark)

Now, r = $\frac{Cov\ (x,y)}{\sigma_x \cdot \sigma_y}$ (1/2 mark)

$\therefore$ 0.3 = $\frac{12}{3 \times \sigma_y}$ (1/2 mark)

$\therefore$ 0.3 = $\frac{4}{\sigma_y}$ $\therefore$ $\sigma_y = \frac{4}{0.3}$

$\therefore$ $\sigma_y$ = 13.33. (1/2 mark)

**(2)**

**ANSWER : 5(A)**

**(i)** Given : $l_0$ = 1000, $l_1$ = 900, $l_2$ = 700, $T_2$ = 11500 (1/2 mark)

We have, $L_x = \frac{l_x + l_{x+1}}{2}$

$\therefore L_0 = \frac{l_0+l_1}{2} = \frac{1000+900}{2} = \frac{1900}{2} = 950$

$L_1 = \frac{l_1+l_2}{2} = \frac{900+700}{2} = \frac{1600}{2} = 800$ (1/2 mark)

We have, $T_x = L_x + T_{x+1}$

$\therefore T_1 = L_1 + T_2$

$\therefore T_1 = 800 + 11500$

$\therefore T_1 = 12300$

and $T_0 = L_0 + T_1$

$\therefore T_0 = 950 + 12300$ (1/2 mark)

$\therefore T_0 = 13250$

Now, we know that,

$e_x^\circ = \frac{T_x}{l_x}$

$\therefore e_0^\circ = \frac{T_0}{l_0} = \frac{13250}{1000} = 13.25$ (1/2 mark)

$e_1^\circ = \frac{T_1}{l_1} = \frac{12300}{900} = 13.667$ (1/2 mark)

$e_2^\circ = \frac{T_2}{l_2} = \frac{11500}{700} = 16.428$ (1/2 mark)

**(3)**

**(ii)**

| Age group (years) | Town I | | Town II | |
|---|---|---|---|---|
| | $P_i$ | $D_i$ | $P_i$ | $D_i$ |
| 0 – 10 | 1500 | 45 | 6000 | 150 |
| 10 – 25 | 5000 | 30 | 6000 | 40 |
| 25 – 45 | 3000 | 15 | 5000 | 20 |
| 45 & above | 500 | 22 | 3000 | 54 |
| Total | $\Sigma P_i = 10000$ | $\Sigma D_i = 112$ | $\Sigma P_i = 20000$ | $\Sigma D_i = 264$ |

**Town I :**

$CDR_I = \frac{\sum D_i}{\sum P_i} \times 1000 = \frac{112}{10000} \times 1000$ (1 mark)

$\therefore CDR_I = 11.2$ per thousand

**Town II :**

$CDR_{II} = \frac{\sum D_i}{\sum P_i} \times 1000 = \frac{264}{20000} \times 1000 = \frac{132}{10}$ (1 mark)

∴ $CDR_{II}$ = 13.2 per thousand

Comment : $CDR_I < CDR_{II}$ .

Town I is more healthy than Town II. (1 mark)

**(3)**

**(iii)**

| Jobs | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $M_1$ | 5 | 11 | 5 | 7 | 6 |
| $M_2$ | 1 | 4 | 2 | 5 | 3 |
| $M_3$ | 1 | 5 | 2 | 3 | 4 |

Here $\min(M_1) = 5$, $\text{Max}(M_2) = 5$ $\text{Min.}(M_3) = 1$

Since, $\text{Min}(M_1) \geq \text{Max}(M_2)$ is satisfied, the problem can be converted into 5 jobs and 2 machines problem.

Now, the two fictitious machines are such that $G = M_1 + M_2$ and
$H = M_2 + M_3$
The problem can be written as the following 5 job and 2 machine problem. (1/2 mark)

| Job | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| G | 6 | 15 | 7 | 12 | 18 |
| H | 2 | 9 | 4 | 8 | 7 |

Min (G, H) = 2, which corresponds To H
Therefore, job 1 is processed in the last (1/2 mark)

| | | | | |
|---|---|---|---|---|
| | | | | 1 |

G → ← H

The problem now reduce to four jobs 2, 3, 4 & 5.
Here, min (G, H) = 4, which corresponds to H

Therefore Job 3 is processed in the Last (1/2 mark)

| | | | | |
|---|---|---|---|---|
| | | | 3 | 1 |

G → ← H

The problem now reduce to three jobs 2, 4 & 5.
Here, min (G, H) = 7, which corresponds to H

Therefore Job 5 is processed in the Last (1/2 mark)

| | | | | |
|---|---|---|---|---|
| | | 5 | 3 | 1 |

G → ← H

The problem now reduce to two 2 & 4. Here min (G, H) = 8, which corresponds to H
Therefor job 4 is processed in the last (1/2 mark)

| | 4 | 5 | 3 | 1 |
|---|---|---|---|---|

G → ← H

Now, the remaining job 2 must be processed next to job 4, Thus, the optimal sequence of jobs is obtained as follows : (1/2 mark)

| 2 | 4 | 5 | 3 | 1 |
|---|---|---|---|---|

G → ← H

**ANSWER : 5 (B)**

**(i)** Let X = Marks in History, Y = Marks in Geography, we construct the following table to compute rank correlation coefficient :

| X | Y | Ranks of X $x_i$ | Ranks of Y $y_i$ | $d_i = (x_i - y_i)$ | $d_i^2$ |
|---|---|---|---|---|---|
| 70 | 80 | 1.5 | 1.5 | 0 | 0 |
| 70 | 60 | 1.5 | 5 | – 3.5 | 12.25 |
| 65 | 80 | 3 | 1.5 | 1.5 | 2.25 |
| 60 | 70 | 4 | 3 | 1 | 1.00 |
| 55 | 65 | 5 | 4 | 1 | 1.00 |
| 50 | 50 | 6 | 6 | 0 | 0 |
| 40 | 42 | 7 | 7 | 0 | 0 |
| 30 | 28 | 8 | 8 | 0 | 0 |
| n = 8 | - | - | - | - | $\sum d_i^2$ = 16.50 |

**(1/2 mark) for table**

Here, in the series of X, 70 is repeated twice. So there is one tie in which $m$ = 2 **(1/2 mark)**

$\therefore T_x = \frac{m(m^2-1)}{12} = \frac{2(2^2-1)}{12} = \frac{2 \times 3}{12} = 0.5$

In the series of Y, 80 is repeated twice. So there is one tie in which m = 2. **(1/2 mark)**

$\therefore T_y = \frac{m(m^2-1)}{12} = \frac{2(2^2-1)}{12} = \frac{2 \times 3}{12} = 0.5$

$\therefore$ Corrected $\sum d_i^2 = \sum d_i^2 + T_x + T_y$ (1/2 mark)

$= 16.50 + 0.5 + 0.5 = 17.50$

Rank correlation coefficient :

$R = 1 - \frac{6[Corrected\ \sum d_i^2]}{n(n^2-1)}$ (1/2 mark)

$= 1 - \frac{6(17.50)}{8(8^2-1)} = 1 - \frac{105}{8 \times 63} = 1 - \frac{105}{504}$

$= 1 - 0.21 = 0.79$ (1/2 mark)

$\therefore$ R = 0.79 ≈ 0.80

**(4)**

**ROW MINIMA**

**(ii)** **Step : 1 :** Subtract the smallest element in each row from every element in that row. We get

| Tasks | Time required (man – hours) | | | |
|---|---|---|---|---|
| | Subordinates | | | |
| | **1** | **2** | **3** | **4** |
| A | 0 | 18 | 9 | 3 |
| B | 9 | 24 | 0 | 22 |
| C | 23 | 4 | 3 | 0 |
| D | 9 | 16 | 14 | 0 |

(1 mark)

**COLUMN MINIMA**

**Step : 2** Subtract the smallest element in each column from every element in that column. We get

| Tasks | Time required (man – hours) | | | |
|---|---|---|---|---|
| | Subordinates | | | |
| | 1 | 2 | 3 | 4 |
| A | 0 | 14 | 9 | 3 |
| B | 9 | 20 | 0 | 22 |
| C | 23 | 0 | 3 | 0 |
| D | 9 | 12 | 14 | 0 |

(1 mark)

**Step : 3** Since the number of straight lines covering all zeros is equal to number of rows / columns, the optimum solution has reached. The optimal assignment can be made as follows :

| Tasks | Time required (man – hours) | | | |
|---|---|---|---|---|
| | Subordinates | | | |
| | 1 | 2 | 3 | 4 |
| A | [0] | 14 | 9 | 3 |
| B | 9 | 20 | [0] | 22 |
| C | 23 | [0] | 3 | ~~0~~ |
| D | 9 | 12 | 14 | [0] |

(1 mark)

Hence, the optimum assignment schedule is obtained as follows :

| Tasks | Subordinate | Time |
|---|---|---|
| | | (man – hours) |
| A | 1 | 8 |
| B | 3 | 4 |
| C | 2 | 19 |
| D | 4 | 10 |

(1 mark)

$\therefore$ **Minimum requirement of man – hours = 41 hours**

**(4)**

**(iii)** Let $u_i = x_i - 45$ and $v_i = y_i - 150$

So $\sum u_i = 45$, $\sum u_i^2 = 4400$,

$\sum v_i = 280$, $\sum v_i^2 = 167432$,

$\sum u_i\, v_i = 21680$ **(0.5 Mark)**

$\therefore \quad \bar{u} = \frac{\sum u_i}{n} = \frac{45}{8} = 5.625$ **(0.5 Mark)**

$\bar{v} = \frac{\sum v_i}{n} = \frac{280}{8} = 35$ **(0.5 Mark)**

$\sigma_u^2 = \frac{\sum u_i^2}{n} - (\bar{u})^2$

$= \frac{4400}{8} - (5.625)^2 = 518.3593$ **(0.5 Mark)**

$\sigma_v^2 = \frac{\sum v_i^2}{n} - (\bar{v})^2$

$= \frac{167432}{8} - (35)^2 = 19704$ **(0.5 Mark)**

Cov $(u, v) = \frac{\sum u_i v_i}{n} - \bar{u}\,\bar{v}$

$= \frac{21680}{8} - (5.625)(35)$

$= 2513.125$ **(0.5 Mark)**

From the properties of regression coefficients, you know that they are independent of change of origin.

So $b_{YX} = b_{VU} = \frac{cov\,(U,V)}{\sigma_U^2} = \frac{2513.125}{518.3593} = 4.8482$ **(0.5 Mark)**

and $b_{XY} = b_{UV} = \frac{cov\,(U,V)}{\sigma_v^2} = \frac{2513.125}{19704} = 0.1275$ **(0.5 Mark)**

**ANSWER : 6(A)**

**(i)** Let $x_1$ : Number of executive single rooms.
and
$x_2$ : Number of executive double rooms. **(0.5 Mark)**

Since manager of hotel plans an extension of not more than 50 rooms, therefore $x_1 + x_2 \leq 50$

Hotel must have at least 5 executive single rooms.

$\therefore x_1 \geq 5$ **(0.5 Mark)**

Since the number of executive double rooms should be at least 3 times, the number of executive single rooms.

$\therefore x_2 \geq 3x_1$ **(0.5 Mark)**

Since hotel manager charges Rs. 3000 for an executive double room and Rs. 1800 for executive single room per day, total profit is Rs. $(1800x_1 + 3000x_2)$. Let us denote this total profit by Z. **(0.5 Mark)**

$\therefore$ Royal hotel manager's problem is to determine $x_1$ and $x_2$ so as to

Maximize $Z = 1800x + 3000x_2$

Subject to constraints

$x_1 + x_2 \leq 50$

$x_1 \geq 5$

$x_2 \geq 3x_1$ **(1 Mark)**

**(3)**

**(ii)** The period of investments of three partners is same.

$\therefore$ the loss of Rs. 24500 is shared in the proportion to their investments.

i.e., in the proportion to 60000 : 40000 : 75000

$\Rightarrow$ 12 : 8 : 15

Total share is 12 + 8 + 15 = 35

$\therefore$ Raghu's share in loss = $\frac{12}{35} \times 24500$ = Rs. 8400

Madhu's share in loss = $\frac{8}{35} \times 24500$ = Rs. 5600

Ramu's share in loss = $\frac{15}{35} \times 24500$ = Rs. 10,500

Hence, the respective shares of loss that Raghu, Madhu and Ramu had to bear are Rs. 8400, Rs. 5600, Rs. 10500.

**(3)**

**(iii)** An article is marked at Rs. 1600.

$\therefore$ list price of the article is Rs. 1600

Trade discount at 25% on Rs. 1600 (1 mark)

= $1600 \times \frac{25}{100}$ = Rs. 400

Now, Invoice Price = List Price – Trade Discount

$\therefore$ invoice price = Rs. (1600 – 400 =) 1200 (1 mark)

Cash discount at 5% on Rs. 1200

= $1200 \times \frac{5}{100}$ = Rs. 60

Now, Net Price = Invoice price – Cash discount

= Rs. 1200 – Rs. 60 = 1140 (1 mark)

Hence, the net price of an article is Rs. 1140.

**(3)**

**ANSWER : 6(B)**

**(i)** Given A = Rs. 20,500

C = Rs. 10,000, n = 2 years (1/2 mark)

To find r
Using accumulated value A

$$A = \frac{C}{i}\left[(1+i)^n - 1\right]$$ (1/2 mark)

∴ $20{,}500 = \frac{10000}{i}\left[(1+i)^2 - 1\right]$

∴ $\frac{20500}{10000} = \frac{1+2i+i^2-1}{i}$ (1/2 mark)

∴ $2.05 = \frac{2i+i^2}{i}$

∴ 2.05 = 2 + i (1/2 mark)

∴ i = 2.05 – 2

∴ i = 0.05

∴ $i = \frac{r}{100}$

$0.05 = \frac{r}{100}$ (1 mark)

∴ r = 5

∴ rate of interest is 5% p.a. (1 mark)

**(4)**

**(ii)** B.G. = Banker's Gain
B.D. = Banker's Discount
T.D. = True Discount (1/2 mark)

Let T.D. = Rs. x

B.G = B. D – TD (1/2 mark)

= Interest on T.D. for 6 months at 4% p.a.

∴ $80 = x \times \frac{1}{2} \times \frac{4}{100} = \frac{x}{50}$

∴ $x = 80 \times 50 = 4000$ (1/2 mark)

∴ True discount is Rs. 4,000

∴ B.D. = B.G. + T.D. (1/2 mark)

= 80 + 4000 = Rs. 4,080.

∴ Banker's discount is Rs. 4,080.

B.D. = interest on F.V. for 6 months at 4% p.a.

Let the face value (F.V.) be y (1/2 mark)

∴ B.D. = y $\times \frac{1}{2} \times \frac{40}{100}$

∴ 4080 = $\frac{y}{50}$ (1/2 mark)

∴ y = 2,04,000

∴ Amount of the bill is Rs. 2,04,000/- (1 mark)

**(4)**

**(iii)** In poisson distribution

x ~ p(m)

P(X = $x$) = $\frac{e^{-m} \cdot m^{x}}{x!}$ (1/2 mark)

Given, p(X = 1) = P(x = 2)

$\therefore \frac{e^{-m} \cdot m^{1}}{1!} = \frac{e^{-m} \cdot m^{2}}{2!}$ (1/2 mark)

$\frac{m}{1} = \frac{m^2}{2}$

$\frac{2}{1} = \frac{m^2}{m}$

2 = m

∴ m = 2 (1 mark)

Now, P(X ≥ 1) = 1 – p(X < 1) (1 mark)

= 1 – P(x = 0)

= 1 – $\frac{e^{-2} \cdot 2^{\circ}}{0!}$

= 1 – $\frac{0.1353 \times 1}{1}$

= 1 – 0.1353

= 0.8647 (1 mark)

**(4)**