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SUGGESTED SOLUTION

SYJC 2020

SUBJECT- MATHS AND STATS AND S.P.

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BRANCH - () (Date :)

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ANSWER-1

(A) Let $y = \sin^{-1}(\cos\ x)$

$$y = \sin^{-1}\left(\sin\left(\frac{\pi}{2} - x\right)\right)$$

$$= \frac{\pi}{2} - x$$

Differentiating both sides with respect to *x*, we get

$$\frac{dy}{dx} = -1$$

(02)

(B) $x^3 + y^2 + xy = 7$

Differentiating both sides w.r.t. *x*, we get,

$$3x^2 + 2y\frac{dy}{dx} + x\frac{dy}{dx} + y\frac{d}{dx}(x) = 0$$

$$\therefore\ 3x^2 + 2y\frac{dy}{dx} + x\frac{dy}{dx} + y \text{ x } 1 = 0$$

$$\therefore\ (2y + x)\frac{dy}{dx} = -3x^2 - y = -(y + 3x^2)$$

$$\therefore\ \frac{dy}{dx} = -\left(\frac{y + 3x^2}{2y + x}\right).$$

(02)

(C) $x^3y^3 = x^2 - y^2$

Differentiating both sides w.r.t. x, we get,

$$x^3\frac{d}{dx}(y^3) + y^3\frac{d}{dx}(x^3) = 2x - 2y\frac{dy}{dx}$$

$$\therefore\quad x^3 \times 3y^2\frac{dy}{dx} + y^3 \times 3x^2 = 2x - 2y\frac{dy}{dx}$$

$$\therefore\quad 3x^3y^2\frac{dy}{dx} + 2y\frac{dy}{dx} = 2x - 3x^2y^3$$

$$\therefore\quad (3x^3y^2 + 2y)\frac{dy}{dx} = 2x - 3x^2y^3$$

$\therefore \quad \frac{dy}{dx} = \frac{2x-3x^2y^3}{3x^3y^2+2y} = \frac{x(2-3xy^3)}{y(3x^3y+2)}$

$= \frac{x}{y}\left(\frac{2-3xy^3}{2+3x^3y}\right)$

(02)

ANSWER-2

(A) $y = \tan^{-1}\left(\frac{x+\sqrt{x}}{1-\sqrt{x^3}}\right)$

$= \tan^{-1}\left(\frac{x+\sqrt{x}}{1-x\sqrt{x}}\right)$

$= \tan^{-1} x + \tan^{-1}\sqrt{x}$

$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\tan^{-1} x\right) + \frac{d}{dx}\left(\tan^{-1}\sqrt{x}\right)$

$= \frac{1}{1+x^2} + \frac{1}{1+\left(\sqrt{x}\right)^2} \cdot \frac{d}{dx}\left(\sqrt{x}\right)$

$= \frac{1}{1+x^2} + \frac{1}{1+x} \text{ x } \frac{1}{2\sqrt{x}}$

$= \frac{1}{1+x^2} + \frac{1}{2\sqrt{x}\left(1+x\right)}$

(03)

(B) g = t^2 + 2t + 1, h = tan $\left(\frac{\pi t}{4}\right)$ $-$cos $\left(\frac{\pi t}{2}\right)$

Differentiating g and h w.r.t. *t*, we get,

$\frac{dg}{dt} = \frac{d}{dt}(t^2 + 2t + 1)$ = 2t + 2 × 1 + 0 = 2t + 2

and $\frac{dh}{dt} = \frac{d}{dt}\left[tan\left(\frac{\pi t}{4}\right) - cos\left(\frac{\pi t}{2}\right)\right]$

$= \sec^2\left(\frac{\pi t}{4}\right) \cdot \frac{d}{dt}\left(\frac{\pi t}{4}\right) - \left[-sin\left(\frac{\pi t}{2}\right)\right] \cdot \frac{d}{dt}\left(\frac{\pi t}{2}\right)$

$= \sec^2\left(\frac{\pi t}{4}\right) \times \frac{\pi}{4} + sin\left(\frac{\pi t}{2}\right) \times \frac{\pi}{2}$

$= \frac{\pi}{4}\sec^2\left(\frac{\pi t}{4}\right) + \frac{\pi}{2}\sin\left(\frac{\pi t}{2}\right)$

$\therefore \frac{dg}{dh} = \frac{(dg/dt)}{(dh/dt)} = \frac{2t+2}{\frac{\pi}{4}sec^2\left(\frac{\pi t}{4}\right)+\frac{\pi}{2}sin\left(\frac{\pi t}{2}\right)}$

$\therefore \left(\frac{dg}{dh}\right)_{at\ t=1} = \frac{2(1)+2}{\frac{\pi}{4}sec^2\frac{\pi}{4}+\frac{\pi}{2}\sin\frac{\pi}{2}}$

$= \frac{4}{\frac{\pi}{4}(\sqrt{2})^2 + \frac{\pi}{2} \times 1} = \frac{4}{\frac{\pi}{2} + \frac{\pi}{2}}$

$= \frac{4}{\pi}.$

(03)

ANSWER-3

(A) $\tan\left(\frac{x+y}{x-y}\right) = a$

$\therefore \frac{x+y}{x-y} = \tan^{-1} a = p$ ....(Say)

$\therefore$ x + y = $px - py$

$\therefore$ y + $py = px - x$

$\therefore$ (1 + p)y = (P – 1) x

$\therefore \frac{y}{x} = \frac{p-1}{1+p}$ ....(A constant)

Differentiating both sides w.r.t. x, we get,

$\frac{d}{dx}\left(\frac{y}{x}\right) = 0$

$\therefore \frac{x\frac{dy}{dx} - y\frac{d}{dx}(x)}{x^2} = 0$

$\therefore x\frac{dy}{dx} - y \times 1 = 0$

$\therefore x\frac{dy}{dx} = y$

$\therefore \frac{dy}{dx} = \frac{y}{x}.$

Alternative Method :

$\tan\left(\frac{x+y}{x-y}\right) = a$

$\therefore \frac{x+y}{x-y} = \tan^{-1} a$

Differentiating both sides w.r.t. *x*, we get

$\frac{d}{dx}\left(\frac{x+y}{x-y}\right) = \frac{d}{dx}(tan^{-1}a)$

$\therefore \frac{(x-y)\cdot\frac{d}{dx}(x+y) - (x+y)\cdot\frac{d}{dx}(x-y)}{(x-y)^2} = 0$

$\therefore$ (x – y) $\left(1 + \frac{dy}{dx}\right)$ – (x + y) $\left(1 - \frac{dy}{dx}\right)$ = 0

$\therefore$ (x – y) + (x – y) $\frac{dy}{dx}$ – (x + y) + (x + y) $\frac{dy}{dx}$ = 0

$\therefore$ (x – y + x + y) $\frac{dy}{dx}$ = – x + y + x + y

$\therefore$ 2x $\frac{dy}{dx} = 2y$

$\therefore$ $\frac{dy}{dx} = \frac{y}{x}$.

(04)

(B) $y = x^x + (7x-1)^x$

$\therefore \; y = u + v$

Differentiating both sides with respect to *x*,

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad \ldots \qquad (1)$$

$u = x^x$

Taking logarithm of both the sides,

Log *u* = *x* log *x*

Differentiating both sides with respect to *x*,

$$\frac{1}{u} \qquad \frac{du}{dx} = 1.\log x + x.\frac{1}{x}$$

$$\frac{du}{dx} = x^x(\log x + 1) \qquad \ldots \qquad (2)$$

As $v = (7x-1)^x$,

Taking logarithm of both the sides,

$\log v = x\log(7x-1)$

Differentiating both sides with respect to *x*,

$$\frac{1}{v}\frac{dv}{dx} = 1.\log(7x-1) + x.\frac{7}{(7x-1)}$$

$$\frac{dv}{dx} = (7x-1)^x \text{ x } \left(\log(7x-1) + \frac{7x}{7x-1}\right) \qquad \ldots \qquad (3)$$

Substituting (2) and (3) in (1) we get

$$\frac{dy}{dx} = x^x(\log \; x+1) +$$

$$(7x-1)^x \text{ x } \left(\log(7x-1) + \frac{7x}{7x-1}\right)$$

(04)

ANSWER-4

ANSWER-A **(2 MARKS)**

1. **Creditor**
2. **Current**

ANSWER-B **(3 MARKS)**

1. **Equity shareholders**
2. **Face value**
3. **Fixed rate bond**

ANSWER-5 **(5 MARKS)**

1. **Shares**

The term 'share' is defined by section 2(46) of the Companies Act 1956 as – "share means a share in the share capital of a company and includes stock except where a distinction between stock and shares is expressed or implied".

Share is a unit by which the share capital is divided. Share is a small unit of the capital of a company. The total capital of company is divided into small parts and each such part is called 'share'. The capital of company is divided into large number of shares. It facilitates the public to subscribe the capital in smaller amount. Thus a share is indivisible unit of share capital.

A person can purchase any number of shares as he wishes. A person who purchases shares of a company is known as a shareholder of that company.

Features of Shares

1. A share is the smallest unit of share capital.
2. It shows the ownership of a shareholder.
3. Each share has distinct number, which is noted in the share certificate.
4. Share is a movable property of a member.
5. A share is not any visible thing. It is shown by share certificate or in the form the 'demat' share.
6. Each share has value expressed in terms of money. There may be
   - **(a) Face value :** This value is written on the share certificate and mentioned in the Memorandum of Association.
   - **(b) Issue value :** It is the price at which company sells its shares. For example, issue at par or at premium or at discount.
   - **(c) Market value :** This value of share is determined by demand and supply forces in the share market.
7. A shareholder is entitled to get a share in the net profit of the company. It is called 'dividend'.
8. A company can issue two types of shares –

(a) Equity shares (b) Preference share

2. Every public company issues shares. These shares are listed and traded on various share markets. Companies in India issue shares which are traded on Indian share markets like BSE(Bombay Stock Exchange) and NSE (National Stock Exchange) etc.

With adoption of free economic policy and due to globalization, shares of some of the Indian companies are also listed and traded on foreign stock exchanges like NYSE (New York Stock Exchange) or NASDAQ (National Association of Securities Dealers Automated Quotation).

To list shares on stock exchanges, a company has to comply with policies of those stock exchanges. These policies are much more different from the policies of Indian stock exchange. Therefore those Indian companies which can not list their shares directly on foreign stock exchange, get listed on these stock exchanges indirectly using GDR and ADR.

Indian companies raise equity capital in international market through GDR and ADR. It is dollar/ Euro denominated instrument traded on stock exchanges of USA and Europe. Company issues shares to an intermediary called 'depository'. Bank of New York, Citygroup etc. act as Foreign Depository Banks. The Depository banks issue GDRs Or ADRs to investors against these shares. Each 'Depository Receipt' is having fixed number of shares. These 'Depository Receipts' are then sold to people of a foreign country. The Depository Receipts are listed on the stock exchanges. These Depository Receipts are traded like ordinary shares. In simple words, the depository bank stores the shares on behalf of the receipt – holder.

Both ADR and GDR are depository receipts. They represent a claim on underlying shares. The only difference is the location where they are traded. If the Depository Receipt is traded in U.S.A, it is called 'American Depository Receipt' (ADR). If the Depository Receipt is traded in a country other than USA, it is called 'Global Depository Receipt (GDR)'.

ADR and GDR are means of investment for NRI (Non – Resident Indians) and foreign nationals, wanting to invest money in India. By purchasing ADR or GDR, they can invest directly in Indian companies. NRI and foreigners can buy Depository Receipts using their regular equity trading account. The companies pay dividend in home currency to depositories and depositories convert it into the currency of investor and pays dividend.

ANSWER-6 **(10 MARKS)**

1 . Preference share :

As the name indicates, these shares have certain privileges and preferential rights distinct from those attaching to equity shares.

The shares which carry following preferential rights are termed as preference shares.

(a) A preferential right as to the payment of dividend during the life time of the company.

(b) A preferential right as to the return of capital in the event of winding up of the company.

Holder of these shares have a prior right to receive the fixed rate of dividend before any dividend is paid to equity shares. The rate of dividend is prescribed in the issue.

Normally preference shares carry no voting power. They have voting right only on those matters which affect their interest such as selling of undertaking or changing rights of preference holder, etc.

The preference shareholders are co – owners of the company but not controllers. These shares are purchased by cautious investors who carve for safety of principal and are satisfied with low but definite and regular income. Preference shares occupy an 'in between' position in the capital structure.

Types of preference shares

Preference shares

- Cumulative & non - cumulative
- Participating and non participating
- Convertible and non - convertible
- Redeemable and irredeemable

1. **Cumulative preference shares :**
   Cumulative preference shares are those shares on which dividend goes on accumulating until it is fully paid. This means, if the dividend is not paid in one or more years due to inadequate profit, then such unpaid dividend gets accumulated. The accumulated dividend is paid when company performs well. The arrears of dividend are paid before making payment to equity shareholders. The preference shares are always cumulative unless otherwise stated in the Articles of Association. It means that if dividend is not paid in any year or falls short of prescribed rate, the unpaid amount is carried forward to next year and so on ; until all arrears have been paid.
2. **Non – cumulative preference shares :** Dividend on these shares does not accumulate. This means, the dividend on shares can be paid only out of profits of that year. The right to claim dividend will lapse, if company does not make profit in that particular year. If dividend is not paid in any year, it is lost.
3. **Participating preference shares :** The holders of these shares are entitled to participate in surplus profit besides preferential dividend. The surplus profit which remains after the dividend has been paid to equity shareholders up to certain limit, is distributed to preference shareholders.
4. **Non – participating preference shares :** The preference shares are deemed to be non – participating, if there is no clear provision in Articles of Association. These shareholders are entitled only to fixed rate of dividend prescribed in the issue.
5. **Convertible preference shares :** These shareholders have a right to convert their preferences shares into equity shares. The conversion takes place within a certain fixed period.
6. **Non – convertible preference shares :** These shares can not be converted into equity shares.
7. **Redeemable preference shares :** Shares which can be redeemed after a certain fixed period are called redeemable preference shares. A company limited by shares, if authorized by Articles of Association, issues redeemable preference shares. Such shares must be fully paid.

These shares are redeemed out of divisible profit only or out of fresh issue of shares made for this purpose.

8. **Irredeemable preference shares :** Shares which are not redeemable i.e. payable only on the winding up of the company are called irredeemable preference shares. As per Companies Act (Amendment made in 1988) the company can not issue irredeemable preference share.

2. Debenture :

Debentures have occupied a significant position in the financial structure of the companies. It is one of the main sources of raising debt capital to meet long term financial needs.

Debentures represent borrowed capital. The debenture holders are creditors of the company. The debenture holder gets a fixed rate of interest as return on his investment. Board of Directors has the power to issue debentures.

The term ‘debenture’ has come from Latin word ‘debare’, which means to ‘owe’.

Palmer defines a debenture as –
“an instrument under seal evidencing debt, the essence of it being admission of indebtedness”.

Tophan defines –

“a debenture is a document given by a company as evidence of debt to the holder, usually arising out of loan and most commonly secured by charge.”

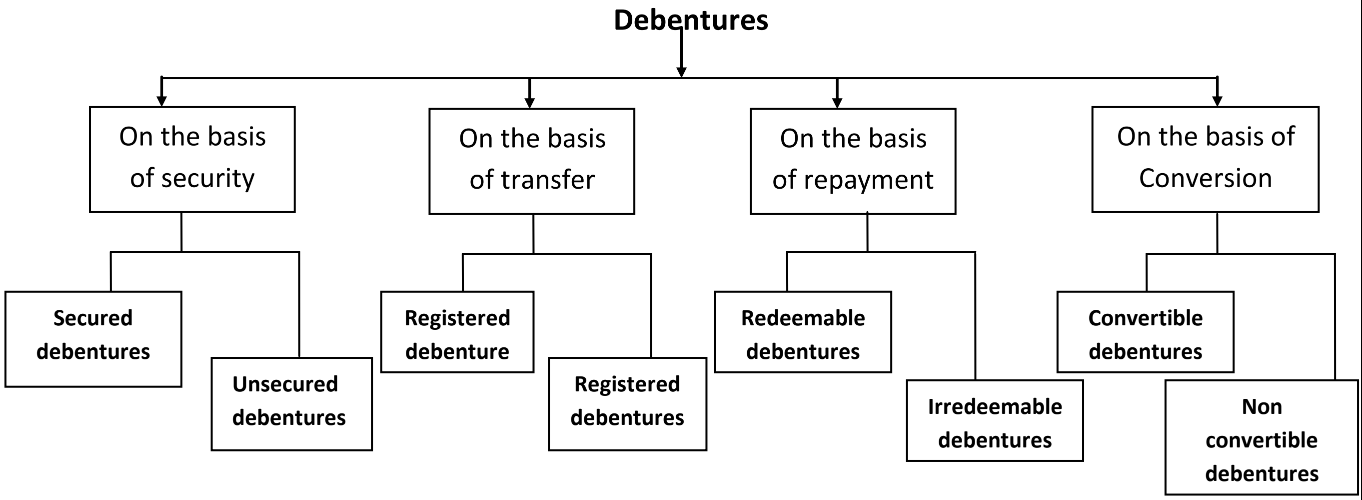
The above definitions bring out the clear meaning of debenture. Firstly it is an instrument issued in the form of ‘debenture certificate’ under the common seal of the company. Being an instrument it is transferable. It is an evidence of indebtedness. Thus debenture holder is creditor of the company. Being a loan it is secured by charge on asset. It is refundable.

The term debenture has not been defined clearly under Companies Act 1956.

Sec 2 (12) of companies Act 1956 only states that, “the word debenture includes debenture stock, bonds and any other security of the company whether constituting a charge on asset of the company or not.”

Under the existing definition debenture means a document which either crates or acknowledges debt. Ordinarily debenture constitutes a charge on company, on some part of its property, but there may be a debenture without such charge.
Here a point to be noted that as noted that as per Companies (Amendment) Act 2000, company can not issue unsecured debentures.

Types of Debentures



The debentures can be of different kinds according to their terms of issue, conversion, provision of security, repayment etc. Let us discuss them in detail.

1. **Secured Debentures :** The debentures can be secured. The property of company may be charged as security for loan. The security may be for some particular asset (fixed charge) or it may be the asset in general (floating charge), The debentures are secured through 'Trust Deed'.
2. **Unsecured debentures :** These are the debenture that have no security. The issue of unsecured debenture is now prohibited by Companies. (Amendment) Act, 2000
3. **Registered debentures :** Registered debentures are those on which the name of holders are recorded. A company maintains a register of debenture holders in which the names, addresses and particulars of holdings of debenture holders are entered. The transfer of debentures in this case requires the execution of regular transfer deed.
4. **Bearer debenture :** Name of holders are not recorded on the bearer debentures. Their names do not appear on the register of debenture holders. Such debentures are transferable by mere delivery. Payment of interest is made by means of coupons attached to debenture certificate.
5. **Redeemable debentures :** Debentures are mostly redeemable i.e. payable at the end of some fixed period, as mentioned on the debenture certificate. Repayment can be made at fixed date at the end of specific period or by installments during the life time of the company. The provision of repayment is normally made in a trust deed.
6. **Irredeemable debentures :** These kind of debentures are not repayable during life time of the company. They are repayable only after the liquidation of the company, or when there is breach of any condition or when some contingency arises.
7. **Convertible debentures :** convertible debentures give the right to the holder to convert them into equity shares after a specific period. Such right is mentioned in the debenture certificate. The issue of convertible debenture must be approved by special resolution in general meeting before they are issued to public. These debentures are advantageous for holder. Because of this conversion right, convertible debenture holder is entitled to equity shares at a rate lower than market value and even he participates in the profit of the company.
8. **Non – convertible debentures :** Non convertible debentures are not convertible into equity shares on maturity. These debentures are normally redeemed on maturity date. These debentures suffer from the disadvantage that there is no appreciation in value.