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# SUGGESTED SOLUTION

SYJC

**SUBJECT- Mathematics & Statistics**

## Test Code – SYJ 6114

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## ANSWER 1 (A)

$f(4) = 8$ (Given) ..................... (1)

$$\lim_{x \to 4} f(x) = \lim_{x \to 4} \frac{x^2 - 16}{x - 4}$$

$$= \lim_{x \to 4} \frac{(x-4)(x+4)}{x-4}$$

$$= \lim_{x \to 4} (x+4) \quad ...[x \to 4, x \neq 4 \quad \therefore x - 4 \neq 0]$$

$= 4 + 4 = 8$ ..................... (2)

From (1) and (2),

$$\lim_{x \to 4} f(x) = f(4)$$

$\therefore$ $f$ is continuous at $x = 4$.

**(2 MARKS)**

## ANSWER 1 (B)

Let $y = \frac{x+2}{x^2+1}$

Differentiating both sides with respect to $x$,

$$\frac{dy}{dx} = \frac{(x_2 + 1) - (x+2)(2x)}{(x^2+1)^2}$$

$$= \frac{x^2 + 1 - 2x^2 - 4x}{(x^2+1)^2}$$

$$= \frac{1 - 4x - x^2}{(x^2+1)^2}$$

$$\therefore \frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)}, \text{ where } \frac{dy}{dx} \neq 0$$

$$= \frac{(x^2+1)^2}{(1 - 4x - x^2)}$$

**(2 MARKS)**

## ANSWER 1 (C)

Since sin $x$ and 9 are continuous functions, sin $x$ – 9 is continuous for all $x \in R$.

Also, $x^2$ – 9 being polynomial function, is continuous for all real $x$.

$\therefore$ $f$ being the ratio of two continuous functions, is continuous except when its denominator

sin $x$ – 9 = 0.

But sin $x$ – 9 ≠ 0 for any $x \in R$, because $-1 \leq \sin x \leq 1$ for all $x \in R$.

**(2 MARKS)**

## ANSWER 1 (D)

$x = e^{2t}, y = e^{\sqrt{t}}$

Differentiating $x$ and $y$ w.r.t. $t$, we get,

$$\frac{dx}{dt} = \frac{d}{dt}\left(e^{2t}\right) = e^{2t}.\frac{d}{dt}(2t)$$

$$= e^{2t} \text{ x } 2 = 2e^{2t}$$

and $\frac{dy}{dt} = \frac{d}{dt}\left(e \quad\right) = e^{\sqrt{t}}.\frac{d}{dt}\left(\sqrt{t}\right)$

$$+ \; e^{\sqrt{t}} \text{ x } \frac{1}{2\sqrt{t}} = \frac{e^{\sqrt{t}}}{2\sqrt{t}}$$

$$\therefore \frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)} = \frac{\left(\frac{e^{\sqrt{t}}}{2\sqrt{t}}\right)}{2e^{2t}}$$

$$= \frac{e^{\sqrt{t}}}{4\sqrt{t}.e^{2t}} = \frac{e^{\sqrt{t}-2t}}{4\sqrt{t}}$$

**(2 MARKS)**

## ANSWER 1 (E)

$f(0) = k$ (Given) ..................... (1)

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{\log(1+3x)}{5x}$$

$$= \lim_{x \to 0} \frac{\log(1+3x)}{3x} \text{ x } \frac{3}{5}$$

$$= \frac{3}{5}.\lim_{x \to 0} \frac{\log(1+3x)}{3x}$$

$= \frac{3}{5}(1)$

$\because \left[ x \to 0, 3x \to 0 \text{ and } \lim_{t \to 0} \frac{\log(1+t)}{t} = 1 \right]$

$\therefore \lim_{x \to 0} f(x) = \frac{3}{5}$ ..................... (2)

Since $f$ is continuous at $x = 0$,

$f(0) = \lim_{x \to 0} f(x)$

$\therefore \ k = \frac{3}{5}$ ..................... [By (1) and (2)]

**(2 MARKS)**

## ANSWER 1 (F)

$x$ = xin$^3$θ, y = cos$^3$θ

Differentiating x and y w.r.t. θ, we get,

$\frac{dx}{du} = \frac{d}{d\theta}(\sin\theta)^3 = 3(\sin\theta)^2, \frac{d}{d\theta}(\sin\theta)$

$= 3\sin^2\theta\cos\theta$

and $\frac{dy}{d\theta} = \frac{d}{d\theta}(\cos\theta)^3 = 3(\cos\theta)^2 \cdot \frac{d}{d\theta}(\cos\theta)$

$= 3\cos^2\theta(-\sin\theta) = -3\cos^2\theta\sin\theta$

$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{-3\cos^2\theta\sin\theta}{3\sin^2\theta\cos\theta}$

$= \frac{-\cos\theta}{\sin\theta} = -\cot\theta.$

**(2 MARKS)**

## ANSWER 2 (A)

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\cos 3x - \cos x}{x^2}$$

$$= \lim_{x\to 0} \frac{-2\sin\left(\frac{3x+x}{2}\right).\sin\left(\frac{3x-x}{2}\right)}{x^2}$$

$$= \lim_{x\to 0} \frac{-2\sin 2x.\sin x}{x^2}$$

$$= \lim_{x\to 0} -2.\frac{\sin 2x}{2x}.\frac{\sin x}{x} \text{ x } 2$$

$$= -4 \lim_{x\to 0} \frac{\sin 2x}{2x}.\lim_{x\to 0} \frac{\sin x}{x}$$

$$= -4(1)(1) \qquad \ldots\left[x\to 0, 2x\to 0 \text{ and } \lim_{\theta} \frac{\sin\theta}{\theta} = 1\right]$$

$$\therefore \lim_{x\to 0} f(x) = -4$$

Now, $f$ is continuous at $x = 0$.

$$\therefore f(0) = \lim_{x\to 0} f(x)$$

$$\therefore f(0) = -4.$$

**(3 MARKS)**

## ANSWER 2 (B)

$$x^2 + (xy)^{\frac{3}{2}} + y^2 = 5$$

Differentiating both sides w.r.t. $x$, we get,

$$2x + \frac{3}{2}(xy)^{\frac{1}{2}}.\frac{d}{dx}(xy) + 2y\frac{dy}{dx} = 0$$

$$\therefore 2x + \frac{3}{2}\sqrt{xy}.\left[x\frac{dy}{dx} + \frac{3}{2} + y \text{ x } 1\right] + 2y\frac{dy}{dx} = 0$$

$$\therefore 2x + \frac{3}{2}.x\sqrt{xy}\frac{dy}{dx} + \frac{3}{2}.y\sqrt{xy} + 2y\frac{dy}{dx} = 0$$

$$\therefore 4x + 3x\sqrt{xy}\frac{dy}{dx} + 3y\sqrt{xy} + 4y\frac{dy}{dx} = 0$$

$$\left(3x\sqrt{xy}+4y\right)\frac{dy}{dx}=-4x-3y\sqrt{xy}$$

$$\therefore\ \sqrt{y}\left(3x\sqrt{x}+4\sqrt{y}\right)\frac{dy}{dx}-\sqrt{x}\left(4\sqrt{x}+3y\sqrt{y}\right)$$

$$\therefore\ \frac{dy}{dx}=\frac{-\sqrt{x}\left(4\sqrt{x}+3y\sqrt{y}\right)}{\sqrt{y}\left(3x\sqrt{x}+4\sqrt{y}\right)}$$

$$= -\sqrt{\frac{x}{y}}\left(\frac{4\sqrt{x}+3y\sqrt{y}}{4\sqrt{y}+3x\sqrt{x}}\right)$$

**(3 MARKS)**

## ANSWER 2 (C)

Given $f$ is continuous at $x = 1$.

$$\therefore\quad f(1) = \lim_{x\to 1} f(x)$$

$$= \lim_{x\to 1}\frac{1+\cos\pi x}{(1-x)^2}$$

Put $(1-x)=t$ $\quad\therefore x=1-t$

as $x\to 1, t\to 0$

$$\therefore\ f(1) = \lim_{t\to 0}\frac{1+\cos\left[\pi(1-t)\right]}{t^2}$$

$$= \lim_{t\to .}\frac{1+\cos(\pi-\pi t)}{t^2}$$

$$= \lim_{t\to 0}\frac{1-\cos\pi t}{t^2}$$

$$= \lim_{t\to 0}\frac{1-\cos\pi t}{t^2}\ \text{x}\ \frac{1+\cos\pi t}{1+\cos\pi t}$$

$\left(\text{as } t\to 0; (1+\cos t)\neq 0\right)$

$$= \lim_{t\to 0}\frac{1-\cos^2\pi t}{(1+\cos\pi t)}$$

$$= \lim_{t\to 0}\frac{\sin^2\pi t}{(1+\cos\pi t)}$$

$$= \lim_{t\to 0}\left(\frac{\sin\pi t}{\pi t}\right)^2\ \text{x}\ \frac{\pi^2}{(1+\cos\pi t)}$$

$$= 1\ \text{x}\ \frac{\pi^2}{2}$$

$$= \frac{\pi^2}{2}$$

$$\therefore \ f(1) = \frac{\pi^2}{2}$$

**(3 MARKS)**

## ANSWER 2 (D)

$x^y = \sin y + 5x$

$\therefore \ \log x^y = \log(\sin y + 5x) \quad \therefore \ y \log x = \log(\sin y + 5x)$

Differentiating both sides w.r.t. x, we get,

$$y \cdot \frac{d}{dx}(\log x) + (\log x)\frac{dy}{dx} = \frac{1}{\sin y + 5x} \cdot \frac{d}{dx}(\sin y + 5x)$$

$$\therefore \ y \times \frac{1}{x} + (\log x)\frac{dy}{dx} = \frac{1}{\sin y + 5x}\left(\cos y \ \frac{dy}{dx} + 5\right)$$

$$\therefore \ \frac{y}{x} + (\log x)\frac{dy}{dx} = \frac{\cos y}{\sin y + 5x}\frac{dy}{dx} + \frac{5}{\sin y + 5x}$$

$$\therefore \ \left(\log x - \frac{\cos y}{\sin y + 5x}\right)\frac{dy}{dx} = \frac{5}{\sin y + 5x} - \frac{y}{x}$$

$$\therefore \ \left[\frac{(\log x)\ (\sin y + 5x) - \cos y}{\sin y + 5x}\right]\frac{dy}{dx} = \frac{5x - y\sin y - 5xy}{x(\sin y + 5x)}$$

$$\therefore \ [(\log x)\ (\sin y + 5x) - \cos y]\ \frac{dy}{dx} = \frac{5x - 5xy - y\sin y}{x}$$

$$\therefore \ \frac{dy}{dx} = \frac{5x(1-y) - y\sin y}{x[(\log x)\ (\sin y + 5x) - \cos y]}.$$

**(3 MARKS)**

## ANSWER 3 (A)

$$\lim_{x \to \left(\frac{\pi}{2}\right)} f(x) = \lim_{x \to \left(\frac{\pi}{2}\right)} \frac{\cos^2 x}{1 - \sin^3 x}$$

$$= \lim_{x \to \left(\frac{\pi}{2}\right)} \frac{1 - \sin^2 x}{1 - \sin^3 x}$$

$$= \lim_{x \to \left(\frac{\pi}{2}\right)} \frac{(1-\sin x)(1+\sin x)}{(1-\sin x)(1+\sin x+\sin^2 x)}$$

$$= \lim_{x \to \left(\frac{\pi}{2}\right)} \frac{1+\sin x}{1+\sin x+\sin^2 x}$$

$$..\left[x \to \frac{\pi}{2}, x \neq \frac{\pi}{2} \quad \therefore \sin x \neq \sin\frac{\pi}{2} = 1 \qquad \therefore 1-\sin x \neq 0\right]$$

$$= \frac{\lim_{x \to \left(\frac{\pi}{2}\right)} (1+\sin x)}{\lim_{x \to \left(\frac{\pi}{2}\right)} (1+\sin x+\sin^2 x)}$$

$$= \frac{1+\sin\frac{\pi}{2}}{1+\sin\frac{\pi}{2}+\sin^2\frac{\pi}{2}}$$

$$= \frac{1+1}{1+1+1} = \frac{2}{3} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\lim_{x \to \left(\frac{\pi}{2}\right)} f(x) = \lim_{x \to \left(\frac{\pi}{2}\right)} \frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x} \text{ x } \frac{\sqrt{2}+\sqrt{1+\sin x}}{\sqrt{2}+\sqrt{1+\sin x}}$$

$$= \lim_{x \to \left(\frac{\pi}{2}\right)} \frac{2-1-\sin x}{(1-\sin^2 x)\left(\sqrt{2}+\sqrt{1+\sin x}\right)}$$

$$= \lim_{x \to \left(\frac{\pi}{2}\right)} \frac{1-\sin x}{(1-\sin x)(1+\sin x)\left(\sqrt{2}+\sqrt{1+\sin x}\right)}$$

$$= \lim_{x \to \left(\frac{\pi}{2}\right)} \frac{1}{(1+\sin x)\left(\sqrt{2}+\sqrt{1+\sin x}\right)}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots \left[x \to \frac{\pi}{2}, x \neq \frac{\pi}{2} \therefore \sin x \neq \sin\frac{\pi}{2} = 1 \qquad \therefore 1-\sin x \neq 0\right]$$

$$= \frac{1}{\lim\limits_{x \to \left(\frac{\pi}{2}\right)} (1+\sin x) \text{ x } \lim\limits_{x \to \left(\frac{\pi}{2}\right)} \left(\sqrt{2}+\sqrt{1+\sin x}\right)}$$

$$= \frac{1}{\left(1+\sin\frac{\pi}{2}\right)\left(\sqrt{2}+\sqrt{1+\sin\frac{\pi}{2}}\right)}$$

$$= \frac{1}{(1+1)\left(\sqrt{2}+\sqrt{1+1}\right)} = \frac{1}{2 \text{ x } 2\sqrt{2}} = \frac{1}{4\sqrt{2}} \qquad \text{.....................} \quad (2)$$

From (1) and (2),

$$\lim_{x \to \left(\frac{\pi}{2}\right)} f(x) \neq \lim_{x \to \left(\frac{\pi}{2}\right)} f(x)$$

$$x \to \left(\frac{\pi}{2}\right)^- \qquad x \to \left(\frac{\pi}{2}\right)^+$$

$$\therefore \lim_{x \to \left(\frac{\pi}{2}\right)^-} f(x) \neq \lim_{x \to \left(\frac{\pi}{2}\right)} f(x)$$

$\therefore \lim\limits_{x \to \frac{\pi}{2}} f(x)$ does not exist

$\therefore$ $f$ is discontinuous at $x = \frac{\pi}{2}$.

This discontinuity cannot be removed.

**(4 MARKS)**

## ANSWER 3 (B)

$x^y = e^{x-y}$

$\therefore \log x^y = \log e^{x-y}$

$\therefore y \log x = (x-y) \log e$

$\therefore y \log x = x - y \qquad ...[\because \log e = 1]$

$\therefore y + y\log x = x \qquad \therefore y(1+\log x) = x$

$\therefore y = \frac{x}{1+\log x}$

$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\frac{x}{1+\log x}\right)$

$= \frac{(1+\log x).\frac{d}{dx}(x) - x\frac{d}{dx}(1+\log x)}{(1+\log x)^2}$

$= \frac{(1+\log x).1 - x\left(0+\frac{1}{x}\right)}{(1+\log x)^2}$

$= \frac{1+\log x - 1}{(1+\log x)^2}$

$= \frac{\log x}{(1+\log x)^2}$

**(4 MARKS)**

## ANSWER 3 (C)

For $x<0, f(x)=\frac{e^{2x}-1}{ax}, a\neq 0$

$\therefore \lim_{x\to 0} f(x) = \lim_{x\to 0}\frac{2^{2x}-1}{2x}$ x $\frac{2}{a}$

$= \frac{2}{a}\lim_{x\to 0}\frac{e^{2x}-1}{2x}$

$= \frac{2}{a}$ x 1 ...$\left[x\to 0, 2x\to 0 \text{ and } \lim_{t\to 0}\frac{e^t-1}{t}=1\right]$

$= \frac{2}{a}$

Also, $f(0)=1$ ..................... (Given)

Now, $f$ is continuous at $x$ = 0

$\therefore \lim_{x\to 0^-} f(x) = f(0)$

$\therefore \frac{2}{a}=1 \qquad \therefore a=2$

For $x>0, f(x)=\frac{\log(1+7x)}{bx}, b\neq 0$

$\therefore \lim_{x\to 0^+} f(x) = \lim_{x\to 0}\frac{\log(1+7x)}{bx}$

$= \lim_{x\to 0}\frac{\log(1+7x)}{7x}$ x $\frac{7}{b}$

$= \frac{7}{b}\lim_{x\to 0}\frac{\log(1+7x)}{7x}$

$= \frac{7}{b} \times 1$

$\ldots\ldots\ldots\ldots\ldots\ldots \left[ x \to 0, 7x \to 0 \text{ and } \lim_{t \to 0} \frac{\log(1+t)}{t} = 1 \right]$

$= \frac{7}{b}$

Since, *f* is continuous at $x = 0$, $\lim_{x \to 0^+} f(x) = f(0)$

$\therefore \frac{7}{b} = 1 \qquad \therefore b = 7$

Hence, *a* = 2 and *b* = 7.

**(4 MARKS)**

## ANSWER 3 (D)

$\log\left(\frac{x^4 - y^4}{x^4 + y^4}\right) = k$

$\therefore \frac{x^4 - y^4}{x^4 - y^4} = e^k = p$ …..(Say)

$\therefore x^4 - y^4 = px^4 + py^4$

$\therefore y^4 + py^4 = x^4 - px^4$

$\therefore (1 + p)y^4 = (1 - p)x^4$

$\therefore \frac{y^4}{x^4} = \frac{1-p}{1+p}$

$\therefore \frac{y}{x} = \sqrt[4]{\frac{1-p}{1+p}}$ ……(A constant)

Differentiating both sides w.r.t. x, we get,

$\frac{d}{dx}\left(\frac{y}{x}\right) = 0$

$\therefore \frac{x\frac{dy}{dx} - y\frac{d}{dx}(x)}{x^2} = 0$

$\therefore x\frac{dy}{dx} - y \times 1 = 0$

$\therefore x\frac{dy}{dx} = y$

$\therefore \frac{dy}{dx} = \frac{y}{x}.$

**(4 MARKS)**

**Alternative Method :**

$\log\left(\frac{x^4-y^4}{x^4+y^4}\right) = k$

$\therefore \log(x^4 - y^4) - \log(x^4 + y^4) = k$

Differentiating both sides w.r.t. x, we get,

$\frac{1}{x^4-y^4}\cdot\frac{d}{dx}(x^4 - y^4) - \frac{1}{x^4+y^4}\cdot\frac{d}{dx}(x^4 + y^4) = 0$

$\therefore \frac{1}{x^4-y^4}\left(4x^3 - 4y^3\frac{dy}{dx}\right) - \frac{1}{x^4+y^4}\left(4x^3 + 4y^3\frac{dy}{dx}\right) = 0$

$\therefore \frac{4x^3}{x^4-y^4} - \frac{4y^3}{x^4-y^4}\frac{dy}{dx} - \frac{4x^3}{x^4+y^4} - \frac{4y^3}{x^4+y^4}\frac{dy}{dx} = 0$

$\therefore \frac{4y^3}{x^4-y^4}\frac{dy}{dx} + \frac{4y^3}{x^4+y^4}\frac{dy}{dx} = \frac{4x^3}{x^4-y^4} - \frac{4x^3}{x^4+y^4}$

$\therefore 4y^3\left(\frac{1}{x^4-y^4} + \frac{1}{x^4+y^4}\right)\frac{dy}{dx} = 4x^3\left(\frac{1}{x^4-y^4} - \frac{1}{x^4+y^4}\right)$

$\therefore 4y^3\left[\frac{x^4+y^4+x^4-y^4}{(x^4-y^4)(x^4+y^4)}\right]\frac{dy}{dx} -4x^3\left[\frac{x^4+y^4-x^4+y^4}{(x^4-y^4)(x^4+y^4)}\right]$

$\therefore 4y^3(2x^4)\frac{dy}{dx} = 4x^3(2y^4)$

$\therefore 8x^4y^3\frac{dy}{dx} = 8x^3y^4$

$\therefore x\frac{dy}{dx} = y$

$\therefore \frac{dy}{dx} = \frac{y}{x}.$

**(4 MARKS)**