

SYJC - MATHEMATICS & STATISTICS

HIGHLIGHTS

- ✓ SOLUTION TO ALL QUESTIONS
- ✓ SOLUTIONS ARE PUT IN WAY THE STUDENT IS EXPECTED TO REPRODUCE IN THE BOARD EXAM
- ✓ TAUGHT IN THE CLASS ROOM THE SAME WAY AS THE SOLUTION ARE PUT UP HERE . THAT MAKES THE STUDENT TO EASILY GO THROUGH THE SOLUTION & PREPARE HIM/HERSELF WHEN HE/SHE SITS BACK TO REVISE AND RECALL THE TOPIC AT ANY GIVEN POINT OF TIME .
- ✓ LASTLY, IF STUDENT DUE TO SOME UNAVOIDABLE REASONS , HAS MISSED THE LECTURE , WILL NOT HAVE TO RUN HERE AND THERE TO UPDATE HIS/HER NOTES .
- ✓ HOWEVER STUDENT IS REQUESTED NOT TO MISUSE THE ABOVE POINT AS CLASS ROOM LECTURES ARE MUST FOR EASY PASSAGE OF UNDERSTANDING & LEARNING THE MINUEST DETAILS OF THE GIVEN TOPIC

PAPER - II **SEQUENCING**

SEQUENCING

N – JOBS ON 2 MACHINES

Q1. Find the sequence that minimizes total elapsed time (in hours) required to complete the following jobs on two machines M_1 and M_2 in the order M_1M_2 . Also find the minimum elapsed time and idle time for two machines

| Job | A | B | C | D | E |
|-------|---|---|----|---|----|
| M_1 | 6 | 2 | 10 | 4 | 11 |
| M_2 | 3 | 7 | 8 | 9 | 5 |

Step 1 : Finding the optimal sequence

Min time = 2 on job B on machine M_1 . Place the job at the start of the sequence

| | | | | |
|----------|--|--|--|--|
| B | | | | |
|----------|--|--|--|--|

Next min time = 3 on job A on machine M_2 . Place the job at the end of the sequence

| | | | | |
|----------|--|--|--|----------|
| B | | | | A |
|----------|--|--|--|----------|

Next min time = 4 on job D on machine M_1 . Place it at the start of the sequence after B

| | | | | |
|----------|----------|--|--|----------|
| B | D | | | A |
|----------|----------|--|--|----------|

Next min time = 5 on job E on machine M_2 . Place it at the end of the sequence before A

| | | | | |
|----------|----------|--|----------|----------|
| B | D | | E | A |
|----------|----------|--|----------|----------|

OPTIMAL SEQUENCE

| | | | | |
|----------|----------|----------|----------|----------|
| B | D | C | E | A |
|----------|----------|----------|----------|----------|

Step 2 : Work table

According to the optimal sequence

| Job | B | D | C | E | A | total process time |
|-------|---|---|----|----|---|--------------------|
| M_1 | 2 | 4 | 10 | 11 | 6 | = 33 hrs |
| M_2 | 7 | 9 | 8 | 5 | 3 | = 32 hrs |

WORK TABLE

| JOBS | MACHINES | | | | Idle time on M ₂ |
|------|----------------|-----|----------------|-----|--------------------------------|
| | M ₁ | | M ₂ | | |
| | IN | OUT | IN | OUT | |
| B | 0 | 2 | 2 | 9 | 2 |
| D | 2 | 6 | 9 | 18 | |
| C | 6 | 16 | 18 | 26 | 1 |
| E | 16 | 27 | 27 | 32 | |
| A | 27 | 33 | 33 | 36 | 1 |

Step 3 :

Total elapsed time T = 36 hrs

Idle time on M₁ = T - [sum of processing time of all jobs on M₁]

$$= 36 - 33$$

$$= 3 \text{ hrs}$$

Idle time on M₂ = T - [sum of processing time of all jobs on M₂]

$$= 36 - 32$$

$$= 4 \text{ hr} \quad (\text{CHECK : } 2 + 1 + 1 = 4)$$

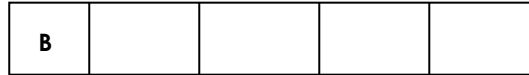
02 .

| Job | A | B | C | D | E |
|----------------|---|---|---|---|----|
| M ₁ | 5 | 1 | 9 | 3 | 10 |
| M ₂ | 2 | 6 | 7 | 8 | 4 |

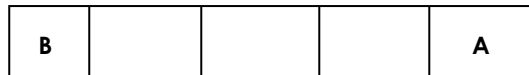
(MARCH 14 , OCT 14)

Step 1 : Finding the optimal sequence

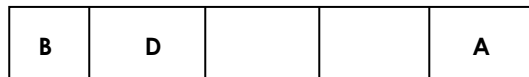
Min time = 1 on job B on machine M₁ . Place the job at the start of the sequence



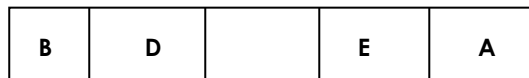
Next min time = 2 on job A on machine M₂ . Place the job at the end of the sequence



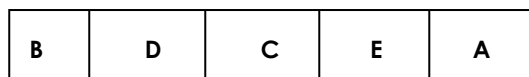
Next min time = 3 on job D on machine M₁ . Place it at the start of the sequence after B



Next min time = 4 on job E on machine M₂ . Place it at the end of the sequence before A



OPTIMAL SEQUENCE



Step 2 : Work table

According to the optimal sequence

| Job | B | D | C | E | A | total process time |
|----------------|---|---|---|----|---|--------------------|
| M ₁ | 1 | 3 | 9 | 10 | 5 | = 28 hrs |
| M ₂ | 6 | 8 | 7 | 4 | 2 | = 27 hrs |

WORK TABLE

| JOBS | MACHINES | | | | Idle time on M2 |
|------|----------|-----|----|-----|--------------------|
| | M1 | | M2 | | |
| | IN | OUT | IN | OUT | |
| B | 0 | 1 | 1 | 7 | 1 |
| D | 1 | 4 | 7 | 15 | |
| C | 4 | 13 | 15 | 22 | 1 |
| E | 13 | 23 | 23 | 27 | |
| A | 23 | 28 | 28 | 30 | 1 |

Step 3 :

Total elapsed time T = 30 hrs

$$\begin{aligned}
 \text{Idle time on M1} &= T - \left(\text{sum of processing time of all jobs on M1} \right) \\
 &= 30 - 28 \\
 &= 2 \text{ hrs}
 \end{aligned}$$

$$\begin{aligned}
 \text{Idle time on M2} &= T - \left(\text{sum of processing time of all jobs on M2} \right) \\
 &= 30 - 27 \\
 &= 3 \text{ hrs} \quad (\text{CHECK : } 1 + 1 + 1 = 3)
 \end{aligned}$$

Q2. The time (in hours) required to perform printing and binding operation (in that order) for each book is given in the following table **(MARCH 2014)**

| Book | I | II | III | IV | V |
|-------------------------|---|----|-----|----|---|
| Printing M ₁ | 3 | 7 | 4 | 5 | 7 |
| Binding M ₂ | 6 | 2 | 7 | 3 | 4 |

Find the sequence that minimizes the total elapsed time to complete the work . Also find the minimum elapsed time T and idle time for two machines

Step 1 : Finding the optimal sequence

Min time = 2 on job II on machine M₂ . Place the job at the end of the sequence

| | | | | |
|--|--|--|--|-----------|
| | | | | II |
|--|--|--|--|-----------|

Next min time = 3 on job I on machine M₁ & on job IV on machine M₂ . Place the job I at the start of the sequence & job IV at the end of the sequence before II

| | | | | |
|----------|--|--|-----------|-----------|
| I | | | IV | II |
|----------|--|--|-----------|-----------|

Next min time = 4 on job III on machine M₁ & on job V on machine M₂ . Place the job III at the start of the sequence after I & job V at the end of the sequence before IV

| | | | | |
|----------|------------|----------|-----------|-----------|
| I | III | V | IV | II |
|----------|------------|----------|-----------|-----------|

OPTIMAL SEQUENCE

| | | | | |
|----------|------------|----------|-----------|-----------|
| I | III | V | IV | II |
|----------|------------|----------|-----------|-----------|

Step 2 : Work table

According to the optimal sequence

| Job | I | III | V | IV | II | total process time |
|----------------|---|-----|---|----|----|--------------------|
| M ₁ | 3 | 4 | 7 | 5 | 7 | = 26 hrs |
| M ₂ | 6 | 7 | 4 | 3 | 2 | = 22 hrs |

WORK TABLE

| JOBS | MACHINES | | | | Idle time on M ₂ |
|------|----------------|-----|----------------|-----|--------------------------------|
| | M ₁ | | M ₂ | | |
| | IN | OUT | IN | OUT | |
| I | 0 | 3 | 3 | 9 | 3 |
| III | 3 | 7 | 9 | 16 | |
| V | 7 | 14 | 16 | 20 | |
| IV | 14 | 19 | 20 | 23 | |
| II | 19 | 26 | 26 | 28 | 3 |

Step 3 :

Total elapsed time T = 28 hrs

$$\begin{aligned}
 \text{Idle time on M}_1 &= T - \left[\text{sum of processing time of all jobs on M}_1 \right] \\
 &= 28 - 26 \\
 &= 2 \text{ hrs}
 \end{aligned}$$

$$\begin{aligned}
 \text{Idle time on M}_2 &= T - \left[\text{sum of processing time of all jobs on M}_2 \right] \\
 &= 28 - 22 \\
 &= 6 \text{ hrs} \quad (\text{CHECK : } 3 + 3 = 6)
 \end{aligned}$$

Q3. Find the sequence that minimizes total elapsed time (in hours) required to complete the following jobs on two machines M_1 and M_2 in the order M_1M_2 . Also find the minimum elapsed time and idle time for two machines

| Job | A | B | C | D | E | F |
|-------|---|---|---|---|---|---|
| M_1 | 5 | 9 | 4 | 7 | 8 | 6 |
| M_2 | 7 | 4 | 8 | 3 | 9 | 5 |

Step 1 : Finding the optimal sequence

Min time = 3 on job D on machine M_2 . Place the job at the end of the sequence

| | | | | | |
|--|--|--|--|--|----------|
| | | | | | D |
|--|--|--|--|--|----------|

Next min time = 4 on job B on machine M_2 & on job C on machine M_1 . Place the job B at the end of the sequence before D & job C at the start of the sequence

| | | | | | |
|----------|--|--|--|----------|----------|
| C | | | | B | D |
|----------|--|--|--|----------|----------|

Next min time = 4 on job A on machine M_1 & on job F on machine M_2 . Place the job A at the start of the sequence after C & job F at the end of the sequence before B

| | | | | | |
|----------|----------|--|----------|----------|----------|
| C | A | | F | B | D |
|----------|----------|--|----------|----------|----------|

OPTIMAL SEQUENCE

| | | | | | |
|----------|----------|----------|----------|----------|----------|
| C | A | E | F | B | D |
|----------|----------|----------|----------|----------|----------|

Step 2 : Work table

According to the optimal sequence

| Job | C | A | E | F | B | D | total process time |
|-------|---|---|---|---|---|---|--------------------|
| M_1 | 4 | 5 | 8 | 6 | 9 | 7 | = 39 hrs |
| M_2 | 8 | 7 | 9 | 5 | 4 | 3 | = 36 hrs |

WORK TABLE

| JOBS | MACHINES | | | | Idle time on M ₂ |
|------|----------------|-----|----------------|-----|--------------------------------|
| | M ₁ | | M ₂ | | |
| | IN | OUT | IN | OUT | |
| C | 0 | 4 | 4 | 12 | 4 |
| A | 4 | 9 | 12 | 19 | |
| E | 9 | 17 | 19 | 28 | |
| F | 17 | 23 | 28 | 33 | |
| B | 23 | 32 | 33 | 37 | |
| D | 32 | 39 | 39 | 42 | 2 |

Step 3 :

Total elapsed time T = 42 hrs

$$\begin{aligned}
 \text{Idle time on M}_1 &= T - \left[\text{sum of processing time of all jobs on M}_1 \right] \\
 &= 42 - 39 \\
 &= 3 \text{ hrs}
 \end{aligned}$$

$$\begin{aligned}
 \text{Idle time on M}_2 &= T - \left[\text{sum of processing time of all jobs on M}_2 \right] \\
 &= 42 - 36 \\
 &= 6 \text{ hrs} \quad (\text{CHECK : } 4 + 2 = 6)
 \end{aligned}$$

Q4. A book binder has one printing press , one binding machine and the manuscripts of a number of books . The time required to perform the printing and binding operations for each book are given below . Determine the order in which books should be processed in order to minimize the total time required to turn out all the books . Also find the idle time for both the machines

| Jobs | | 1 | 2 | 3 | 4 | 5 | 6 |
|-----------|--|----|-----|----|----|----|-----|
| Machine A | | 30 | 120 | 50 | 20 | 90 | 110 |
| Machine B | | 80 | 100 | 90 | 60 | 30 | 10 |

Step 1 : Finding the optimal sequence

Min time = 10 on job 6 on machine B . Place the job at the end of the sequence



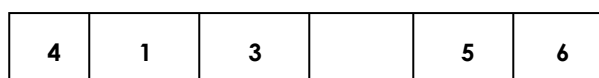
Next Min time = 20 on job 4 on machine A . Place the job at the end of the sequence



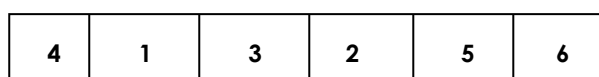
Next min time = 30 on job 1 on machine A & on job 5 on machine B . Place the job 1 at the start of the sequence after 4 & job 5 at the end of the sequence before 6



Next min time = 50 on job 3 on machine A . Place the job at the start of the sequence after 1



OPTIMAL SEQUENCE



Step 2 : Work table

According to the optimal sequence

| Job | 4 | 1 | 3 | 2 | 5 | 6 | total process time |
|----------------|----|----|----|-----|----|-----|--------------------|
| M ₁ | 20 | 30 | 50 | 120 | 90 | 110 | = 420 |
| M ₂ | 60 | 80 | 90 | 100 | 30 | 10 | = 370 |

WORK TABLE

| JOBS | MACHINES | | | | Idle time on M ₂ |
|------|----------------|-----|----------------|-----|--------------------------------|
| | M ₁ | | M ₂ | | |
| | IN | OUT | IN | OUT | |
| 4 | 0 | 20 | 20 | 80 | 20 |
| 1 | 20 | 50 | 80 | 160 | |
| 3 | 50 | 100 | 160 | 250 | |
| 2 | 100 | 220 | 250 | 350 | |
| 5 | 220 | 310 | 350 | 380 | |
| 6 | 310 | 420 | 420 | 430 | 40 |

Step 3 :

Total elapsed time T = 430 units

$$\begin{aligned}
 \text{Idle time on M}_1 &= T - \left[\text{sum of processing time of all jobs on M}_1 \right] \\
 &= 430 - 420 \\
 &= 10 \text{ units}
 \end{aligned}$$

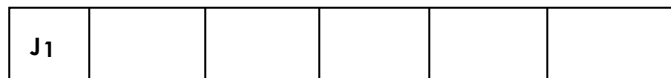
$$\begin{aligned}
 \text{Idle time on M}_2 &= T - \left[\text{sum of processing time of all jobs on M}_2 \right] \\
 &= 430 - 370 \\
 &= 60 \text{ units} \quad (\text{CHECK : } 20 + 40 = 60)
 \end{aligned}$$

Q5. In a factory there are six jobs to be performed , each of which should go through machines A and B in the order A – B . Determine the sequence for performing the jobs that would minimize the total elapsed time T . Find T and the idle time on the two machines **(OCT 2014)**

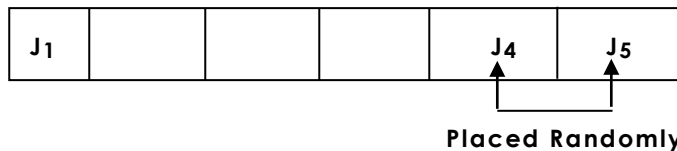
| Job | J1 | J2 | J3 | J4 | J5 | J6 |
|-----|----|----|----|----|----|----|
| MA | 1 | 3 | 8 | 5 | 6 | 3 |
| MB | 5 | 6 | 3 | 2 | 2 | 10 |

Step 1 : Finding the optimal sequence

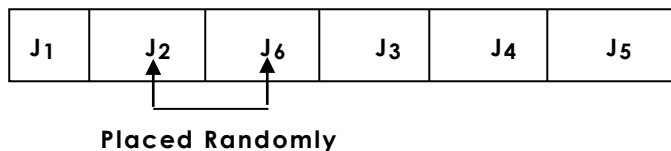
Min time = 1 on job J1 on machine MA . Place the job at the start of the sequence



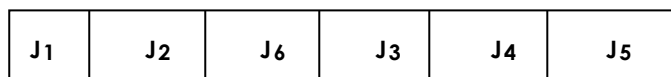
Next min time= 2 on jobs J4 & J5 on machine MB . Place the jobs at the end of the sequence randomly



Next min time = 3 on jobs J2 & J6 on machine MA and on job J3 on machine MB respectively . Place J2 & J6 at the start next to J1 randomly and J3 at the end before J4



OPTIMAL SEQUENCE



Step 2 : Work table

According to the optimal sequence

| Job | J1 | J2 | J6 | J3 | J4 | J5 | total process time |
|-----|----|----|----|----|----|----|--------------------|
| MA | 1 | 3 | 3 | 8 | 5 | 6 | = 26 hrs |
| MB | 5 | 6 | 10 | 3 | 2 | 2 | = 28 hrs |

WORK TABLE

| JOBS | MACHINES | | | | Idle time on M _B |
|----------------|----------------|-----|----------------|-----|--------------------------------|
| | M _A | | M _B | | |
| | IN | OUT | IN | OUT | |
| J ₁ | 0 | 1 | 1 | 6 | 1 |
| J ₂ | 1 | 4 | 6 | 12 | |
| J ₆ | 4 | 7 | 12 | 22 | |
| J ₃ | 7 | 15 | 22 | 25 | |
| J ₄ | 15 | 20 | 25 | 27 | |
| J ₅ | 20 | 26 | 27 | 29 | |

Step 3 :

Total elapsed time T = 29 hrs

$$\begin{aligned}
 \text{Idle time on } M_A &= T - \left(\text{sum of processing time of all 6 jobs on } M_1 \right) \\
 &= 29 - 26 \\
 &= 3 \text{ hrs}
 \end{aligned}$$

$$\begin{aligned}
 \text{Idle time on } M_B &= T - \left(\text{sum of processing time of all 6 jobs on } M_2 \right) \\
 &= 29 - 28 \\
 &= 1 \text{ hr}
 \end{aligned}$$

Step 4 : All possible optimal sequences :

J₁ - J₂ - J₆ - J₃ - J₄ - J₅

OR

J₁ - J₆ - J₂ - J₃ - J₄ - J₅

OR

J₁ - J₂ - J₆ - J₃ - J₅ - J₄

OR

J₁ - J₆ - J₂ - J₃ - J₅ - J₄

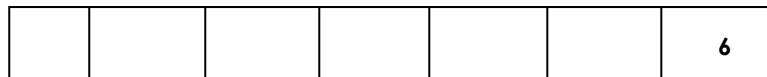
Q6. We have seven jobs each of which has to go through the machines M_1 and M_2 in the order M_1M_2 . Processing time in hours are given below

| Jobs | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|-------|---|----|----|---|----|----|---|
| M_1 | 3 | 12 | 15 | 6 | 10 | 11 | 9 |
| M_2 | 8 | 10 | 10 | 6 | 12 | 1 | 3 |

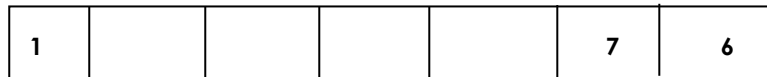
Determine a sequence of these jobs that will minimize the total elapsed T , idle time for machine M_1 and idle time for Machine M_2 **(MARCH 2017)**

Step 1 : Finding the optimal sequence

Min time = 1 on job 6 on machine M_2 . Place the job at the end of the sequence



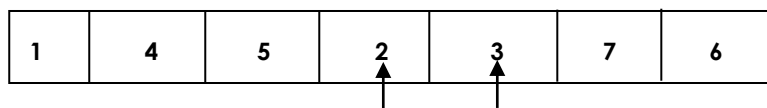
Next min time= 3 on job 1 on machine M_1 & on job 7 on machine M_2 . Place job 1 at the start of the sequence and job 7 at the end of the sequence before job 6



Next min time= 6 on job 4 on machine M_1 & M_2 . Place job 4 at the start of the sequence after job 1

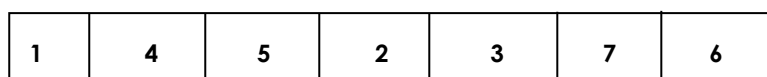


Next min time = 10 on jobs 2 & 3 on machine M_2 and on job 5 on machine M_1 respectively. Place 2 & 3 at the end of the sequence randomly before job 7. and job 5 at the start of the sequence after 1



Placed Randomly

OPTIMAL SEQUENCE



Step 2 : Work table

According to the optimal sequence

| Job | 1 | 4 | 5 | 2 | 3 | 7 | 6 | total process time |
|----------------|---|---|----|----|----|---|----|--------------------|
| M ₁ | 3 | 6 | 10 | 12 | 15 | 9 | 11 | = 66 hrs |
| M ₂ | 8 | 6 | 12 | 10 | 10 | 3 | 1 | = 50 hrs |

WORK TABLE

| JOBS | MACHINES | | | | Idle time on M ₂ |
|------|----------------|-----|----------------|-----|--------------------------------|
| | M ₁ | | M ₂ | | |
| | IN | OUT | IN | OUT | |
| 1 | 0 | 3 | 3 | 11 | 3 |
| 4 | 3 | 9 | 11 | 17 | 2 |
| 5 | 9 | 19 | 19 | 31 | |
| 2 | 19 | 31 | 31 | 41 | 5 |
| 3 | 31 | 46 | 46 | 56 | |
| 7 | 46 | 55 | 56 | 59 | 7 |
| 6 | 55 | 66 | 66 | 67 | |

Step 3 :

Total elapsed time T = 67 hrs

$$\begin{aligned}
 \text{Idle time on M}_1 &= T - \left[\text{sum of processing time of all jobs on M}_1 \right] \\
 &= 67 - 66 \\
 &= 1 \text{ hrs}
 \end{aligned}$$

$$\begin{aligned}
 \text{Idle time on M}_2 &= T - \left[\text{sum of processing time of all jobs on M}_2 \right] \\
 &= 67 - 50 \\
 &= 17 \text{ hrs} \quad (\text{CHECK : } 3 + 2 + 5 + 7 = 17)
 \end{aligned}$$

SEQUENCING

N – JOBS ON 3 MACHINES

Q1. Find the sequence that minimizes total elapsed time (in hours) required to complete the following jobs on three machines M_1 , M_2 and M_3 in the order M_1 - M_2 - M_3 . Also find the minimum elapsed time and idle time for all three machines

| Job | A | B | C | D | E |
|-------|---|---|---|---|---|
| M_1 | 5 | 7 | 6 | 9 | 5 |
| M_2 | 2 | 1 | 4 | 5 | 3 |
| M_3 | 3 | 7 | 5 | 6 | 7 |

STEP 1 : Min time on $M_1 = 5$;

Max time on $M_2 = 5$

Min time on $M_3 = 3$

Min (M_1) \geq Max (M_2) condition satisfied to convert 3 m/c's to 2 m/c's

STEP 2 : CONVERTING TO 2 FICTITIOUS M/C'S G & H

G = $M_1 + M_2$

H = $M_2 + M_3$

| Job | A | B | C | D | E |
|-----|---|---|----|----|----|
| G | 7 | 8 | 10 | 14 | 8 |
| H | 5 | 8 | 9 | 11 | 10 |

STEP 3 : OPTIMAL SEQUENCE

Min time = 5 on job A on machine H . Place the job at the end of the sequence

| | | | | | |
|--|--|--|--|--|----------|
| | | | | | A |
|--|--|--|--|--|----------|

Next min time = 8 on job B & E on machine G . Place them randomly at the start of the sequence

| | | | | | |
|----------|----------|--|--|--|----------|
| B | E | | | | A |
|----------|----------|--|--|--|----------|

Next min time = 9 on job C on machine H . Place it at the end of the sequence before A

| | | | | | |
|----------|----------|--|----------|--|----------|
| B | E | | C | | A |
|----------|----------|--|----------|--|----------|

OPTIMAL SEQUENCE

| | | | | |
|----------|----------|----------|----------|----------|
| B | E | D | C | A |
|----------|----------|----------|----------|----------|

STEP 4 : WORK TABLE

| Job | B | E | D | C | A | total process time |
|-----|---|---|---|---|---|--------------------|
| M1 | 7 | 5 | 9 | 6 | 5 | = 32 hrs |
| M2 | 1 | 3 | 5 | 4 | 2 | = 15 hrs |
| M3 | 7 | 7 | 6 | 5 | 3 | = 28 hrs |

| JOBS | M1 | | IDLE TIME | M2 | | IDLE TIME | M3 | | IDLE TIME |
|------|----|-----|--------------|----|-----|--------------|----|-----|--------------|
| | IN | OUT | | IN | OUT | | IN | OUT | |
| | | | | | | 7 | | | 8 |
| B | 0 | 7 | -- | 7 | 8 | 4 | 8 | 15 | -- |
| E | 7 | 12 | -- | 12 | 15 | 6 | 15 | 22 | 4 |
| D | 12 | 21 | -- | 21 | 26 | 1 | 26 | 32 | -- |
| C | 21 | 27 | -- | 27 | 31 | 1 | 32 | 37 | -- |
| A | 27 | 32 | 8 | 32 | 34 | 6 | 37 | 40 | -- |

STEP 5 : Total elapsed time T = 40 hrs

$$\begin{aligned}
 \text{Idle time on M}_1 &= T - \left(\text{sum of processing time of all 5 jobs on M}_1 \right) \\
 &= 40 - 32 \\
 &= 8 \text{ hrs}
 \end{aligned}$$

$$\begin{aligned}
 \text{Idle time on M}_2 &= T - \left(\text{sum of processing time of all 5 jobs on M}_2 \right) \\
 &= 40 - 15 \\
 &= 25 \text{ hrs} \quad (\text{CHECK} - 7 + 4 + 6 + 1 + 1 + 6 = 25)
 \end{aligned}$$

$$\begin{aligned}
 \text{Idle time on M}_3 &= T - \left(\text{sum of processing time of all 5 jobs on M}_3 \right) \\
 &= 40 - 28 \\
 &= 12 \text{ hrs} \quad (\text{CHECK} - 8 + 4 = 12)
 \end{aligned}$$

Q2. 1. There are five jobs , each of which is to be processed through three machines A , B and C in the order ABC . Processing time in hours are shown in the following table .Determine the optimal sequence for the five jobs and the minimum elapsed time . Also find the idle time for three machines

| Job | 1 | 2 | 3 | 4 | 5 |
|-----|---|---|----|---|---|
| A | 3 | 8 | 7 | 5 | 2 |
| B | 3 | 4 | 2 | 1 | 5 |
| C | 5 | 8 | 10 | 7 | 6 |

HOMEWORK

STEP 1 : Min time on m/c A = 2 ;

Max time on m/c B = 5

Min (m/c C) \geq Max (m/c B)

Min time on m/c C = 5 condition satisfied to convert 3 m/c's to 2 m/c's

STEP 2 : CONVERTING TO 2 FICTITIOUS M/C'S G & H

$G = A + B$

$H = B + C$

| Job | 1 | 2 | 3 | 4 | 5 |
|-----|---|----|----|---|----|
| G | 6 | 12 | 9 | 6 | 7 |
| H | 8 | 12 | 12 | 8 | 11 |

STEP 3 : OPTIMAL SEQUENCE

Min time = 6 on job 1 & 4 on machine G . Place them **RANDOMLY** at the start of the sequence

| | | | | |
|---|---|--|--|--|
| 1 | 4 | | | |
|---|---|--|--|--|

Next min time = 7 on job 5 on machine H . Place it at the end of the sequence

| | | | | |
|---|---|---|--|--|
| 1 | 4 | 5 | | |
|---|---|---|--|--|

Next min time = 9 on job 3 on machine G . Place it at the start of the sequence after 5

| | | | | |
|---|---|---|---|--|
| 1 | 4 | 5 | 3 | |
|---|---|---|---|--|

OPTIMAL SEQUENCE

| | | | | |
|---|---|---|---|---|
| 1 | 4 | 5 | 3 | 2 |
|---|---|---|---|---|

STEP 4 : WORK TABLE

| Job | | | | | | TOTAL PROCESSING | |
|-----|---|---|---|----|---|------------------|--------|
| | 1 | 4 | 5 | 3 | 2 | TIME | |
| A | 3 | 5 | 2 | 7 | 8 | = | 25 HRS |
| B | 3 | 1 | 5 | 2 | 4 | = | 15 HRS |
| C | 5 | 7 | 6 | 10 | 8 | = | 36 HRS |

| JOBS | M/c A | | IDLE TIME | M/c B | | IDLE TIME | M/c C | | IDLE TIME |
|------|-------|-----|--------------|-------|-----|--------------|-------|-----|--------------|
| | IN | OUT | | IN | OUT | | IN | OUT | |
| | | | | | | 3 | | | 6 |
| 1 | 0 | 3 | -- | 3 | 6 | 2 | 6 | 11 | -- |
| 4 | 3 | 8 | -- | 8 | 9 | 1 | 11 | 18 | -- |
| 5 | 8 | 10 | -- | 10 | 15 | 2 | 18 | 24 | -- |
| 3 | 10 | 17 | -- | 17 | 19 | 6 | 24 | 34 | -- |
| 2 | 17 | 25 | 17 | 25 | 29 | 13 | 34 | 42 | -- |

STEP 5 : Total elapsed time T = 42 hrs

$$\begin{aligned}
 \text{Idle time on M/c A} &= T - \left(\text{sum of processing time of all 5 jobs on M/c A} \right) \\
 &= 42 - 25 \\
 &= 17 \text{ hrs}
 \end{aligned}$$

$$\begin{aligned}
 \text{Idle time on M/c B} &= T - \left(\text{sum of processing time of all 5 jobs on M/c B} \right) \\
 &= 42 - 15 \\
 &= 27 \text{ hrs} \quad (\text{CHECK} - 3 + 2 + 1 + 2 + 6 + 13 = 27)
 \end{aligned}$$

$$\begin{aligned}
 \text{Idle time on M/c C} &= T - \left(\text{sum of processing time of all 5 jobs on M/c C} \right) \\
 &= 42 - 36 \\
 &= 6 \text{ hrs}
 \end{aligned}$$

Q2. 2. There are five jobs , each of which is to be processed through three machines A , B and C in the order ABC . Processing time in hours are shown in the following table .Determine the optimal sequence for the five jobs and the minimum elapsed time . Also find the idle time for three machines

| Job | 1 | 2 | 3 | 4 | 5 |
|-----|---|---|---|---|----|
| A | 3 | 8 | 7 | 5 | 4 |
| B | 4 | 5 | 1 | 2 | 3 |
| C | 7 | 9 | 5 | 6 | 10 |

(JULY 2016)

STEP 1 : Min time on m/c A = 3 ;
 Max time on m/c B = 5 Min (m/c C) ≥ Max (m/c B)
 Min time on m/c C = 5 condition satisfied to convert 3 m/c's to 2 m/c's

STEP 2 : CONVERTING TO 2 FICTITIOUS M/C'S G & H

G = A + B

H = B + C

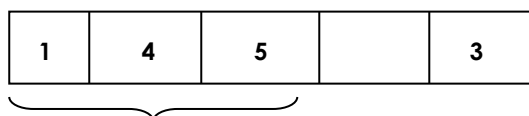
| Job | 1 | 2 | 3 | 4 | 5 |
|-----|----|----|---|---|----|
| G | 7 | 13 | 8 | 7 | 7 |
| H | 11 | 14 | 6 | 8 | 13 |

STEP 3 : OPTIMAL SEQUENCE

Min time = 6 on job 3 on machine H . Place the job at the end of the sequence

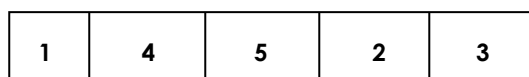


Next min time = 7 on jobs 1 , 4 & 5 on machine G . Place them randomly at the start of the sequence



RANDOM

OPTIMAL SEQUENCE



STEP 4 : WORK TABLE

| Job | 1 | 4 | 5 | 2 | 3 | TOTAL PROCESSING | |
|-----|---|---|----|---|---|------------------|--------|
| | | | | | | TIME | |
| A | 3 | 5 | 4 | 8 | 7 | = | 27 hrs |
| B | 4 | 2 | 3 | 5 | 1 | = | 15 hrs |
| C | 7 | 6 | 10 | 9 | 5 | = | 37 hrs |

| JOBS | M/c A | | IDLE TIME | M/c B | | IDLE TIME | M/c C | | IDLE TIME |
|------|-------|-----|--------------|-------|-----|--------------|-------|-----|--------------|
| | IN | OUT | | IN | OUT | | IN | OUT | |
| | | | | | | 3 | | | 7 |
| 1 | 0 | 3 | -- | 3 | 7 | 1 | 7 | 14 | -- |
| 4 | 3 | 8 | -- | 8 | 10 | 2 | 14 | 20 | -- |
| 5 | 8 | 12 | -- | 12 | 15 | 5 | 20 | 30 | -- |
| 2 | 12 | 20 | -- | 20 | 25 | 2 | 30 | 39 | -- |
| 3 | 20 | 27 | 17 | 27 | 28 | 16 | 39 | 44 | -- |

STEP 5 : Total elapsed time T = 44 hrs

$$\begin{aligned}
 \text{Idle time on M/c A} &= T - \left(\text{sum of processing time of all 5 jobs on M/c A} \right) \\
 &= 44 - 27 \\
 &= 17 \text{ hrs}
 \end{aligned}$$

$$\begin{aligned}
 \text{Idle time on M/c B} &= T - \left(\text{sum of processing time of all 5 jobs on M/c B} \right) \\
 &= 44 - 15 \\
 &= 29 \text{ hrs} \quad (\text{CHECK} - 3 + 1 + 2 + 5 + 2 + 16 = 29)
 \end{aligned}$$

$$\begin{aligned}
 \text{Idle time on M/c C} &= T - \left(\text{sum of processing time of all 5 jobs on M/c C} \right) \\
 &= 44 - 37 \\
 &= 7 \text{ hrs}
 \end{aligned}$$

Q3. Determine the optimal sequence involving 5 jobs and three machines M_1 , M_2 and M_3 . The jobs are processed on three machines in the order $M_1M_2M_3$. Also find the minimum total elapsed time T and idle time for three machines. Processing time in minutes are

| Job | J1 | J2 | J3 | J4 | J5 |
|-----|----|----|----|----|----|
| M1 | 7 | 12 | 11 | 9 | 8 |
| M2 | 8 | 9 | 5 | 6 | 7 |
| M3 | 11 | 13 | 9 | 10 | 14 |

STEP 1 : Min time on $M_1 = 7$;

Max time on $M_2 = 9$

Min time on $M_3 = 9$

Min (M_3) \geq Max (M_2) condition satisfied to convert 3 m/c's to 2 m/c's

STEP 2 : CONVERTING TO 2 FICTITIOUS M/C'S G & H

$$G = M_1 + M_2$$

$$H = M_2 + M_3$$

| Job | J1 | J2 | J3 | J4 | J5 |
|-----|----|----|----|----|----|
| G | 15 | 21 | 16 | 15 | 15 |
| H | 19 | 22 | 14 | 16 | 21 |

STEP 3 : OPTIMAL SEQUENCE

Min time = 14 on job J3 on machine H. Place the job at the end of the sequence

| | | | | |
|--|--|--|--|-----------|
| | | | | J3 |
|--|--|--|--|-----------|

Next min time = 15 on job J1, J4 & J5 on machine G. Place them randomly at the start of the sequence

| | | | | |
|-----------|-----------|-----------|--|-----------|
| J1 | J4 | J5 | | J3 |
|-----------|-----------|-----------|--|-----------|

OPTIMAL SEQUENCE

| | | | | |
|-----------|-----------|-----------|-----------|-----------|
| J1 | J4 | J5 | J2 | J3 |
|-----------|-----------|-----------|-----------|-----------|

STEP 4 : WORK TABLE

| Job | J1 | J4 | J5 | J2 | J3 | total processing time |
|-----|----|----|----|----|----|-----------------------|
| M1 | 7 | 9 | 8 | 12 | 11 | = 47 min |
| M2 | 8 | 6 | 7 | 9 | 5 | = 35 min |
| M3 | 11 | 10 | 14 | 13 | 9 | = 57 min |

| JOBS | M1 | | IDLE TIME | M2 | | IDLE TIME | M3 | | IDLE TIME |
|------|----|-----|-----------|----|-----|-----------|----|-----|-----------|
| | IN | OUT | | IN | OUT | | IN | OUT | |
| | | | | | | 7 | | | 15 |
| J1 | 0 | 7 | -- | 7 | 15 | 1 | 15 | 26 | -- |
| J4 | 7 | 16 | -- | 16 | 22 | 2 | 26 | 36 | -- |
| J5 | 16 | 24 | -- | 24 | 31 | 5 | 36 | 50 | -- |
| J2 | 24 | 36 | -- | 36 | 45 | 2 | 50 | 63 | -- |
| J3 | 36 | 47 | 25 | 47 | 52 | 20 | 63 | 72 | -- |

STEP 5 : Total elapsed time $T = 72$ min

$$\begin{aligned} \text{Idle time on } M_1 &= T - \left[\text{sum of processing time of all 5 jobs on } M_1 \right] \\ &= 72 - 47 \\ &= 25 \text{ min} \end{aligned}$$

$$\begin{aligned} \text{Idle time on } M_2 &= T - \left[\text{sum of processing time of all 5 jobs on } M_2 \right] \\ &= 72 - 35 \\ &= 37 \text{ min} \quad (\text{CHECK} - 7 + 1 + 2 + 5 + 2 + 20 = 37) \end{aligned}$$

$$\begin{aligned} \text{Idle time on } M_3 &= T - \left[\text{sum of processing time of all 5 jobs on } M_3 \right] \\ &= 72 - 57 \\ &= 15 \text{ min} \end{aligned}$$

NOTE : ALL POSSIBLE OPTIMAL SEQUENCES

- i) J1 - J4 - J5 - J2 - J3
- ii) J1 - J5 - J4 - J2 - J3
- iii) J4 - J1 - J5 - J2 - J3
- iv) J4 - J5 - J1 - J2 - J3
- v) J5 - J1 - J4 - J2 - J3
- vi) J5 - J4 - J4 - J2 - J3

However there are 6 possible different optimal sequences but the total elapsed time T and the idle time on all three m/c's will remain the same

Q4. A machine operator has to perform three operations : turning , threading and finishing on 6 different jobs . The time required to perform these operations (in minutes) for each job is given below . Determine the order in which the jobs should be processed in order to minimize the total time required to complete all the jobs . Also find the idle time three operation

| Jobs | 1 | 2 | 3 | 4 | 5 | 6 |
|-----------|----|----|---|----|---|----|
| Turning | 3 | 12 | 5 | 2 | 9 | 11 |
| Threading | 8 | 6 | 4 | 6 | 3 | 1 |
| Finishing | 13 | 14 | 9 | 12 | 8 | 13 |

STEP 1 : Min time on Turning (m/c A) = 2 ;
 Max time on Threading(m/c B) = 8
 Min time on Finishing (m/c C) = 8 Min (m/c C) ≥ Max (m/c B)

STEP 2 : CONVERTING TO 2 FICTITIOUS M/C'S G & H

G = A + B
 H = B + C

| Job | 1 | 2 | 3 | 4 | 5 | 6 |
|-----|----|----|----|----|----|----|
| G | 11 | 18 | 9 | 8 | 12 | 12 |
| H | 21 | 20 | 13 | 18 | 11 | 14 |

STEP 3 : OPTIMAL SEQUENCE

Min time = 8 on job 4 on machine G . Place the job at the start of the sequence

| | | | | | |
|---|--|--|--|--|--|
| 4 | | | | | |
|---|--|--|--|--|--|

Next min time = 9 on job 3 on machine G . Place it at start of sequence after 4

| | | | | | |
|---|---|--|--|--|--|
| 4 | 3 | | | | |
|---|---|--|--|--|--|

Next min time = 11 on job 1 on G & on job 5 on H . Place job 1 at start of sequence after 3 and job 5 at the end of the sequence

| | | | | | |
|---|---|---|--|--|---|
| 4 | 3 | 1 | | | 5 |
|---|---|---|--|--|---|

Next min time = 12 on job 6 on machine G . Place it at start of sequence after 1

| | | | | | |
|---|---|---|---|--|---|
| 4 | 3 | 1 | 6 | | 5 |
|---|---|---|---|--|---|

OPTIMAL SEQUENCE

| | | | | | |
|---|---|---|---|---|---|
| 4 | 3 | 1 | 6 | 2 | 5 |
|---|---|---|---|---|---|

STEP 4 : WORK TABLE

| M/C | JOBS | | | | | TOTAL PROCESSING | |
|-----------|------|---|----|----|----|------------------|----------|
| | 4 | 3 | 1 | 6 | 2 | 5 | TIME |
| CUTTING | 2 | 5 | 3 | 11 | 12 | 9 | = 42 min |
| THREADING | 6 | 4 | 8 | 1 | 6 | 3 | = 28 min |
| FINISHING | 12 | 9 | 13 | 13 | 14 | 8 | = 69 min |

| JOBS | CUTTING | | IDLE TIME | THREADING | | IDLE TIME | FINISHING | | IDLE TIME |
|------|---------|-----|--------------|-----------|-----|--------------|-----------|-----|--------------|
| | IN | OUT | | IN | OUT | | IN | OUT | |
| | | | | | | 2 | | | 8 |
| 4 | 0 | 2 | -- | 2 | 8 | -- | 8 | 20 | -- |
| 3 | 2 | 7 | -- | 8 | 12 | -- | 20 | 29 | -- |
| 1 | 7 | 10 | -- | 12 | 20 | 1 | 29 | 42 | -- |
| 6 | 10 | 21 | -- | 21 | 22 | 11 | 42 | 55 | -- |
| 2 | 21 | 33 | -- | 33 | 39 | 3 | 55 | 69 | -- |
| 5 | 33 | 42 | 35 | 42 | 45 | 32 | 69 | 77 | -- |

STEP 5 : Total elapsed time T = 77 min

$$\begin{aligned} \text{Idle time on CUTTING} &= T - \left(\text{sum of processing time of all 5 jobs on M/c A} \right) \\ &= 77 - 42 \\ &= 35 \text{ min} \end{aligned}$$

$$\begin{aligned} \text{Idle time on THREADING} &= T - \left(\text{sum of processing time of all 5 jobs on M/c B} \right) \\ &= 77 - 28 \\ &= 49 \text{ min} \quad (\text{CHECK} - 2 + 1 + 11 + 3 + 32 = 49) \end{aligned}$$

$$\begin{aligned} \text{Idle time on FINISHING} &= T - \left(\text{sum of processing time of all 5 jobs on M/c C} \right) \\ &= 77 - 69 \\ &= 8 \text{ min} \end{aligned}$$