Marks : 40	SYJC March' 19 Subject : Maths – I Matrices / Continuity	Duration : 1.5 Hours. Solution

Q.1. Attempt any Two: (2 Marks each) **(04)**

1. Since A is a singular matrix, $|A| = 0$

$$\therefore \begin{vmatrix} 1 & 2 & 3 \\ 2 & a & 2 \\ 5 & 7 & 3 \end{vmatrix} = 0$$

$\therefore$ 1 (3a - 14) – 2 (6 – 10) + 3 (14 – 5a) = 0

$\therefore$ 3a – 14 + 8 + 42 – 15a = 0

$\therefore$ - 12a + 36 = 0

$\therefore$ - 12a = -36

$\therefore$ a = 3.

2.
$$2A - 3B = 2\begin{bmatrix} 1 & 2 & -1 \\ 3 & 4 & 5 \end{bmatrix} - 3\begin{bmatrix} -1 & 2 & 3 \\ 0 & 9 & 1 \end{bmatrix}$$

$$2A - 3B = \begin{bmatrix} 2 & 4 & -2 \\ 6 & 8 & 10 \end{bmatrix} - \begin{bmatrix} -3 & 6 & 9 \\ 0 & 27 & 3 \end{bmatrix}$$

$$2A - 3B = \begin{bmatrix} 2+3 & 4-6 & -2-9 \\ 6-0 & 8-27 & 10-3 \end{bmatrix}$$

$$2A - 3B = \begin{bmatrix} 5 & -2 & -11 \\ 6 & -19 & 7 \end{bmatrix}$$

3. Let A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 3 & 4 & 6 \end{bmatrix}$

It is a square matrix.

$$\because A' = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 3 & 4 & 6 \end{bmatrix} = A$$

$\therefore$ A is symmetric matrix.

Q.2 Attempt any Four : (3 Marks each) **(12)**

1.
$$\begin{bmatrix} -1 & 1 & 4 \end{bmatrix}\left\{2\begin{bmatrix} 5 & 5 \\ 6 & 6 \\ -1 & 2 \end{bmatrix} + 3\begin{bmatrix} 3 & 4 \\ 4 & 1 \\ 1 & -1 \end{bmatrix}\right\} = \begin{bmatrix} x & y \end{bmatrix}$$

$$\therefore \begin{bmatrix} -1 & 1 & 4 \end{bmatrix}\left\{\begin{bmatrix} 10 & 10 \\ 12 & 12 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} 9 & 12 \\ 12 & 3 \\ 3 & -3 \end{bmatrix}\right\} = \begin{bmatrix} x & y \end{bmatrix}$$

$$\therefore\begin{bmatrix}-1 & 1 & 4\end{bmatrix}\begin{bmatrix}19 & 22\\ 24 & 15\\ 1 & 1\end{bmatrix}=\begin{bmatrix}x & y\end{bmatrix}$$

$$\therefore\begin{bmatrix}-19+24+4 & -22+15+4\end{bmatrix}=\begin{bmatrix}x & y\end{bmatrix}$$

$$\therefore\begin{bmatrix}9 & -3\end{bmatrix}=\begin{bmatrix}x & y\end{bmatrix}$$

$\therefore$ by equality of matrices , x = 9, y = -3.

2. $$A^2=A.A=\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & -3\\ 1 & -1 & 0\end{bmatrix}\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & -3\\ 1 & -1 & 0\end{bmatrix}$$

$$A^2=A.A=\begin{bmatrix}4+0+1 & 0+0-1 & 2+0+0\\ 4+2-3 & 0+1+3 & 2-3-0\\ 2-2+0 & 0-1-0 & 1+3+0\end{bmatrix}$$

$$A^2=A.A=\begin{bmatrix}5 & -1 & 2\\ 3 & 4 & -1\\ 0 & -1 & 4\end{bmatrix}$$

$$A^2-5A+6I=\begin{bmatrix}5 & -1 & 2\\ 3 & 4 & -1\\ 0 & -1 & 4\end{bmatrix}-5\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & -3\\ 1 & -1 & 0\end{bmatrix}+6\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$A^2-5A+6I=\begin{bmatrix}5 & -1 & 2\\ 3 & 4 & -1\\ 0 & -1 & 4\end{bmatrix}-\begin{bmatrix}10 & 0 & 5\\ 10 & 5 & -15\\ 5 & -5 & 0\end{bmatrix}+\begin{bmatrix}6 & 0 & 0\\ 0 & 6 & 0\\ 0 & 0 & 6\end{bmatrix}$$

$$A^2-5A+6I=\begin{bmatrix}5-10+6 & -1-0+0 & 2-5+0\\ 3-10+0 & 4-5+6 & -1+15+0\\ 0-5+0 & -1+5+0 & 4-0+6\end{bmatrix}$$

$$A^2-5A+6I=\begin{bmatrix}1 & -1 & -3\\ -7 & 5 & 14\\ -5 & 4 & 10\end{bmatrix}$$

3. Let A = $\begin{pmatrix}2 & 0 & -1\\ 5 & 1 & 0\\ 0 & 1 & 3\end{pmatrix}$

$$\therefore|A|=\begin{vmatrix}2 & 0 & -1\\ 5 & 1 & 0\\ 0 & 1 & 3\end{vmatrix}$$

$$\therefore|A|=2(3-0)-0-1(5-0)$$

$$\therefore|A|=6-0-5$$

$$\therefore|A|=1\neq 0$$

$\therefore$ A^{-1}exists.

We write $AA^{-1} = I$

$$\therefore \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 \leftrightarrow R_2$,

$$\begin{bmatrix} 5 & 1 & 0 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 - 2R_2$,

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 - 2R_1$, $R_2 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_2 + 3R_3$,

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 4 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 - R_2$ and $R_3 - R_2$,

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} -7 & 3 & -3 \\ 5 & -2 & 3 \\ -5 & 2 & -2 \end{bmatrix}$$

By $(-R_3)$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -7 & 3 & -3 \\ 5 & -2 & 3 \\ 5 & -2 & 2 \end{bmatrix}$$

By $R_1 + 2R_3$ and $R_2 - 4R_3$,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

4. $A = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \cos\theta\,(\cos\theta - 0) + \sin\theta\,(\sin\theta - 0) + 0$

$= \cos^2\theta + \sin^2\theta = 1 \neq 0$

$\therefore A^{-1}$ exists.

(i) We write $AA^{-1} = I$

$$\therefore \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $\cos\theta \times R_1$,

$$\begin{bmatrix} \cos^2\theta & -\sin\theta\cos\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \cos\theta & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 + \sin\theta \times R_2$,

$$\begin{bmatrix} 1 & 0 & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_2 - \sin\theta \times R_1$,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta\cos\theta & \cos^2\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $\left(\dfrac{1}{\cos\theta}\right) \times R_2$,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5. Let $A = \begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} -1 & 5 \\ -3 & 2 \end{vmatrix} = -2 + 15 = 13 \neq 0$

$\therefore A^{-1}$ exists.

First we have to find the3 cofactor matrix

= $[A_{ij}]_{2\times 2}$, where $A_{ij} = (-1)^{i+j}M_{ij}$

Now, $A_{11} = (-1)^{1+1}M_{11} = 2$

$A_{12} = (-1)^{1+2}M_{12} = -(-3) = 3$

$A_{21} = (-1)^{2+1}M_{21} = -5$

$A_{22} = (-1)^{2+2}M_{22} = -1$

Hence, the cofactor matrix.

$$= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -5 & -1 \end{bmatrix}$$

$$\therefore \text{ adj } A = \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{13}\begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$$

Q.3. Attempt any One : (4 Marks each) (04)

1. The given equation can be written in the matrix form as:

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 3 & -1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \\ 5 \end{bmatrix}$$

By $R_2 - 2R_1$ and $R_3 - 3R_1$,

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -3 \\ 0 & -7 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \\ -19 \end{bmatrix}$$

By $R_3 - 7R_2$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -3 \\ 0 & 0 & 16 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \\ 16 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x + 2y + z \\ 0 - y - 3z \\ 0 + 0 + 16z \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \\ 16 \end{bmatrix}$$

By equality of matrices,

$x + 2y + z = 8$ ... (1)

$-y - 3z = -5$ … (2)

$16z = 16$ … (3)

From (3), $z = 1$

Substituting $z = 1$ in (2), we get,

$-y - 3 = -5$, $\therefore y = 2$

Substituting $y = 2$, $z = 1$ in (1), we get,

$x + 4 + 1 = 8$ $\therefore x = 3$

Hence, $x = 3$, $y = 2$, $z = 1$ is the required solution.

2. The given equations can be written in the matrix form as:

$$\begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$

This is of the form AX = B, where

$$A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$

Let us find A^{-1},

$$|A| = \begin{vmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{vmatrix}$$

$$= 5(18 + 10) + 1(12 - 25) + 4(-4 - 15)$$

$$= 140 - 13 - 76 = 51 \neq 0$$

$\therefore A^{-1}$ exists.

We write $AA^{-1} = I$

$$\therefore \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 - 2R_2$,

$$\begin{bmatrix} 1 & -7 & -6 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_2 - 2R_1$ and $R_3 - 5R_1$,

$$\begin{bmatrix} 1 & -7 & -6 \\ 0 & 17 & 17 \\ 0 & 33 & 36 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ -5 & 10 & 1 \end{bmatrix}$$

By $\left(\frac{1}{17}\right)R_2$,

$$\begin{bmatrix} 1 & -7 & -6 \\ 0 & 1 & 1 \\ 0 & 33 & 36 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -2 & 0 \\ \frac{-2}{17} & \frac{5}{17} & 0 \\ -5 & 10 & 1 \end{bmatrix}$$

By $R_1 + 7R_2$ and $R_3 - 33R_2$,

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{3}{17} & \frac{1}{17} & 0 \\ \frac{-2}{17} & \frac{5}{17} & 0 \\ \frac{-19}{17} & \frac{5}{17} & 1 \end{bmatrix}$$

By $\left(\frac{1}{3}\right)R_3$,

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{3}{17} & \frac{1}{17} & 0 \\ \frac{-2}{17} & \frac{5}{17} & 0 \\ \frac{-19}{51} & \frac{5}{51} & \frac{1}{3} \end{bmatrix}$$

By $R_1 - R_3$, $R_2 - R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{28}{51} & \frac{-2}{51} & \frac{-1}{3} \\ \frac{-13}{51} & \frac{10}{51} & \frac{-1}{3} \\ \frac{-19}{51} & \frac{5}{51} & \frac{1}{3} \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{51}\begin{bmatrix} 28 & -2 & -17 \\ 13 & 10 & -17 \\ -19 & 5 & 17 \end{bmatrix}$$

Now, pre multiply AX = B by A^{-1}, we get.

$A^{-1}(AX) = A^{-1}B$

$\therefore (A^{-1}A)\, X = A^{-1}B$

$\therefore IX = A^{-1}B$

$$\therefore X = \frac{1}{51}\begin{bmatrix} 28 & -2 & -17 \\ 13 & 10 & -17 \\ -19 & 5 & 17 \end{bmatrix}\begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$

$$\therefore X = \frac{1}{51}\begin{bmatrix} 140-4+17 \\ 65+20+17 \\ -95+10-17 \end{bmatrix}$$

$$\therefore X = \frac{1}{51}\begin{bmatrix} 153 \\ 102 \\ -102 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix}$$

By equality of matrices,

x = 3, y = 2, z = -2 is the required solution.

Q.4. Attempt any Two : (2 Marks each) **(04)**

1. $\lim_{x\to 0}\left(1+\frac{5x}{3}\right)^{\frac{1}{x}}$

$= \lim_{x\to 0}\left[\left(1+\frac{5x}{3}\right)^{\frac{3}{5x}}\right]^{\frac{5}{3}}$

Applying Limits, we get

$= e^{\frac{5}{3}}$ $\left\{\because \lim_{x\to 0}(1+x)^{\frac{1}{x}} = e\right\}$

$f(0) = \lim_{x\to 0} f(x)$

$\therefore$ Function f(x) is continuous at x = 0

2. $\lim_{x\to 1}\frac{\sqrt{x+8}-3}{x^3-1}$

$=\lim_{x\to 1}\frac{(\sqrt{x+8}-3)(\sqrt{x+8}+3)}{(x^3-1^3)\,(\sqrt{x+8}+3)}$

$=\lim_{x\to 1}\frac{x+8-9}{(x-1)(x^2+x+1)(\sqrt{x+8}+3)}$

$=\lim_{x\to 1}\frac{(x-1)}{(x-1)(x^2+x+1)(\sqrt{x+8}+3)}$

$=\lim_{x\to 1}\frac{1}{(x^2+x+1)(\sqrt{x+8}+3)}$

$=\frac{1}{(1+1+1)(\sqrt{1+8}+3)}$

$=\frac{1}{3\ (3+3)}$

$=\frac{1}{3+6}=\frac{1}{18}$

$f(1) \neq \lim_{x\to 0} f(x)$

$\therefore$ Function f(x) is discontinuous at x = 1

3. $f(0) = k$ (Given)

$\lim_{x\to 0} f(x) = \lim_{x\to 0}\frac{(e^x - 1)\sin x}{x^2}$

$= \lim_{x\to 0}\left(\frac{e^x-1}{x}\right).\frac{\sin x}{x}$

$= \lim_{x\to 0}\frac{e^x-1}{x}\times\lim_{x\to 0}\frac{\sin x}{x}$

$= 1 \times 1 = 1$

Since f is continuous at x = 0,

$f(0) = \lim_{x\to 0} f(x)$

$\therefore$ k = 1.

Q.5. Attempt any Four : (3 Marks each) (12)

1. $f(x) = \dfrac{3x^2 - 2x - 1}{2x^2 - x - 15}$

To find the value of x , for which f(x) has no finite value take

$2x^2-x-15 = 0$

$\therefore \quad 2x^2-6x+5x-15 = 0$

$2x(x-3) +5(x-3) = 0$

$(2x+5)(x-3) = 0$

$x = \dfrac{-5}{2}$, $x = 3$

$\therefore$ f(x) is discontinue at $x = \dfrac{-5}{2}$ & x = 3

2. Here $f(0) = 0$ (given)

Consider $\lim_{x\to 0} f(x) = \lim_{x\to 0} \dfrac{\sqrt{4+x}-2}{3x}$

$= \lim_{x\to 0} \dfrac{\sqrt{4+x}-2}{3x} \times \dfrac{\sqrt{4+x}+2}{\sqrt{4+x}+2}$

$= \lim_{x\to 0} \dfrac{4+x-4}{3x(\sqrt{4+x}+2)}$

$= \lim_{x\to 0} \dfrac{x}{3x(\sqrt{4+x}+2)}$ $(\because x \to 0, x \neq 0)$

$= \dfrac{1}{12} \neq f(0)$

$\therefore$ The function f has removable discontinuity at x = 0

Redefine f as $f(x) = \dfrac{\sqrt{4+x}-2}{3x}$, for $x \neq 0$

$= \dfrac{1}{12}$, for x = 0

Then f is continuous at x = 0.

3. f is continuous at x = 0

$\therefore \quad \lim_{x\to 0} f(x) = f(0)$

$\therefore \quad \lim_{x\to 0} \dfrac{(e^{3x}-1)}{ax} = 1$

$\therefore \quad \lim_{x\to 0} \dfrac{1}{a}\left[\dfrac{e^{3x}-1}{3x} \times 3\right] = 1$

$\dfrac{1}{a}[\log e \times 3] = 1 \qquad \left[\because \lim_{x\to 0}\dfrac{(a^x-1)}{x} = \log a\right]$

$\frac{1}{a} \times (3) = 1$ [$\because \log_e e = 1$]

$\frac{3}{a} = 1$

$\Rightarrow$ a = 3

4. $f(x) = 3x + 2, \quad 2 \le x \le 4$

$\therefore f(2) = 3(2) + 2 = 8$

$f(x) = x^2 + ax + b, \quad x < 2$

$\therefore \lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (x^2 + ax + b)$

$= 2^2 + a(2) + b = 4 + 2a + b$

Since f is continuous at x = 2, $\lim_{x\to 2^-} f(x) = f(2)$

$\therefore 4 + 2a + b = 8$

$\therefore 2a + b = 4$ ... (1)

$f(4) = 3(4) + 2 = 14$

Also, $f(x) = 2ax + 5b, 4 < x$

$\therefore \lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} (2ax + 5b)$

$= 2a(4) + 5b = 8a + 5b$

Since f is continuous at x = 4,

$\lim_{x\to 4^+} f(x) = f(4)$

$\therefore 8a + 5b = 14$ ... (2)

Multiplying equation (1) by 5, we get,

10a + 5b = 20

Subtracting equation (2) from this equation, we get,

2a = 6 $\therefore$ a = 3

$\therefore$ from (1), 2(3) + b = 4

$\therefore$ 6 + b = 4 $\therefore$ b = -2

Hence, a = 3 and b = -2.

5. $f(0) = 5$ (given) ... (1)

$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\sin^2 5x}{x^2}$

$= \lim_{x\to 0} \left(\frac{\sin 5x}{5x}\right)^2 \times 25$

$= 25 \left(\lim_{x\to 0} \frac{\sin 5x}{5x}\right)^2$

$= 25(1)^2$ ... [$x \to 0, 5x \to 0$ and $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$]

$= 25$ ... (2)

From (1) and (2),

$\lim_{x\to 0} f(x) \ne f(0)$

$\therefore f$ is discontinuous at x = 0.

Here $\lim_{x\to 0} f(x)$ exists but not equal to $f(0)$.

Hence, the discontinuity at x = 0 is removable and can be removed by redefining the function as follows :

$$f(x) = \frac{\sin^2 5x}{x^2}, \text{ for } x \neq 0$$
$$= 25, \quad \text{for } x = 0.$$

Q.6. Attempt any One : (4 Marks each) (04)

1. f(x) is continuous at x = 0

$$\therefore f(0) = \lim_{x\to 0} f(x)$$
$$= \lim_{x\to 0} \frac{(3^{\sin x} - 1)^2}{x\log(1+x)}$$
$$f(0) = \lim_{x\to 0} \frac{\frac{(3^{\sin x} - 1)^2}{x^2}}{\frac{x\log(1+x)}{x^2}}$$
$$= \lim_{x\to 0} \frac{\frac{(3^{\sin x} - 1)^2}{x^2}}{\frac{1}{x}\log(1+x)}$$
$$= \frac{\lim_{x\to 0} \frac{(3^{\sin x} - 1)^2}{x^2}}{\lim_{x\to 0} \log(1+x)^{1/x}}$$
$$= \frac{\lim_{x\to 0} \left(\frac{3^{\sin x} - 1}{x}\right)^2}{\lim_{x\to 0} \log(1+x)^{1/x}}$$
$$= \frac{\lim_{x\to 0} \left(\frac{3^{\sin x} - 1}{\sin x} \cdot \frac{\sin x}{x}\right)^2}{\log e}$$
$$= \frac{(\log 3 \times 1)^2}{1} = (\log 3)^2$$

2. $$\lim_{x\to 0} \frac{2 - \sqrt{3 + \cos kx}}{x^2}$$
$$= \lim_{x\to 0} \frac{(2 - \sqrt{3 + \cos kx})\,(2 + \sqrt{3 + \cos kx})}{x^2(2 + \sqrt{3 + \cos kx})}$$
$$= \lim_{x\to 0} \frac{4 - (3 + \cos kx)}{x^2(2 + \sqrt{3 + \cos ks})}$$
$$= \lim_{x\to 0} \frac{1 - \cos kx}{x^2} \times \frac{1}{2 + \sqrt{3 + \cos kx}}$$

$$= \lim_{x\to 0} \frac{2\sin^2(\frac{kx}{2})}{x^2} \times \frac{1}{2+\sqrt{3+\cos kx}}$$

$$= \lim_{x\to 0} \frac{2\sin^2\left(\frac{kx}{2}\right)}{\frac{k^2}{4}x^2} \times \frac{1}{\frac{4}{k^2}} \times \frac{1}{2+\sqrt{3+\cos kx}}$$

$$= \lim_{x\to 0} \frac{2\sin^2(\frac{kx}{2})}{(\frac{kx}{2})^2} \times \frac{k^2}{4} \times \frac{1}{2+\sqrt{3+\cos kx}}$$

$$= \frac{2k^2}{4} \lim_{x\to 0} \left(\frac{\sin(\frac{kx}{2})}{(\frac{kx}{2})}\right)^2 \times \frac{1}{2+\sqrt{3+\cos kx}}$$

$$= \frac{k^2}{2} \times 1^2 \times \frac{1}{2+\sqrt{3+\cos(k\times 0)}}$$

$$= \frac{k^2}{2} \times \frac{1}{2+\sqrt{3+1}}$$

$$= \frac{k^2}{2} \times \frac{1}{2+2}$$

$$= \frac{k^2}{2\times 4} = \frac{k^2}{8}$$

f(0) = 2

$\lim_{x\to 0} f(x) = f(0)$ since f is continuous at x = 0

$$\therefore \frac{k^2}{8} = 2$$

$$\therefore k^2 = 16$$

k = 4

3. $$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{(2^x - 1)^2}{\tan x \cdot \log(1+x)}$$

$$= \lim_{x\to 0} \frac{\left(\frac{2^x - 1}{x}\right)^2}{\left(\frac{\tan x}{x}\right)\frac{1}{x}\log(1+x)}$$

$$= \frac{\lim_{x\to 0}\left(\frac{2^x - 1}{x}\right)^2}{\left(\lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{1}{\cos x}\right)\left[\lim_{x\to 0} \log(1+x)^{1/x}\right]}$$

$= \frac{(\log 2)^2}{1 \times 1 \times 1}$

$= (\log 2)^2$

But f (0) = log 4 = log $(2)^2$ = 2 log 2 (1)

$\therefore$ from (1) and (2) $\lim_{x \to 0} f(x) \neq f(0)$

$\therefore$ f is discontinuous at x = 0

4. If f is continuous at x = 0 then

$\lim_{x\to 0} f(x) = f(0)$

$\lim_{x\to 0} f(x) = \lim_{x\to 0} \cdot \frac{\sin kx}{x}$

$= \left[\lim_{x\to 0} \frac{\sin kx}{kx}\right] k = 1 \times k = k$

$f(0) = 7 + 0 = 7$

$\lim_{x\to 0} f(x) = f(0)$

$\therefore$ k = 7.