PAPER – II : MODEL PAPER – 08

(SPECIMEN PAPER)

MATHEMATICS & STATISTICS

COMMERCE

TIME : 1 HR 30 MIN **MARKS : 40**

Q4. Attempt any six of the following **(12)**

01. Oliver spends 30% of his income on food items and 15% on conveyance . If he spent ₹ 1800 on conveyance , find his expenditure on food items during the same month

SOLUTION

$$\frac{\text{Expenditure on food}}{\text{Expenditure on conveyance}} = \frac{30}{15}$$

$$\frac{\text{Expenditure on food}}{1800} = 2$$

Expenditure on food = ₹ 3600

02. an annuity immediate is to be paid for certain number of years at 12% p.a. Its present value is ₹ 5000 and the accumulated value is ₹ 10,000 . Find the amount of each annuity payment

SOLUTION

P = ₹ 5000 , A = ₹ 10000 , i = 0.12

$$\frac{1}{P} - \frac{1}{A} = \frac{i}{C}$$

$$\frac{1}{5000} - \frac{1}{10000} = \frac{0.12}{C}$$

$$\frac{2-1}{10000} = \frac{0.12}{C}$$

C = 0.12 x 10000 = ₹ 1200

03.

A person holding a life policy of ₹ 60,000 for a term of 25 years wishes to discontinue after paying premium for 8 years at the rate of ₹ 58 per thousand per annum . What paid up value will he get on the policy

SOLUTION

Paid Up Value

$$= \frac{\text{No. of Premiums paid}}{\text{total no. of Premiums originally stipulated in the policy}} \times \text{Policy value}$$

$$= \frac{8}{25} \times 60{,}000$$

= ₹ 19,200

04.

a certain sum due 3 months hence is 21/20 of the present worth . What is the rate of interest

SOLUTION

F.V. = P.W.+ INT ON P.W. FOR 3 MONTHS @ r %P.A

$$\frac{21}{20}\text{P.W.} = \text{P.W.} + \text{P.W.} \times \frac{3}{12} \times \frac{r}{100}$$

$$\frac{21}{20}\text{P.W.} - \text{P.W.} = \text{P.W.} \times \frac{r}{400}$$

$$\frac{1}{20}\text{P.W.} = \text{P.W.} \times \frac{r}{400}$$

$$\frac{1}{20} = \frac{r}{400}$$

r = 20% p.a.

05.

coefficient of correlation between variables X and Y is 0.3 and their covariance is 12 . The variance of X is 9 . Find standard deviation of Y

SOLUTION

$r = 0.3$, $cov(x,y) = 12$, $\sigma x^2 = 9$,

$$r = \frac{cov(x,y)}{\sigma x \cdot \sigma y} \qquad \sigma y = \frac{12}{3 \times 0.3}$$

$$0.3 = \frac{12}{3 \times \sigma y} \qquad = \frac{120}{3 \times 3}$$

$$= \frac{40}{3} = 13.33$$

06.

Age Group	No. of persons	Deaths
0 – 10	600	18
10 – 25	1000	5
25 – 65	3000	24
65 & above	400	20

Compute Age specific death rate

SOLUTION

Age Group	SDR $= \frac{D}{P} \times 1000$ (per 000)
0 – 10	$\frac{18}{600} \times 1000 = 30$
10 – 25	$\frac{5}{1000} \times 1000 = 5$
25 – 65	$\frac{24}{3000} \times 1000 = 8$
65&above	$\frac{20}{400} \times 1000 = 50$

07 . Bring out the fallacy if any in the following statement

"the mean of the binomial dist. is 15 and standard deviation 5"

SOLUTION

In Binomial distribution

Mean = np = 15

Variance = npq = 25

$$\frac{npq}{np} = \frac{25}{15}$$

$q = \frac{5}{3} > 1$. This is not possible

Hence the given statement is INCORRECT

08.

Three person X , Y and Z started a business in partnership by investing ₹ 24,000 , ₹ 52,000 and ₹ 80,000 respectively . At the end of the year they Y earned a profit of ₹ 2,600 in the business . Find the X's share of profit

SOLUTION

STEP 1 :

Profits will be shared in the **'RATIO OF THE INVESTMENT'**

	X		Y		Z	
=	24,000	:	52,000	:	80,000	
=	24	:	52	:	80	
=	6	:	13	:	20	TOTAL = 39

STEP 2 :

$$\frac{X\text{'s share of profit}}{Y\text{'s share of profit}} = \frac{6}{13}$$

$$\frac{X\text{'s share of profit}}{2600} = \frac{6}{13}$$

$$X\text{'s share of profit} = \frac{6}{13} \times 2600 = ₹\ 1200$$

Q5.

(A) Attempt any TWO of the following (06)

01.

r = 0.8 , $\Sigma xiyi = 60$, $\sigma y = 2.5$, $\Sigma xi^2 = 90$,
where xi and yi are the deviations from the respective means . Find the number of pair of observations

SOLUTION

$r = 0.8$, $\Sigma(x-\bar{x})(y-\bar{y}) = 60$, $\Sigma(x-\bar{x})^2 = 90$,

$\sigma y = 2.5$

$$\sigma y = \sqrt{\frac{\Sigma(y-\bar{y})^2}{n}}$$

$$2.5 = \sqrt{\frac{\Sigma(y-\bar{y})^2}{n}}$$

$$\sqrt{\Sigma(y-\bar{y})^2} = 2.5\sqrt{n}$$

Now

$$r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}}$$

$$0.8 = \frac{60}{\sqrt{90} \times 2.5\sqrt{n}}$$

$$\sqrt{n} = \frac{60}{\sqrt{90} \times 2.5 \times 0.8}$$

$$\sqrt{n} = \frac{60}{\sqrt{90} \times 2}$$

$$\sqrt{n} = \frac{30}{\sqrt{90}}$$

$$n = \frac{900}{90}$$

$$= \mathbf{10}$$

02.

in binomial distribution with five independent trials , probabilities of one and two successes are 0.4096 and 0.2048 respectively . Find the probability of success

SOLUTION

$n = 5$

r.v.x = no of successes = 0, 1, 2, ... , 5

$X \sim B(5,p)$

$P(x = 1) = 0.4096 \quad \therefore {}^5C_1p^1q^4 = 0.4096$

$P(x = 2) = 0..2048 \quad \therefore {}^5C_2p^2q^3 = 0.2048$

$$\frac{{}^5C_1p^1q^4}{{}^5C_2p^2q^3} = \frac{0.4096}{0.2048}$$

$$\frac{5\,p^1q^4}{10\,p^2q^3} = 2$$

$$\frac{q}{2p} = 2 \quad \therefore \quad q = 4p$$

$$1 - p = 4p$$

$$1 = 5p$$

$$p = {}^1/_5$$

03.

stocks in shop and godown costing ₹ 50,000 and ₹ 1,00,000 respectively were insured through an agent who was paid 12% of the total premium . If the former was insured for 80% and the later for 60% of the value , find the agents commission when the rate of premium was 80 paise percent less 20%

SOLUTION

Stock
Value = 50,000
Insured value = $\frac{80}{100} \times 50{,}000$ = ₹ 40,000

Godown
Value = 1,00,000
Insured value = $\frac{60}{100} \times 1{,}00{,}000$ = ₹ 60,000

Total insured value = 1,00,000

Rate of premium = 80 paise percent
= 0.80%

Premium = $\frac{8}{1000} \times 1{,}00{,}000$
= 800
Less 20% 160
Net premium = 640

Agents commission @ 12% = $\frac{12}{100} \times 640$
= ₹ 76.80

Q5.

(B) Attempt any TWO of the following (08)

01 income of Mr Shah , Mr Patel and Mr Mehta are in the ratio 1 : 2 : 3 while their expenditures are in the ratio 2 : 3 :4 . If Mr Shah saves 20% of his income , find the ratio of their savings

SOLUTION

STEP 1

	SHAH	PATEL	MEHTA
Salary	x	2x	3x
Expense's	2y	3y	4y
Saving's	x – 2y	2x – 3y	3x – 4y

STEP 2

Mr Shah saves 20% of his income

$$x - 2y = \frac{20}{100}x$$

$$x - \frac{20x}{100} = 2y$$

$$\frac{80x}{100} = 2y$$

$$x = \frac{5y}{2}$$

STEP 3

SHAH'S SAVINGS	PATEL'S SAVING'S	MEHTA 'S SAVING 'S
= x – 2y	= 2x – 3y	= 3x – 4y
$= \frac{5y}{2} - 2y$	$= 2.\frac{5y}{2} - 3y$	$= 3.\frac{5y}{2} - 4y$
$= \frac{y}{2}$	= 2y	$= \frac{7y}{2}$

STEP 4 : RATIO OF SAVING'S

SHAH	:	PATEL	:	MEHTA
$\frac{y}{2}$	:	2y	:	$\frac{7y}{2}$
$\frac{1}{2}$	:	2	:	$\frac{7}{2}$
1	:	4	:	7

02. the following is the pdf of a continuous random variable X

$$f(x) = \frac{x^2}{3} \quad ; \quad -1 < x < 2$$

$$= 0 \quad ; \quad \text{otherwise}$$

(i) Find cdf of X

(ii) Hence find $P(X > 0)$; $P(1 < X < 2)$

cdf

i) $F(x) = \int_{-1}^{x} \frac{x^2}{3}\,dx$

$$= \left(\frac{x^3}{9}\right)_{-1}^{x}$$

$$= \left(\frac{x^3}{9}\right) - \left(\frac{-1}{9}\right)$$

$$= \frac{x^3}{9} + \frac{1}{9}$$

ii) $P(X > 0) = 1 - P(x \le 0)$

$$= 1 - F(0)$$

$$= 1 - \left(\frac{0}{9} + \frac{1}{9}\right)$$

$$= 1 - \frac{1}{9}$$

$$= \frac{8}{9}$$

iii) $P(1 < X < 2) = F(2) - F(1)$

$$= \left(\frac{8}{9} + \frac{1}{9}\right) - \left(\frac{1}{9} + \frac{1}{9}\right)$$

$$= \frac{9}{9} - \frac{2}{9}$$

$$= \frac{7}{9}$$

03.

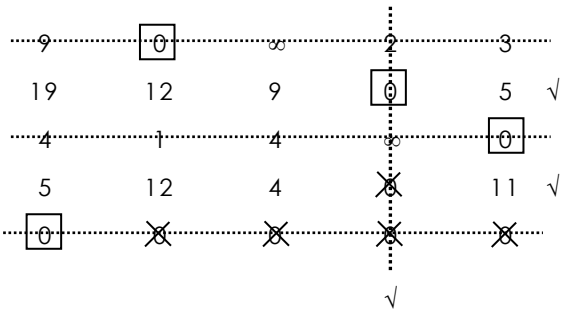
there are four capsulation machines available in a pharmaceutical company , namely C_1 , C_2 , C_3 , C_4 and company has five types of antibiotic products A , B , C , D , E to be filled in capsules . The cost of performance of various products on different capsulation machines is given below in the matrix .

		ANTIBIOTICS				
		A	B	C	D	E
CAPSUALTION MACHINES	C_1	27	18	---	20	21
	C_2	31	24	21	12	17
	C_3	20	17	20	---	16
	C_4	21	28	20	16	27

Find the optimal assignments of antibiotic products to different capsulation machines if capsule C cannot be filled on machine C_1 and capsule D cannot be filled on machine C_3

27	18	∞	20	21	– C cannot be filled on C_1 and D cannot be filled
31	24	21	12	17	on C_3 . Hence ∞
20	17	20	∞	16	– Adding a DUMMY machine C_5 with zero cost to
21	28	20	16	27	BALANCE the matrix
0	0	0	0	0	
9	0	∞	2	3	
19	12	9	0	5	Reducing the matrix using ROW MINIMUM
4	1	4	∞	0	
5	12	4	0	11	
0	0	0	0	0	
9	[0]	∞	2	3	
19	12	9	[0]	5	Allocation using SINGLE ZERO ROW – COLUMN METHOD
4	1	4	∞	[0]	
5	12	4	~~0~~	11	allocation INCOMPLETE
[0]	~~0~~	~~0~~	~~0~~	~~0~~	

REVISE THE MATRIX



STEP 1 – Drawing minimum lines to cover ALL '0's

9	0	∞	6	3
15	8	5	0	1
4	1	4	∞	0
1	8	0	0	7
0	0	0	4	0

STEP 2 – REVISE THE MATRIX

reduce all the uncovered elements by it minimum and add the same at the intersection

9	[0]	∞	6	3
15	8	5	[0]	1
4	1	4	∞	[0]
1	8	[0]	~~0~~	7
[0]	~~0~~	~~0~~	4	~~0~~

Allocation once again using SINGLE ZERO ROW – COLUMN METHOD

Since every row and every column contains an assigned zero ,

the ASSIGNMNET PROBLEM IS SOLVED

OPTIMAL ASSIGNMENT : C_1 – B , C_2 – D , C_3 – E , C_4 – C , C_5 – A (DUMMY) ,

min cost = 18 + 12 + 16 + 20

= 66

Q6.

(A) Attempt any TWO of the following (06)

01. Determine I_1 ; I_2 ; I_3 given that $I_0 = 100$; $q_0 = 0.10$; $q_1 = 1/9$; $p_2 = 15/16$

STEP 1 :

$$p_x = 1 - q_x$$

$$p_0 = 1 - q_0$$

$$= 1 - 0.1$$

$$\mathbf{p_0 = 0.9}$$

$$p_1 = 1 - q_1$$

$$= 1 - \frac{1}{9}$$

$$\mathbf{p_1 = \frac{8}{9}}$$

STEP 2 :

$$p_x = \frac{I_{x+1}}{I_x}$$

$p_0 = \frac{I_1}{I_0}$	$p_1 = \frac{I_2}{I_1}$	$p_2 = \frac{I_3}{I_2}$
$0.9 = \frac{I_1}{100}$	$\frac{8}{9} = \frac{I_2}{90}$	$\frac{15}{16} = \frac{I_3}{80}$
$\mathbf{I_1 = 90}$	$\mathbf{I_2 = 80}$	$\mathbf{I_3 = 75}$

2. the coefficient of rank correlation of ranks obtained by 10 students in Statistics and Mathematics was found to be 0.2. It was later found that the difference in ranks in the two subjects obtained by one of the students was wrongly taken as 9 instead of 7 . Find the correct coefficient of rank correlation

SOLUTION

N = 10 , R = 0.2

Incorrect d = 9 while correct d = 7

STEP – 1

$$\mathbf{R} = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

$$0.2 = 1 - \frac{6\Sigma d^2}{10(100 - 1)}$$

$$0.2 = 1 - \frac{6\Sigma d^2}{10(99)}$$

$$0.2 = 1 - \frac{\Sigma d^2}{165}$$

$$\frac{\Sigma d^2}{165} = 1 - 0.2$$

$$\frac{\Sigma d^2}{165} = 0.8$$

$$\Sigma d^2 = 132$$

STEP 2

$$\Sigma d^2 = 132$$

$$-9^2 = -81$$

$$+7^2 = +49$$

$$-32$$

$$\Sigma d^2_{\text{correct}} = 100$$

STEP 3

$$\mathbf{R}_{\text{correct}} = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

$$= 1 - \frac{6(100)}{10(100 - 1)}$$

$$= 1 - \frac{6(100)}{10(99)}$$

$$= 1 - \frac{20}{33}$$

$$= \frac{13}{33}$$

$$= 0.3939$$

03.

a shopkeeper sold a TV set for ₹ 8,832/- after allowing 8% trade discount and 4% cash discount . If he made 38% profit , find the cost price and the market price of the TV set

SOLUTION

PART A :

List Price	= ₹	100
Less 8% T.D.	–	8
Invoice Price	= ₹	92
Less 4% C.D.	–	3.68
Net Selling Price	= ₹	88.32

Now When ;
Net SP = 88.32 ; List. Price = 100

Net SP =₹ 8832 ; List Price $= \frac{100}{\cancel{8832}} \times \frac{100}{\cancel{88.32}}$

= ₹ 10,000

PART B :

SP = CP+ Profit @ 38%

8832 = CP+ $\frac{38}{100}$ CP

8832 = $\frac{138}{100}$ CP

CP = $\frac{64}{8832 \times 100}{\cancel{138}}$

= ₹ 6,400

Q6.

(B) Attempt any TWO of the following (08)

01

the following results were obtained from record of age X and systolic blood pressure (Y) of a group of 10 men

	X	y
Mean	50	140
Variance	130	165

, $\Sigma(x-\bar{x})(y-\bar{y}) = 1220$

Obtain the regression line to estimate blood pressure of a man of age 40 years

SOLUTION

$$byx = \frac{cov(x,y)}{\sigma x^2}$$

$$= \frac{\frac{\Sigma((x-\bar{x})(y-\bar{y})}{n}}{\sigma x^2}$$

$$= \frac{\frac{1220}{10}}{130}$$

$$= \frac{122}{130}$$

= 0.9387

LOG CALC
2.0864
– 2.1139
AL 1.9725
0.9387

Y on X

$y - \bar{y} = byx\,(x - \bar{x})$

y – 140 = 0.9387(x – 50)

y – 140 = 0.9387x – 46.935

y = 0.9387x + 93.065

Put x = 40

y = 0.9387(40) + 93.065

y = 37.548 + 93.065

= 130.613

ans : estimated blood pressure of a man aged 40 years is 130.613

02 Suppose X is a random variable with pdf

$$f(x) = \frac{k}{\sqrt{x}} \quad ; \quad 0 < x < 4$$

$$= 0 \quad ; \quad \text{otherwise}$$

Find k & E(X)

i) $$\int_0^4 \frac{k}{\sqrt{x}}\,dx = 1$$

$$k \int_0^4 \frac{1}{\sqrt{x}}\,dx = 1$$

$$k \left[2\sqrt{x}\right]_0^4 = 1$$

$$k \left[2\sqrt{4}\right] - \left[0\right] = 1$$

$$k(4) = 1$$

$$k = 1/4$$

Hence

the pdf of a continuous random variable X is given by

$$f(x) = \frac{1}{4\sqrt{x}} \quad ; \quad 0 < x < 4$$

$$= 0 \quad ; \quad \text{otherwise}$$

ii) $$E(X) = \int_0^4 x.f(x)\,dx$$

$$= \int_0^4 x \cdot \frac{1}{4\sqrt{x}}\,dx$$

$$= \int_0^4 \frac{x}{4\sqrt{x}}\,dx$$

$$= \int_0^4 \frac{\sqrt{x}}{4}\,dx$$

$$= \frac{1}{4}\int_0^4 x^{1/2}\,dx$$

$$= \frac{1}{4}\left[\frac{x^{3/2}}{3/2}\right]_0^4$$

$$= \frac{1}{4}\frac{2}{3}\left[x^{3/2}\right]_0^4$$

$$= \frac{1}{6}\left[4^{3/2}\right]$$

$$= \frac{1}{6}\,2^3$$

$$= \frac{8}{6} = \frac{4}{3}$$

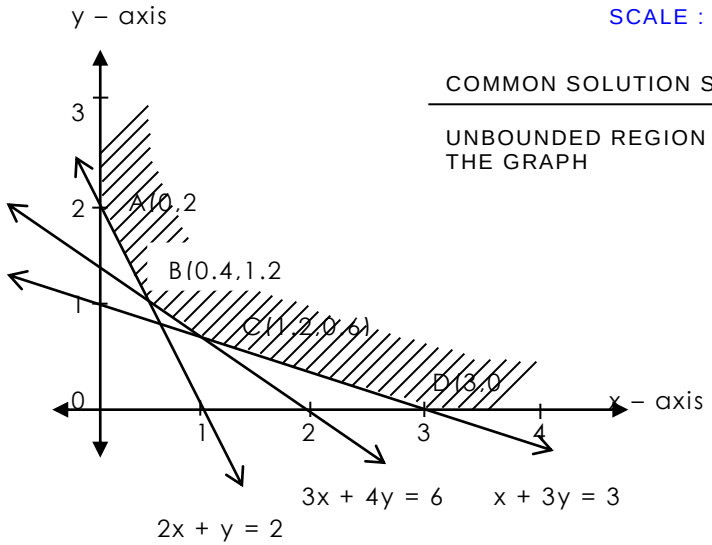
03. Minimize $z = 2x + 4y$, subject to

$2x + y \geq 2$, $x + 3y \geq 3$, $3x + 4y \geq 6$, $x , y \geq 0$

STEP 1 :

$2x + y \geq 2$	$2x + y = 2$ cuts x–axis at (1,0) cuts y–axis at (0,2)	Put (0,0) in $2x + y \geq 2$ $0 \geq 2$ (NOT SATISFIED) SS : NON-ORIGIN SIDE
$x + 3y \geq 3$	$x + 3y = 3$ cuts x–axis at (3,0) cuts y–axis at (0,1)	Put (0,0) in $x + 3y \geq 3$ $0 \geq 3$ (NOT SATISFIED) SS : NON-ORIGIN SIDE
$3x + 4y \geq 6$	$3x + 4y = 6$ cuts x–axis at (2,0) cuts y–axis at (0,1.5)	Put (0,0) in $3x + 4y \geq 6$ $0 \geq 6$ (NOT SATISFIED) SS : NON-ORIGIN SIDE

STEP 2



COMMON SOLUTION SET

UNBOUNDED REGION AS SHOWN IN THE GRAPH

FOR B :

$$\begin{array}{rcll} 3x + 4y & = & 6 & \ldots\ldots (1) \\ 4x\ 2x + y & = & 2 & \ldots\ldots\ldots (2) \end{array}$$

$$\begin{array}{rcl} 3x + 4y & = & 6 \\ 8x + 4y & = & 8 \\ - \quad - & & - \\ \hline 5x & = & 2 \end{array}$$

$x = 0.4$

subs in (2)

$y = 1.2$

B(0.4,1.2)

FOR C :

$$\begin{array}{rcll} 3x + 4y & = & 6 & \ldots\ldots (1) \\ 3x\ x + 3y & = & 3 & \ldots\ldots (2) \end{array}$$

$$\begin{array}{rcl} 3x + 4y & = & 6 \\ 3x + 9y & = & 9 \\ - \quad - & & - \\ \hline 5y & = & 3 \end{array}$$

$y = 0.6$

subs in (2)

$x = 1.2$

C(1.2,0.6)

STEP 3 :

CORNERS	Z	=	2x + 4y			
A(0,2)	Z	=	2(0) + 4(2)	=	8	
B(0.4,1.2)	Z	=	2(0.4)+4(1.2)	=	0.8+4.8	= 5.6
C(1.2,0.6)	Z	=	2(1.2)+4(0.6)	=	2.4+2.4	= 4.8
D(3,0)	Z	=	2(3) + 4(0)	=	6	

STEP 4 :

Optimal Solution : Z min = 4.8 at (1.2,0.6)