Marks : 40	SYJC March' 19 Subject : Maths – I Logic & Differentiation	Duration : 1.5 Hours.

Q.1. Attempt any One : (2 Marks each) **(04)**

1. Let p: $\sqrt{2}$
   Let q : 4 – 3i is a complex number
   Then the symbolic form of the given statement is $p \vee q$.
   The truth values of p and q are F and T respectively.
   $\therefore$ the truth value of $p \vee q$ is T. [$F \vee T \equiv T$]

2. Let p: 6 is an even number
   Let q : Pune is a harbour
   Then the symbolic form of the given statement is $p \vee q$.
   The truth values of p and q are T and F respectively.
   $\therefore$ the truth value of $p \vee q$ is T. [$T \vee F \equiv T$]

3. Let p : Re(z) < $|z|$, where z is a complex number.
   The truth value of p is T.
   Therefore, the truth value of $\sim p$ is F.

Q.2. Attempt any Four : (3 Marks each) **(12)**

1. Let p: Party's name must be given in a credit transaction
   Let q: single entry system is costly in a credit transaction.
   Then the symbolic form of the given statement is $p \wedge q$.
   The truth values of p and q are T and F respectively.
   $\therefore$ the truth value of $p \wedge q$ is F. [$T \wedge F \equiv F$]

2. (i) Yuvraj has sufficient money and he will buy a car.
   The symbolic form of given statement is $p \wedge q$)

p	q	$p \wedge q$
T	T	T

$\therefore$truth value of the given statement is 'T'.

(ii) If Yuvraj has sufficient money then he will not buy a car.
The symbolic form of given statement is $p \rightarrow \sim q$

p	q	$\sim q$	$p \rightarrow \sim q$
T	T	F	F

$\therefore$truth value of the given statement is 'F'.

(iii) Yuvraj does not have sufficient money or he will buy a car.
The symbolic form of given statement is $\sim p \vee q$

p	q	$\sim p$	$\sim p \vee q$
T	T	F	T

$\therefore$truth value of the given statement is 'T'.

3. Let p: Every accountant is free to apply his own accounting rules.
Let q: Machinery is an asset
Then the symbolic form of the given statement is $p \leftrightarrow q$.
The truth values of p and q are F and T respectively.
$\therefore$ the truth value of $p \leftrightarrow q$ is F. $[F \leftrightarrow T \equiv F]$

4. The dual of the statement
$p \vee (q \vee r) \equiv p \wedge (q \wedge r)$

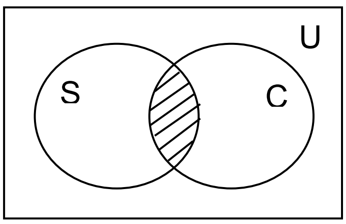
1	2	3	4	5	6	7
p	q	r	$q \wedge r$	$p \wedge (q \wedge r)$	$p \wedge q$	$(p \wedge q) \wedge r$
T	T	T	T	T	T	T
T	T	F	F	F	T	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	T	F	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

The entries in column 5 for $p \wedge (q \wedge r)$ and those in the column 7 for $(p \wedge q) \wedge r$ are identical.
Hence, the statements $p \wedge (q \wedge r)$ and $(p \wedge q) \wedge r$ are logically equivalent.
$\therefore p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$.

5. Let p: Total assets minus capital is equal to liabilities
Let q: Book-keeping is the language of the business.
Then the symbolic form of the given statement is $p \vee q$.
Since, $\sim (p \vee q) \equiv \sim p \wedge \sim q$, the negation of the given statement is:
"Total assets minus capital is not equal to liabilities and book-keeping is not the language of the business."

6. L.H.S. $\equiv p \vee \{ [\sim p \wedge (p \vee q)] \vee (q \wedge p) \}$
L.H.S. $\equiv p \vee \{ [(\sim p \wedge p) \vee (\sim p \wedge q)] \vee (q \wedge p) \}$ (Distributive Law)
L.H.S. $\equiv p \vee \{ [c \vee (\sim p \wedge q)] \vee (q \wedge p) \}$ (Complement Law)
L.H.S. $\equiv p \vee \{ (\sim p \wedge q) \vee (q \wedge p) \}$ (Identify Law)
L.H.S. $\equiv p \vee \{ (q \wedge \sim p) \vee (q \wedge p) \}$ (Commutative Law)
L.H.S. $\equiv p \vee \{ q \wedge (\sim p \vee p) \}$ (Distributive Law)
L.H.S. $\equiv p \vee (q \wedge t)$ (Complement Law)
L.H.S. $\equiv p \vee q$ (Identify Law)
L.H.S. = R.H.S.

7. Let U : set of all human beings
Let S : set of all shareholders
Let C : set of all chartered accountant
Then the Venn diagram represents the truth of the given statement is given below :



$S \cap C \neq \phi$

Q.3. Attempt any One : (4 Marks each). **(04)**

1. $(p \vee q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$

1	2	3	4	5	6	7	8
p	q	r	$p \vee q$	$(p \vee q) \rightarrow r$	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

The entries in columns 5 and 8 identical.

$\therefore (p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

2. $(\sim p \wedge \sim q) \wedge (q \wedge r)$

p	q	r	~p	~q	$\sim p \wedge \sim q$	$q \wedge r$	$(\sim p \wedge \sim q) \wedge (q \wedge r)$
T	T	T	F	F	F	T	F
T	T	F	F	F	F	F	F
T	F	T	F	T	F	F	F
T	F	F	F	T	F	F	F
F	T	T	T	F	F	T	F
F	T	F	T	F	F	F	F
F	F	T	T	T	T	F	F
F	F	F	T	T	T	F	F

The entries in the last column of the above truth table are F.

$\therefore$ $(\sim p \wedge \sim q) \wedge (q \wedge r)$ is a contradiction.

3. $(p \leftrightarrow r) \wedge (q \leftrightarrow p)$

p	q	r	$p \leftrightarrow r$	$q \leftrightarrow p$	$(p \leftrightarrow r) \wedge (q \leftrightarrow p)$
T	T	T	T	T	T
T	T	F	F	T	F
T	F	T	T	F	F
T	F	F	F	F	F
F	T	T	F	F	F
F	T	F	T	F	F
F	F	T	F	T	F
F	F	F	T	T	T

Q.4. Attempt any Two : (2 Marks each) **(04)**

1. $y = 5 + x^2e^{-x} + 2x$

$\therefore \quad \frac{dy}{dx} = \frac{d}{dx}\left(5 + x^2e^{-x} + 2x\right)$

$\frac{dy}{dx} = \frac{d}{dx}(5) + \frac{d}{dx}\left(x^2e^{-x}\right) + 2\frac{d}{dx}(x)$

$\frac{dy}{dx} = 0 + x^2\frac{d}{dx}\left(e^{-x}\right) + e^{-x}\frac{d}{dx}\left(x^2\right) + 2(1)$

$\frac{dy}{dx} = x^2.e^{-x}.(-1) + e^{-x}2(x) + 2$

$\frac{dy}{dx} = -x^2e^{-x} + 2xe^{-x} + 2$

$\frac{dy}{dx} = xe^{-x}(2 - x) + 2$

Rate of change of demand (x) w.r.t. price (y) $= \frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)} = \frac{1}{xe^{-x}(2 - x) + 2}$

2. $y = \frac{5x + 7}{2x - 13}$

$\therefore \quad \frac{dy}{dx} = \frac{d}{dx}\left(\frac{5x + 7}{2x - 13}\right)$

$\therefore \quad \frac{dy}{dx} = \frac{(2x - 13).\frac{d}{dx}(5x + 7) - (5x + 7).\frac{d}{dx}(2x - 13)}{(2x - 13)^2}$

$\frac{dy}{dx} = \frac{(2x - 13).(5 \times 1 + 0) - (5x + 7).(2 \times 1 - 0)}{(2x - 13)^2}$

$\frac{dy}{dx} = \frac{10x - 65 - 10x - 14}{(2x - 13)^2} = \frac{-79}{(2x - 13)^2}$

Marginal demand $= \frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)} = \frac{-(2x - 13)^2}{79}$

3. $y = \cos 3x$

Differentiating w.r.t.x. we get,

$\frac{dy}{dx} = \frac{d}{dx}(\cos 3x) = -\sin 3x.\frac{d}{dx}(3x) = -\sin 3x\ (3) = -3\sin 3x$

Differentiating again w.r.t.x. we get,

$\frac{d^2y}{dx^2} = \frac{d}{dx}(-3\sin 3x) = -3.\frac{d}{dx}(\sin 3x)$

$\frac{d^2y}{dx^2} = -3\cos 3x\frac{d}{dx}(3x) = -3\cos 3x\ (3)$

$\therefore \frac{d^2y}{dx^2} = -9\cos 3x$

Q.5. Attempt any Four : (3 Marks each) **(12)**

1. $y = x^x.(\cos x)^{(x+3)}.\log x$

$\therefore \log y = \log [x^x.(\cos x)^{(x+3)}.\log x]$

$\therefore \log y = \log x^x + \log(\cos x)^{(x+3)} + \log(\log x)$

$\therefore \log y = x\log x + (x+3).\log\cos x + \log(\log x)$

Differentiating both sides w.r.t.x, we get

$$\frac{1}{y}.\frac{dy}{dx}=\frac{d}{dx}(x\log x)+\frac{d}{dx}\left[(x+3)\log\cos x\right]+\frac{d}{dx}\left[\log(\log x)\right]$$

$$=x.\frac{d}{dx}(\log x)+(\log x).\frac{d}{dx}(x)+(x+3).\frac{d}{dx}(\log\cos x)+(\log\cos x).\frac{d}{dx}(x+3)+\frac{1}{\log x}.\frac{d}{dx}(\log x)$$

$$=x\times\frac{1}{x}+(\log x)(1)+(x+3)\times\frac{1}{\cos x}.\frac{d}{dx}(\cos x)+(\log\cos x).(1+0)+\frac{1}{\log x}\times\frac{1}{x}$$

$$=1+\log x+(x+3)\,x\,\frac{1}{\cos x}.(-\sin x)+\log\cos x+\frac{1}{x\log x}$$

$$\therefore\ \frac{dy}{dx}=y\left[1+\log x-(x+3)\tan x+\log\cos x+\frac{1}{x\log x}\right]$$

$$=x^x.(\cos x)^{(x+3)}.\log x\left[1+\log x-(x+3)\tan x+\log\cos x+\frac{1}{x\log x}\right]$$

2. Let $y = \sin^{-1}\left(\frac{5x+12\sqrt{1-x^2}}{13}\right)$

Put $x = \sin\theta$. Then $\theta = \sin^{-1}x$

$$\therefore y=\sin^{-1}\left(\frac{5\sin\theta+12\sqrt{1-\sin^2\theta}}{13}\right)$$

$$\therefore y=\sin^{-1}\left(\frac{5\sin\theta+12\cos\theta}{13}\right)$$

Since $\left(\frac{5}{13}\right)^2+\left(\frac{12}{13}\right)^2=\frac{25}{169}+\frac{144}{169}=1$

We can write $\frac{5}{13}=\cos\alpha$ and $\frac{12}{13}=\sin\alpha$

$$\therefore y=\sin^{-1}\left[(\sin\theta)\left(\frac{5}{13}\right)+(\cos\theta)\left(\frac{12}{13}\right)\right]$$

$$\therefore y=\sin\left[\sin\theta\cos\alpha+\cos\theta\sin\alpha\right]$$

$$\therefore y=\sin^{-1}\left[\sin(\theta+\alpha)\right]=\theta+\alpha$$

$\therefore y=\sin^{-1}x+\alpha$ (α is constant)

$$\therefore\frac{dy}{dx}=\frac{d}{dx}(\sin^{-1}x)+\frac{d}{dx}(\alpha)$$

$$=\frac{1}{\sqrt{1-x^2}}+0=\frac{1}{\sqrt{1-x^2}}$$

3. Let $y = \left(\frac{x^2}{1+x}\right)^x$

$\therefore \log y = x.\log\left(\frac{x^2}{x+1}\right)$

$= x\,[2\log x - \log(x+1)]$

$= (2x)\log x - x\log(x+1)$

Differentiating both sides w.r.t. x, we get,

$$\frac{1}{y}.\frac{dy}{dx} = 2\frac{d}{dx}(x\log x) - \frac{d}{dx}[x.\log(x+1)]$$

$$= \left\{2\left[x\frac{d}{dx}(\log x) + (\log x)\frac{d}{dx}(x)\right] - \left[x\frac{d}{dx}\log(x+1) + \log(x+1).\frac{d}{dx}(x)\right]\right\}$$

$$= 2\left[x \times \frac{1}{x} + (\log x)(1)\right] - \left[x.\frac{1}{x+1}.\frac{d}{dx}(x+1) + \log(x+1).(1)\right]$$

$$= 2 + 2\log x - \frac{x}{x+1}(1+0) - \log(x+1)$$

$$\therefore \frac{dy}{dx} = y\left[2 - \frac{x}{x+1} + \log x^2 - \log(x+1)\right]$$

$$= \left(\frac{x^2}{x+1}\right)^x\left[2 - \frac{x}{x+1} + \log\left(\frac{x^2}{x+1}\right)\right]$$

4. $y = \sin^{-1}\left(\frac{8x}{1+16x^2}\right) = \sin^{-1}\left[\frac{2\,x\,4x}{1+(4x)^2}\right]$

Put $4x = \tan\theta$. Then $\theta = \tan^{-1}4x$

$$y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$= \sin^{-1}(\sin 2\theta)$

$= 2\theta = 2\tan^{-1}4x$

$$\frac{dy}{dx} = 2\frac{d}{dx}\left(\tan^{-1}4x\right)$$

$$\frac{dy}{dx} = 2\,x\,\frac{1}{1+(4x)^2}.\frac{d}{dx}(4x)$$

$$\frac{dy}{dx} = \frac{2}{1+16x^2}\,x\,4 = \frac{8}{1+16x^2}$$

5. $u = 1 + \sin\theta$, $\nu = \theta - \cos\theta$.

Differentiating u and ν w.r.t. θ, we get

$$\frac{du}{d\theta} = \frac{d}{d\theta}(1+\sin\theta) = 0 + \cos\theta$$

$$\frac{du}{d\theta} = \cos\theta$$

And $\frac{dv}{d\theta} = \frac{d}{d\theta}(\theta - \cos\theta) = 1 - (-\sin\theta)$

$$\frac{dv}{d\theta} = 1 + \sin\theta$$

$$\frac{du}{dv} = \frac{(du/d\theta)}{(dv/d\theta)} = \frac{\cos\theta}{1+\sin\theta}$$

$$\left(\frac{du}{dv}\right)_{at\,\theta=\frac{\pi}{4}} = \frac{\cos\frac{\pi}{4}}{1+\sin\frac{\pi}{4}}$$

$$\left(\frac{du}{dv}\right)_{at\,\theta=\frac{\pi}{4}} = \frac{\left(\frac{1}{\sqrt{2}}\right)}{1+\frac{1}{\sqrt{2}}} = \frac{\left(\frac{1}{\sqrt{2}}\right)}{\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)}$$

$$\left(\frac{du}{dv}\right)_{at\,\theta=\frac{\pi}{4}} = \frac{1}{\sqrt{2}+1} = \frac{1}{\sqrt{2}+1} x \frac{\sqrt{2}-1}{\sqrt{2}-1}$$

$$\left(\frac{du}{dv}\right)_{at\,\theta=\frac{\pi}{4}} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2}-1.$$

6. $x^y = \sin y + 5x$

$\therefore$ $\log x^y = \log(\sin y + 5x)$

$\therefore$ $y \log x = \log(\sin y + 5x)$

Differentiating both sides w.r.t.x, we get,

$$y.\frac{d}{dx}(\log x) + (\log x)\frac{dy}{dx} = \frac{1}{\sin y + 5x}.\frac{d}{dx}(\sin y + 5x)$$

$$\therefore\ y x \frac{1}{x} + (\log x)\frac{dy}{dx} = \frac{1}{\sin y + 5x}\left(\cos y\frac{dy}{dx} + 5\right)$$

$$\therefore\ \frac{y}{x} + (\log x)\frac{dy}{dx} = \frac{\cos y}{\sin y + 5x}\frac{dy}{dx} + \frac{5}{\sin y + 5x}.$$

$$\therefore\ \left(\log x - \frac{\cos y}{\sin y + 5x}\right)\frac{dy}{dx} = \frac{5}{\sin y + 5x} - \frac{y}{x}$$

$$\therefore\ \left[\frac{(\log x)(\sin y + 5x) - \cos y}{\sin y + 5x}\right]\frac{dy}{dx} = \frac{5x - y\sin y - 5xy}{x(\sin y + 5x)}$$

$$\therefore\ \left[(\log x)(\sin y + 5x) - \cos y\right]\frac{dy}{dx} = \frac{5x - 5xy - y\sin y}{x}$$

$$\therefore\ \frac{dy}{dx} = \frac{5x(1-y) - y\sin y}{x\left[(\log x)(\sin y + 5x) - \cos y\right]}$$

7. Find $\frac{dy}{dx}$, if $x = a(\sin\theta - \theta\cos\theta)$ and $y = a(\cos\theta + \theta\sin\theta)$

$x = a(\sin\theta - \theta\cos\theta)$, $y = a(\cos\theta + \theta\sin\theta)$

Differentiating x and y w.r.t. θ, we get,

$$\frac{dx}{d\theta} = a\frac{d}{d\theta}(\sin\theta - \theta\cos\theta)$$

$$= a\left[\frac{d}{d\theta}(\sin\theta) - \frac{d}{d\theta}(\theta\cos\theta)\right]$$

$$= a\left[\cos\theta - \left\{\theta\frac{d}{d\theta}(\cos\theta) + \cos\theta\frac{d}{d\theta}(\theta)\right\}\right]$$

$$= a\left[\cos\theta - \left\{\theta(-\sin\theta) + \cos\theta x 1\right\}\right]$$

$$= a\left[\cos\theta + \theta\sin\theta - \cos\theta\right] = a\theta\sin\theta$$

And, $\frac{dy}{d\theta} = a\frac{d}{d\theta}(\cos\theta + \theta\sin\theta)$

$= a\left[\frac{d}{d\theta}(\cos\theta) + \frac{d}{d\theta}(\theta\sin\theta)\right]$

$= a\left[-\sin\theta + \theta\frac{d}{d\theta}(\sin\theta) + \sin\theta\frac{d}{d\theta}(\theta)\right]$

$= a[-\sin\theta + \theta\cos\theta + \sin\theta \times 1] = a\theta\cos\theta$

$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\theta\cos\theta}{a\theta\sin\theta} = \cot\theta$

Q.6. Attempt any One : (4 Marks each) (04)

1. $y = \tan^{-1}\left[\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right]$

$$y = \tan^{-1}\left[\frac{\sqrt{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2} + \sqrt{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2}}{\sqrt{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2} - \sqrt{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2}}\right]$$

$$= \tan^{-1}\left[\frac{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right) + \left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right) - \left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)}\right]$$

$$= \tan^{-1}\left[\frac{2\cos\left(\frac{x}{2}\right)}{2\sin\left(\frac{x}{2}\right)}\right]$$

$$= \tan^{-1}\left[\cot\left(\frac{x}{2}\right)\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{2} - \frac{x}{2}\right)\right]$$

$$y = \frac{\pi}{2} - \frac{x}{2}$$

Differentiating w.r.t. x,

$\frac{dy}{dx} = 0 - \frac{1}{2}$

$= \frac{-1}{2}$

2. If $x=\frac{4t}{1+t^2}$, $y=3\left(\frac{1-t^2}{1+t^2}\right)$

Differentiating x and y w.r.t. θ, we get

$$\frac{dx}{dt}=\frac{d}{dt}\left(\frac{4t}{1+t^2}\right)=\frac{\left(1+t^2\right).\frac{d}{dt}(4t)-4t.\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}$$

$$\frac{dx}{dt}=\frac{\left(1+t^2\right)(4)-4t(0+2t)}{\left(1+t^2\right)^2}$$

$$=\frac{4+4t^2-8t^2}{\left(1+t^2\right)^2}=\frac{4-4t^2}{\left(1+t^2\right)^2}$$

$$=\frac{4\left(1-t^2\right)}{\left(1+t^2\right)^2}$$

And $\frac{dy}{dt}=3\frac{d}{dt}\left(\frac{1-t^2}{1+t^2}\right)$

$$\frac{dy}{dt}=3\left[\frac{\left(1+t^2\right).\frac{d}{dt}\left(1-t^2\right)-\left(1-t^2\right).\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}\right]$$

$$\frac{dy}{dt}=3\left[\frac{\left(1+t^2\right)(0-2t)-\left(1-t^2\right)(0+2t)}{\left(1+t^2\right)^2}\right]$$

$$\frac{dy}{dt}=3\left[\frac{-2t-2t^3-2t+2t^3}{\left(1+t^2\right)^2}\right]$$

$$\frac{dy}{dt}=\frac{-12t}{\left(1+t^2\right)^2}$$

$$\therefore\ \frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{\left[\frac{-12t}{\left(1+t^2\right)^2}\right]}{\left[\frac{4\left(1-t^2\right)}{\left(1+t^2\right)^2}\right]}$$

$$\therefore\ \frac{dy}{dx}=\frac{-3t}{1-t^2} \qquad \text{......(1)}$$

$$\therefore\ -\frac{9}{4}.\frac{x}{y}=-\frac{9}{4}.\frac{\left(\frac{4t}{1+t^2}\right)}{3\left(\frac{1-t^2}{1+t^2}\right)}=\frac{-3t}{1-t^2} \qquad (2)$$

From (1) & (2)

$$\frac{dy}{dx}=-\frac{9}{4}.\frac{x}{y}$$

3. If $\log\left(\frac{x^4-y^4}{x^4+y^4}\right)=k$, then show that $\frac{dy}{dx}=\frac{y}{x}$.

$\therefore \frac{x^4-y^4}{x^4+y^4}=e^k=p$ (Say)

$\therefore x^4-y^4=px^4+py^4$

$\therefore y^4+py^4=x^4-px^4$

$\therefore (1+p)y^4=(1-p)x^4$

$\therefore \frac{y^4}{x^4}=\frac{1-p}{1+p}$

$\therefore \frac{y}{x}=\sqrt[4]{\frac{1-p}{1+p}}$ (p is a constant)

Differentiating both sides w.r.t.x, we get,

$\frac{d}{dx}\left(\frac{y}{x}\right)=0$

$\therefore \frac{x\frac{dy}{dx}-y\frac{d}{dx}(x)}{x^2}=0$

$\therefore x\frac{dy}{dx}-y\times 1=0$

$\therefore x\frac{dy}{dx}=y$

$\therefore \frac{dy}{dx}=\frac{y}{x}$

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