Marks : 40	SYJC March' 19 Subject : Maths – I Logic & Integration	Duration : 1.5 Hours. Set – A SOLUTION

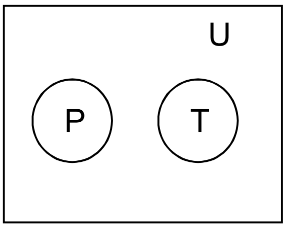
Q.1. Attempt any Two : (2 Marks each). **(04)**

1. a. Not a Statement
   b. Not a Statement
2. Truth table

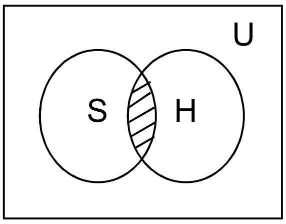
p	q	$p \wedge q$	$p \vee (p \wedge q)$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

3.
(i) Truth Value
   a. Statement – F
   b. Statement – T

(ii)
a. P = set of policemen U = set of people
   T = set of thieves



b. U = set of people
   S = set of students
   H = set of hardworkers



Q.2. Attempt any Four : (3 Marks each). **(12)**

1. Truth table

p	q	~p	~q	$p \rightarrow q$	$\sim q \rightarrow \sim p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

2. a. $\sim [(P \wedge q) \rightarrow p]$
   $\equiv (p \wedge q) \wedge p$
   b. $\sim [\sim p \leftrightarrow \sim q]$
   $\equiv (\sim p \wedge q) \vee (\sim q \wedge p)$

3. Let p = You go to Himalayas
   q = You will get a peace of mind

Converse : q → p : If you will get a peace of mind then you go to Himalayas
Contra positive: ~ q → ~ p : If you will not get a peace of mind then you do not go to Himalayas.
Inverse : ~p → ~ q : If you do not go to Himalayas then you will not get a peace of mind

4. (pÙ ~ q)® p

p	q	~q	(pÙ ~ q)	(pÙ ~ q)® p
T	T	F	F	F
T	F	T	T	F
F	T	F	F	F
F	F	T	F	F

5. The truth table is

p	q	~ q	p ∧ q	(p ∧ q) v ~ q	p ∨ ~ q
T	T	F	T	T	T
T	F	T	F	T	T
F	T	F	F	F	F
F	F	T	F	T	T

From the truth table we get (p ∧ q) v ~ q ≡ p ∨ ~ q

6.

p	q	p → q	~ p	~ q	~ q → ~ p	(p → q) ↔ (~q → ~p)
T	T	T	F	F	T	T
T	F	F	F	T	F	T
F	T	T	T	F	T	T
F	F	T	T	T	T	T

From the truth table we get (p → q) ↔ (~q → ~p) is a tautology.

Q.3. Attempt any One : (4 Marks each). **(04)**

1. (a) q ∧ ~ p = A number is greater than 10 but not greater than 20.
   (b) p → q = If a number is greater than 20 then it is greater than 10.

2. (a) (p ∧ ~ q) ∧ (q ∧ ~ r)
   (b) ~ p ∨ (~ q ∧ ~ r)

Q.4. Attempt any Two : (2 Marks each) **(04)**

1. $x \int \sin x\, dx - \int \left[\frac{d}{dx}(x) \int \sin x\, dx \right] dx$

$= -x \cos x + \int \cos x\, dx$

$= -x \cos x + \sin x + c$

2. $I = \int_0^{\pi/2} \frac{\sin\left(\pi/2 - x\right)}{\sin\left(\pi/2 - x\right) + \cos\left(\pi/2 - x\right)} dx \quad \ldots(i)$

$$I = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x}\, dx \qquad \ldots(ii)$$

Add (i) & (ii)

$$2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\cos x + \sin x}\, dx$$

$$2I = \int_0^{\pi/2} 1\, dx$$

$$2I = \left[x\right]_0^{\pi/2}$$

$$2I = \pi/2$$

$$\therefore \quad I = \pi/4$$

3. Required area = $\int_1^3 y\, dx$, where $y = x^2$

$$= \int_1^3 x^2\, dx$$

$$= \left[\frac{x^3}{3}\right]_1^3$$

$$= \frac{1}{3}\left[27 - 1\right]$$

$$= \frac{26}{3} \text{ sq.units}$$

Q.5. Attempt any Four : (3 Marks each). (12)

1. Let, $I = \int (x^3 \log x)\, dx$

$$= (\log x)\int x^3\, dx - \int \left[\frac{d}{dx}(\log x) \int x^3\, dx\right] dx$$

$$= (\log x) \times \frac{x^4}{4} - \int \frac{1}{x} \cdot \frac{x^4}{4}\, dx$$

$$= \frac{x^4 \log x}{4} - \frac{1}{4}\int x^3\, dx$$

$$= \frac{1}{4}x^4 \log x - \frac{1}{16}x^4 + c$$

2. $$\frac{x+1}{(2x+1)(2x+3)} = \frac{A}{(2x+1)} + \frac{B}{(2x+3)}$$

$x + 1 = A(2x + 3) + B(2x + 1)$

$= 2Ax + 3A + 2Bx + B$

$x + 1 = (2A + 2B)\, x + 3A + B$

Comparing coefficient

$2A + 2B = 1 \qquad . . . (1)$

$3A + B = 1 \qquad . . . (2)$

$2A + 2B = 1$

$3A + B = 1 \times 2$

$2A + 2B = 1$

$6A + 2B = 2$

$- \quad\quad -$

$-4A \quad = -1$

$3A + B = 1$

$\frac{3}{4} + B = 1$

$B = 1 - \frac{3}{4}$

$B = \frac{1}{4}$

$= \frac{1}{4}\int \frac{1}{(2x+1)}dx + \frac{1}{4}\int \frac{1}{(2x+3)}dx$

$= \frac{1}{2}\frac{1}{4}\log(2x+1) + \frac{1}{4}\frac{1}{2}\log(2x+3) + C$

$= \frac{1}{8}\log(2x+1) + \frac{1}{8}\log(2x+3) + C$

3. Let $I = \int \tan^{-1}\sqrt{x}\,dx$

Put $\sqrt{x} = t \quad \therefore x = t^2 \quad \therefore dx = 2t\,dt$

$\therefore \quad I = \int (\tan^{-1} t)(2t)\,dt$

$=[(\tan^{-1}t)].\int 2t\,dt - \int\left[\frac{d}{dt}\left(\tan^{-1}t\right).\int 2t\,dt\right]dt$

$= t^2.\tan^{-1}t - \int \frac{1}{1+t^2}.t^2 dt$

$= t^2.\tan^{-1}t - \int \frac{\left(1+t^2\right)-1}{1+t^2}dt$

$= t^2.\tan^{-1}t - \int 1\,dt + \int \frac{dt}{1+t^2}$

$= t^2.\tan^{-1} t - t + \tan^{-1} t + c$

$= (t^2 + 1)\tan^{-1} t - t + c$

$= (x + 1)\tan^{-1}\sqrt{x} - \sqrt{x} + c$

4. Let $I = \int_0^1 \frac{\log(1+x)}{1+x}dx$

$= \int_0^1 \log(1+x) \times \frac{1}{1+x}dx$

Put $\log(1 + x) = t$

$\therefore \quad \frac{1}{1+x}dx = dt$

When $x = 0$, $t = \log 1 = 0$

When $x = 1$, $t = \log 2$

$\therefore \quad I = \int_0^{\log 2} t\,dt = \left[\frac{t^2}{2}\right]_0^{\log 2}$

$= \frac{1}{2}\left[(\log 2)^2 - 0\right]$

$$= \frac{1}{2}(\log 2)^2$$

5. Required area = $\int_1^4 x.\,dy$

$\therefore$ A $= \int_1^4 \sqrt{16y}.dy$

$$= 4.\left[\frac{2}{3}.y^{\frac{3}{2}}\right]_1^4$$

$$= \frac{8}{3} \text{ x } 7 = \frac{56}{3} \text{ sq.units}$$

Q.6. Attempt any One : (4 Marks each). **(04)**

1. $\int e^x \sqrt{5e^{2x} - 4e^x - 3}\, dx$

Put $t = e^x$

$\therefore$ $dt = e^x\, dx$

$$= \int \sqrt{5t^2 - 4t - 3}\, dt$$

$$= \sqrt{5} \int \sqrt{t^2 - \frac{4}{5}t - \frac{3}{5}}\, dt$$

$$= \sqrt{5} \int \sqrt{t^2 - \frac{4t}{5} + \frac{4}{25} - \frac{3}{5} - \frac{4}{25}}\, dt$$

$$= \sqrt{5} \int \sqrt{\left(t - \frac{2}{5}\right)^2 - \left(\frac{\sqrt{19}}{5}\right)^2}$$

$$= \sqrt{5}\left[\frac{t - \frac{2}{5}}{2}\sqrt{t^2 - \frac{4t}{5} - \frac{3}{5}} - \frac{19}{50}\log\left|t - \frac{2}{5} + \sqrt{t^2 - \frac{4}{5}t - \frac{3}{5}}\right|\right] + c$$

$$= \sqrt{5}\left[\frac{5t - 2}{10}\sqrt{t^2 - \frac{4}{5}t - \frac{3}{5}} - \frac{19}{50}\log\left|t - \frac{2}{5} + \sqrt{t^2 - \frac{4}{5}t - \frac{3}{5}}\right|\right] + c$$

$$= \frac{\sqrt{5}\left(5e^x - 2\right)}{10}\sqrt{e^{2x} - \frac{4}{5}e^x - \frac{3}{5}} - \frac{19\sqrt{5}}{50}\log\left|e^x - \frac{2}{5} + \sqrt{e^{2x} - \frac{4}{5}e^x - \frac{3}{5}}\right| + c$$

2. Let, $\frac{(x^2 - 1)}{(x^2 - 4)(x^2 - 9)} = \left[\frac{A}{(x^2 - 4)} + \frac{B}{(x^2 - 9)}\right]$

Put, $x^2 = t$

$\therefore$ $\frac{(t - 1)}{(t - 4)(t - 9)} = \frac{A}{(t - 4)} + \frac{B}{(t - 9)}$

$\therefore$ $\frac{(t - 1)}{(t - 4)(t - 9)} = \frac{A(t - 9) + B(t - 4)}{(t - 4)(t - 9)}$

$\therefore$ $(t - 1) = A(t - 9) + B(t - 4)$

Sub t = 4	Sub t = 9
$\therefore 3 = A(-5) + B(0)$	$\therefore 8 = A(0) + B(5)$
$\therefore A = \frac{-3}{5}$	$\therefore B = \frac{8}{5}$

$$\int \frac{(x^2-1)}{(x^2-4)(x^2-9)}dx = \left[\frac{-3}{5(x^2-4)} + \frac{8}{5(x^2-9)}\right]dx = \frac{-3}{5}\int \frac{1}{(x^2-2^2)}dx + \frac{8}{5}\int \frac{1}{x^2-3^2}dx$$

$$= \frac{-3}{5}\frac{1}{2(2)}\log\left|\frac{x-2}{x+2}\right| + \frac{8}{5}\frac{1}{2(3)}\log\left|\frac{x-3}{x+3}\right| + c$$

$$= \frac{-3}{20}\log\left|\frac{x-2}{x+2}\right| + \frac{4}{15}\log\left|\frac{x-3}{x+3}\right| + c$$

3. Let V be the volume of the solid obtained by revolving the region OACO about X-axis.

Volume $= \pi\int_a^b y^2\,dx$

$= \pi\int_0^1 x\,dx$

$= \pi\left[\frac{x^2}{2}\right]_0^1$

$= \pi\left[x^2\right]_0^1$

$= \frac{\pi}{2}(1-0)$

$= \frac{\pi}{2}\text{cu. units}$