PAPER – II : MODEL PAPER – 01

(BASED ON MARCH 2014**)**

MATHEMATICS & STATISTICS

COMMERCE

TIME : 1 HR 30 MIN **MARKS : 40**

Q4. Attempt any six of the following **(12)**

01. Oliver spends 30% of his income on food items and 15% on conveyance . If in the particular month he spent ₹ 1800 on conveyance , find his expenditure on food items during the same month

SOLUTION :

$$\frac{\text{expenses on food items}}{\text{Expenses on conveyance}} = \frac{30}{15}$$

$$\frac{\text{expenses on food items}}{1800} = 2$$

$$\therefore \text{ expenses on food items} = ₹\ 3600$$

02. a building worth ₹ 5,00,000 is insured for 4/5 of its value at a premium of 5% . What is the amount of premium .

SOLUTION :

property value = ₹ 5,00,000

Insured value $= \frac{4}{5} \times 5{,}00{,}000 = 4{,}00{,}000$

rate of premium = 5%

Premium $= \frac{5}{100} \times 4{,}00{,}000 = 20{,}000$

03.

Age of wives	Age of Husbands (in yrs)			
(in yrs)	20 – 30	30 – 40	40 – 50	50 – 60
15 – 25	5	9	3	–
25 – 35	–	10	25	2
35 – 45	–	1	12	2
45 – 55	–	–	4	16
55 – 65	–	–	–	4

Find conditional distribution of age of husbands when age of wives lies in 25 – 35

CONDITIONAL FREQ. DIST. OF AGE OF HUSBANDS WHEN AGE OF WIVES LIES IN 25 – 35

CI	20 – 30	30 – 40	40 – 50	50 – 60	TOTAL
F	0	10	25	2	37

04. For a bivariate data : $\overline{x} = 53$; $\overline{y} = 28$; $byx = -1.5$; $bxy = -0.2$

Estimate y when x = 50

SOLUTION : Y ON X

$$y - \overline{y} = byx(x - \overline{x})$$

$$y - 28 = -1.5(x - 53)$$

Put x = 50

$$y - 28 = -1.5(50 - 53)$$

$$y - 28 = -1.5(-3)$$

$$y - 28 = 4.5$$

$$y = 32.5 \quad \text{when } x = 50$$

05. Values of two regression coefficients between the variables X and Y are $byx = -0.6$ and $bxy = -0.3$ respectively . Obtain the value of correlation coefficient

SOLUTION :

$$r^2 = byx \times bxy$$

$$r^2 = -0.6 \times -0.3$$

$$r^2 = \frac{6}{10} \times \frac{3}{10}$$

$$r^2 = \frac{18}{100}$$

$$r = \pm \frac{18}{100}$$

$$r = -\sqrt{\frac{18}{100}} \quad (\text{ byx \& bxy are } -\text{ve})$$

$$\log r' = \frac{1}{2}\left(\log 18 - \log 100\right)$$

$$\log r' = \frac{1}{2}\left(1.2553 - 2.0000\right)$$

$$\log r' = \frac{1.2553}{2} - \frac{2.0000}{2}$$

$$\log r' = 0.6277 - 1.0000$$

$$\log r' = \overline{1}.6277$$

$$r' = AL(\overline{1}.6277)$$

$$r' = 0.4243 \qquad \therefore r = -0.4243$$

06. Verify whether the following function can be regarded as probability mass function (p.m.f.) f or the given values of X

X = x	− 2	−1	1	2
P(X = x)	0.5	−0.1	0.6	0

SOLUTION : $P(-1) = -0.1$

$p(x) \geq 0 \ \forall \ x$ is not satisfied . Hence function is not a pmf

07. let the pmf of a random variable X be

$$P(x) = \frac{3-x}{10} \quad ; \quad x = -1, 0, 1, 2$$
$$= 0 \quad ; \quad \text{otherwise}$$

Calculate E(x)

SOLUTION :

x	$p(x) = \frac{3-x}{10}$	pi.xi
−1	$\frac{3+1}{10} = \frac{4}{10}$	$\frac{-4}{10}$
0	$\frac{3-0}{10} = \frac{3}{10}$	$\frac{0}{10}$
1	$\frac{3-1}{10} = \frac{2}{10}$	$\frac{2}{10}$
2	$\frac{3-2}{10} = \frac{1}{10}$	$\frac{2}{100}$
	$\Sigma pi.xi = \frac{0}{10}$	

$E(x) = \Sigma pi.xi = 0$

08. The time (in hours) required to perform the printing and binding operations (in that order) for each book is given in the following table :

Books	:	I	II	III	IV	V
Printing Machine M_1	:	3	7	4	5	7
Binding Machine M_2	:	6	2	7	3	4

Find the sequence to minimizes the total elapsed time (in hours) to complete the work

SOLUTION : min time = 2 on job II on m/c M_2

				II

next min time = 3 on job I on m/c M_1 , on job IV on m/c M_2

I			IV	II

next min time = 4 on job III on m/c M_1 , on job V on m/c M_2

I	III	V	IV	II

OPTIMAL SEQUENCE

Q5. (A) Attempt any TWO of the following (06)

01. Find the present value of annuity immediate of ₹ 20,000 per annum for 3 years at 10% p.a. compounded annually ($1.1^{-3} = 0.7513$)

SOLUTION :

$C = ₹\ 20,000$; $i = 10\% = 0.1$; $n = 3$

$$P = C\left(\frac{1-(1+i)^{-n}}{i}\right)$$

$$= 20000 \left(\frac{1-(1+0.1)^{-3}}{0.1}\right)$$

$$= 20000 \left(\frac{1 - 1.1^{-3}}{0.1}\right)$$

$$= 20000 \left(\frac{1-0.7513}{0.1}\right)$$

$$= 20000 \ \frac{0.2487}{0.1}$$

$$= 20000 \ \frac{2.487}{1}$$

$$= 20000 \times 2.487 = ₹\ 49,740$$

02. The coefficient of rank correlation for a certain group of data is 0.5 . If $\Sigma d^2 = 42$, assuming no ranks are repeated ; find the no. of pairs of observation

SOLUTION

$R = 0.5$; $\Sigma d^2 = 42$

$$R = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$$

$$0.5 = 1 - \frac{6(42)}{n(n^2-1)}$$

$$\frac{6(42)}{n(n^2-1)} = 1 - 0.5$$

$$\frac{6(42)}{n(n^2-1)} = 0.5$$

$$\frac{6(42)}{n(n^2-1)} = \frac{1}{2}$$

$$n(n^2-1) = 6 \times 42 \times 2$$

$$n(n^2-1) = 3 \times 2 \times 3 \times 2 \times 7 \times 2$$

$$(n-1).n.(n+1) = 7 \times 8 \times 9$$

On comparing , $n = 8$

03. Ranking of 8 trainees at the beginning (X) and at the end (Y) of a certain course are given below

Trainees	:	A	B	C	D	E	F	G	H
X	:	1	2	4	5	6	8	3	7
Y	:	2	4	3	7	8	1	5	6

SOLUTION

	x	y	$d = \|x - y\|$	d^2
A	1	2	1	1
B	2	4	2	4
C	4	3	1	1
D	5	7	2	4
E	6	8	2	4
F	8	1	7	49
G	3	5	2	4
H	7	6	1	1
				$\Sigma d^2 = 68$

$$R = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

$$= 1 - \frac{6(68)}{8(64 - 1)}$$

$$= 1 - \frac{6(68)}{8(63)}$$

$$= 1 - \frac{17}{21}$$

$$= \frac{4}{21}$$

$$= 0.19$$

(B) Attempt any TWO of the following **(08)**

01. Calculate the quantities indicated by '?' for the following part of a life table

x	lx	dx	qx	px	Lx	Tx	e_x^o
70	5000	?	?	?	?	50000	?
71	4800	?	?	?	?	?	?
72	4400						

$d_x = l_x - l_{x+1}$

✓ $d_{70} = l_{70} - l_{71}$
$= 5000 - 4800$
$= 200$

✓ $d_{71} = l_{71} - l_{72}$
$= 4800 - 4400$
$= 400$

$q_x = \dfrac{d_x}{l_x}$

✓ $q_{70} = \dfrac{d_{70}}{l_{70}} = \dfrac{200}{5000} = \dfrac{2}{50}$
$= 0.04$

✓ $q_{71} = \dfrac{d_{71}}{l_{71}} = \dfrac{400}{4800} = \dfrac{1}{12}$
$= 0.083$

$p_x = 1 - q_x$

✓ $p_{70} = 1 - q_{70} = 1 - 0.04$
$= 0.96$

✓ $p_{71} = 1 - q_{71} = 1 - 0.083$
$= 0.917$

$L_x = \dfrac{l_x + l_{x+1}}{2}$

✓ $L_{70} = \dfrac{l_{70} + l_{71}}{2} = \dfrac{5000 + 4800}{2}$
$= 4900$

✓ $L_{71} = \dfrac{l_{71} + l_{72}}{2} = \dfrac{4800 + 4400}{2}$
$= 4600$

$T_{x+1} = T_x - L_x$

✓ $T_{71} = T_{70} - L_{70} = 50000 - 4900$
$= 45100$

✓ $T_{72} = T_{71} - L_{71} = 45100 - 4600$
$= 40500$

$e_x^0 = \dfrac{T_x}{l_x}$

✓ $e_{70}^0 = \dfrac{T_{70}}{l_{70}} = \dfrac{50000}{5000} = 10$

✓ $e_{71}^0 = \dfrac{T_{71}}{l_{71}} = \dfrac{45100}{48\cancel{00}} = 9.937$

✓ $e_{72}^0 = \dfrac{T_{72}}{l_{72}} = \dfrac{40500}{4400} = 9.204$

LOG CALCULATIONS FOR 'e_x^0'

LOG 451 – LOG 48	LOG 405 – LOG 44
2.6542	2.6075
– 1.6812	– 1.6435
AL 0.9730	AL 0.9640
9.397	9.204

ans :

x	**lx**	**dx**	**qx**	**px**	**Lx**	**Tx**	**e_x^o**
70	5000	**200**	**0.04**	**0.96**	**4900**	50000	**10**
71	4800	**400**	**0.083**	**0.917**	**4600**	**45100**	**9.396**
72	4400	---	---	---	---	**40500**	**9.204**

02. Calculate CDR for town I and town II and comment on the results

SOLUTION

Age Group (Years)	Town I		Town II	
	POPULATION	NO. OF DEATHS	POPULATION	NO. OF DEATHS
0 – 10	1500	45	6000	150
10 – 25	5000	30	6000	40
25 – 45	3000	15	5000	20
45 & above	500	22	3000	54
	$\Sigma P = 10000$	$\Sigma D = 112$	$\Sigma P = 20000$	$\Sigma D = 264$

$$\text{CDR}_{(\text{TOWN I})} = \frac{\Sigma D}{\Sigma P} \times 1000 = \frac{112}{10000} \times 1000 = 11.2$$

(DEATHS PER THOUSAND)

$$\text{CDR}_{(\text{TOWN II})} = \frac{\Sigma D}{\Sigma P} \times 1000 = \frac{264}{20000} \times 1000 = 13.2$$

(DEATHS PER THOUSAND)

COMMENT : CDR(TOWN I) < CDR(TOWN II) . HENCE TOWN I IS HEALTHIER THAN TOWN II

03. A card is drawn at random and replaced four times from a well shuffled pack of 52 cards . Find the probability that

a) two diamond cards are drawn b) at least one diamond card is drawn

SOLUTION

A card is drawn 4 times from a well shuffled pack of 52 cards with replacement , n = 4

For a trial Success – a diamond card

$$p = 13/52 = 1/4 \quad ; \quad q = 3/4$$

r.v. X – no of successes = 0 , 1 , 2 , 3 , 4 , **X ~ B (4 , 1/4)**

P(at least 1 diamond card)

$= P(X \geq 1)$

$= P(1) + P(2) + \ldots\ldots\ldots\ldots + P(4)$

$= 1 - P(0)$

$= 1 - {}^4C_0 \, . \, p^0 . \, q^4$

$= 1 - {}^4C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^4$

$= 1 - \frac{3^4}{4^4}$

$= 1 - {}^{81}/_{256} = {}^{175}/_{256}$

P(2 diamond cards are drawn)

$= P(X = 2)$

$= {}^4C_2 \, . \, p^2 . \, q^2$

$= {}^4C_2 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^2$

$= \frac{6 \, . \, 1 . \, 9}{256}$

$= \frac{54}{256}$

$= \frac{27}{128}$

Q6. (A) Attempt any TWO of the following (06)

01. Mr. Ahuja and Mr. Sinha started a business with a capital investment of ₹ 75,000 and ₹ 50,000 respectively . After 5 months Mr. Ahuja put in ₹ 5,000 more as capital , while Mr. Sinha withdrew ₹ 10,000 from his existing capital . At the end of the year the profit was ₹ 11,720 . Find the share of each in the profit .

SOLUTION

PARTNER's NAME	CAPITAL INVESTED	PERIOD OF INVESTMENT
AHUJA	₹ 75,000	5 MONTHS
	+ ₹ 80,000	7 MONTHS
SINHA	₹ 50,000	5 MONTHS
	+ ₹ 40,000	7 MONTHS

STEP 1 :

Profits will be shared in the
'RATIO OF PRODUCT OF CAPITAL INVESTED & PERIOD OF INVESTMENT'

$$\begin{array}{cccc} & \text{AHUJA} & & \text{SINHA} \\ = & \left(\begin{array}{c} 75{,}000 \text{ X } 5 \\ + \\ 80{,}000 \text{ X } 7 \end{array}\right) & : & \left(\begin{array}{c} 50{,}000 \text{ X } 5 \\ + \\ 40{,}000 \text{ X } 7 \end{array}\right) \end{array}$$

$$= (375 + 560) : (250 + 280)$$

$$= 935 : 530$$

$$= 187 : 106 \qquad \text{TOTAL} = 293$$

STEP 2 :

PROFIT = ₹ 11,720

Ahuja's share of profit $= \frac{187}{293} \times 11{,}720 =$ ₹ 7,480

Sinha's share of profit $= \frac{106}{293} \times 11{,}720 =$ ₹ 4,240

02. A bill of ₹ 21,900 drawn on July 10 for 6 months was discounted for ₹ 21,720 at 5% p.a. On which day the bill was discounted

SOLUTION

due 6 months @ 5% p.a.

d days

10^{th} July ? ← 13^{th} Jan

₹ 21,720 ₹ 21,900

STEP 1 :

Date of drawing	=	10 / 07
Add period of bill	+	6 months
Nominal due date	=	10 / 01
Add Grace days	+	3 days
Legal due date	=	13 / 01 ……. 13^{th} January

STEP 2 :

Let Unexpired period = d days

STEP 3 :

B.D. = F.V. – C.V.

= 21,900 – 21,720

= ₹ 180

STEP 4 :

B.D. = Interest on F.V. for 'd' days @ 5% p.a.

$$180 = 21900 \times \frac{d}{365} \times \frac{5}{10}$$

d = 60

STEP 5 :

Dt. of Discount

= 13^{th} Jan – 60 days

	Jan		Dec	Nov
=	13	+	31	+ 16

= 14^{th} November

03. Let X be the number of matches played by the player and Y be the number of matches in which he scored more than 50 runs . The following data shown is for 5 players

No of Matches Played	X	:	21	25	26	24	19
Scored more than 50	Y	:	19	20	24	21	16

Find the regression line of X on Y

SOLUTION

x	y	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$	$(y - \bar{y})^2$	$(x - \bar{x})(y - \bar{y})$
21	19	– 2	– 1		1	2
25	20	2	0		0	0
26	24	3	4		16	12
24	21	1	1		1	1
19	16	– 4	– 4		16	16
115	100	0	0		34	31
$\bar{x} = 23$	$\bar{y} = 20$			$\Sigma(x - \bar{x})^2$	$\Sigma(y - \bar{y})^2$	$\Sigma(x - \bar{x})(y - \bar{y})$

X ON Y

$$bxy = \frac{\Sigma(x - \bar{x})(y - \bar{y})}{\Sigma(y - \bar{y})^2}$$

$$= \frac{31}{34}$$

$$= 0.91$$

$$x - \bar{x} = bxy\ (y - \bar{y})$$

$$x - 23 = 0.91(y - 20)$$

$$x - 23 = 0.91y - 18.2$$

$$x = 0.91y - 18.2 + 23$$

$$x = 0.91y + 4.8$$

(B) Attempt any TWO of the following **(08)**

01. Find the sequence that minimizes total elapsed time (in hours) required to complete the following jobs on two machines M_1 and M_2 in the order M_1M_2 . Also find the minimum elapsed time and idle time for two machines

Job	A	B	C	D	E
M_1	6	2	10	4	11
M_2	3	7	8	9	5

STEP 1 FINDING THE OPTIMAL SEQUENCE

Min time = 2 on job B on machine M_1 .

B				

Next min time = 3 on job A on machine M_2 .

B				**A**

Next min time = 4 on job D on machine M_1

B	**D**			**A**

Next min time = 5 on job E on machine M_2 .

B	**D**	**C**	**E**	**A**

OPTIMAL SEQUENCE

STEP 2 WORK TABLE

Job	B	D	C	E	A	**total process time**	
M_1	2	4	10	11	6	=	33 hrs
M_2	7	9	8	5	3	=	32 hrs

WORK TABLE

	MACHINES				
	M_1		**M_2**		**Idle time**
JOBS	**IN**	**OUT**	**IN**	**OUT**	**on M_2**
B	0	2	2	9	2
D	2	6	9	18	
C	6	16	18	26	
E	16	27	27	32	1
A	27	33	33	36	1

STEP 3

Total elapsed time T = 36 hrs

Idle time on M_1 = T – (sum of processing time of all jobs on M1)

= 36 – 33 = 3 hrs

Idle time on M_2 = T – (sum of processing time of all jobs on M2)

= 36 – 32 = 4 hr (**CHECK** : 2 + 1 + 1 = 4)

02. Minimize $z = 3x_1 + 2x_2$, subject to

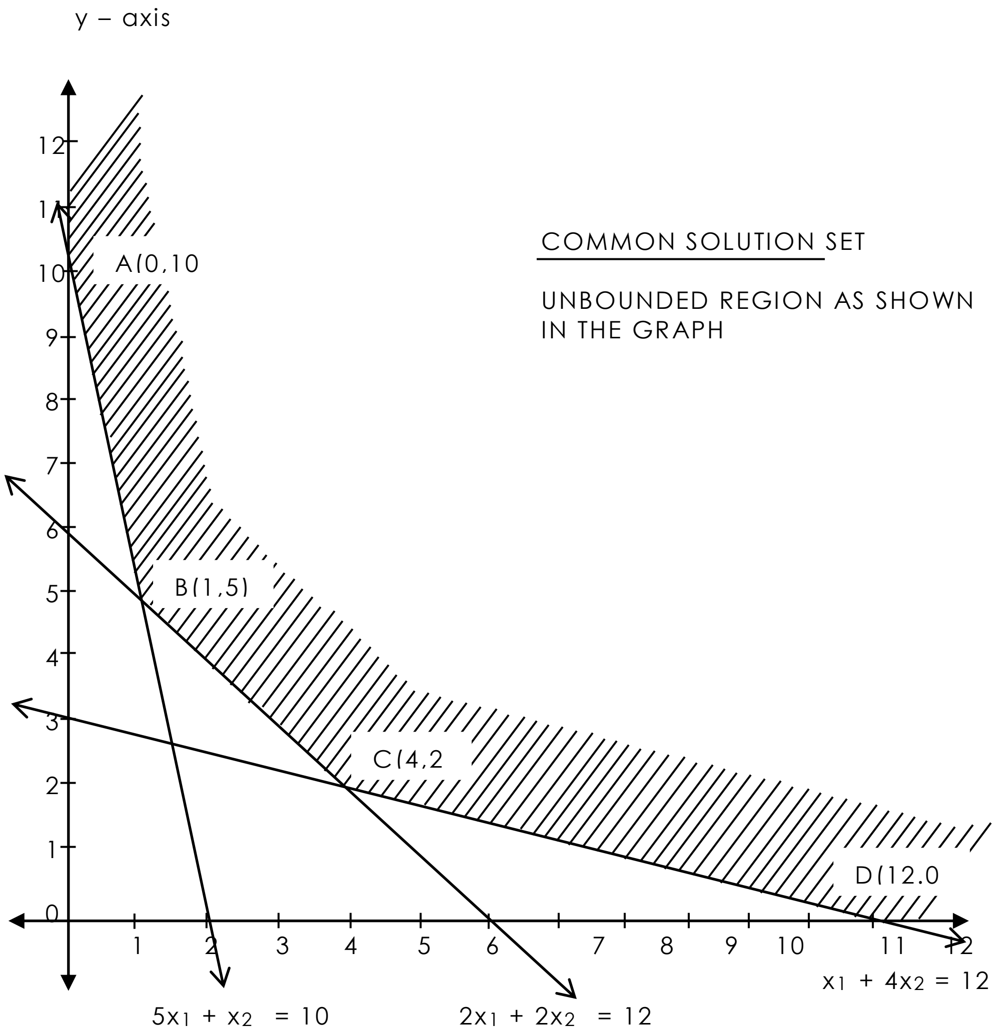
$5x_1 + x_2 \geq 10$, $2x_1 + 2x_2 \geq 12$, $x_1 + 4x_2 \geq 12$, $x_1 , x_2 \geq 0$

STEP 1 :

$5x_1 + x_2 \geq 10$	$5x_1 + x_2 = 10$ cuts x_1–axis at (2,0) cuts x_2–axis at (0,10)	Put (0,0) in $5x_1 + x_2 \geq 10$ $0 \geq 10$ (NOT SATISFIED) SS :NON-ORIGIN SIDE
$2x_1 + 2x_2 \geq 12$	$2x_1 + 2x_2 = 12$ cuts x_1–axis at (6,0) cuts x_2–axis at (0,6)	Put (0,0) in $2x_1 + 2x_2 \geq 12$ $0 \geq 12$ (NOT SATISFIED) SS : NON-ORIGINSIDE
$x_1 + 4x_2 \geq 12$	$x_1 + 4x_2 = 12$ cuts x_1–axis at (12,0) cuts x_2–axis at (0,3)	Put (0,0) in $x_1 + 4x_2 \geq 12$ $0 \geq 12$ (NOT SATISFIED) SS : NON-ORIGINSIDE
$x_1 , x_2 \geq 0$		SS : I QUADRANT

STEP 2 :

SCALE : 1 CM = 1 UNIT



COMMON SOLUTION SET

UNBOUNDED REGION AS SHOWN IN THE GRAPH

STEP 3 :

CORNERS	$Z = 3x_1 + 2x_2$	
A(0,10)	$Z = 3(0) + 2(10)$	$= 20$
B(1,5)	$Z = 3(1) + 2(5)$	$= 13$
C(4,2)	$Z = 3(4) + 2(2)$	$= 16$
D(12,0)	$Z = 3(12) + 2(0)$	$= 36$

STEP 4 :

Optimal Solution : Zmin = 13 at (1,5)

03. Maximize : $z = 4x + 10y$

subject to : $2x + 5y \le 10$; $5x + 3y \le 15$; $x \ge 0$; $y \ge 0$

STEP 1

$2x + 5y \le 10$	$2x + 5y = 10$ cuts x – axis at (5,0) cuts y – axis at (0,2)	Put (0,0) in $2x + 5y \le 10$ $0 \le 10$ SS : ORIGIN SIDE
$5x + 3y \le 15$	$5x + 3y = 15$ cuts x – axis at (3,0) cuts y – axis at (0,5)	Put (0,0) in $5x + 3y \le 15$ $0 \le 15$ SS : ORIGIN SIDE
$x, y \ge 0$		SS : I QUADRANT

FOR C

$5 \times \quad 2x + 5y = 10 \quad \ldots\ldots. (1)$

$2 \times \quad 5x + 3y = 15 \quad \ldots\ldots. (2)$

$$\begin{aligned} 10x + 25y &= 50 \\ \underline{-\ 10x \pm 6y} &= \underline{-30} \\ 19y &= 20 \\ y &= 20/19 \end{aligned}$$

subs in (1)

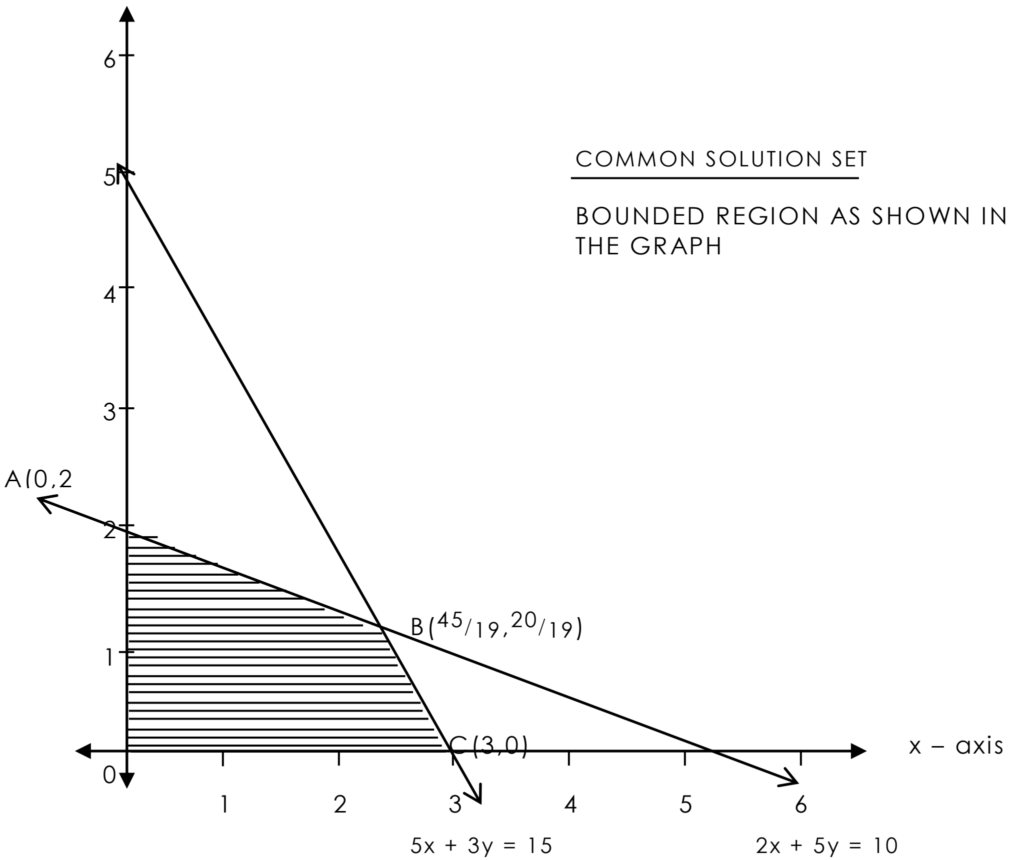
$$2x + \frac{100}{19} = 10$$

$$2x = \frac{90}{19}$$

$$x = \frac{45}{19} \qquad B \equiv \left(\frac{45}{19}, \frac{20}{19}\right)$$

STEP 2

y – axis SCALE : 2CM = 1 UNIT



STEP 3 :

CORNERS	$Z = 4x + 10y$		
A(0,2)	$Z = 4(0) + 10(2)$	$= 20$	
B(45/19,20/19)	$Z = \frac{180}{19} + \frac{200}{19}$	$= \frac{380}{19}$	$= 20$
C(3.0)	$Z = 4(3) + 10(0)$	$= 12$	

STEP 4

Zmin = 20 at all points on seg AB **(INFINITE OPTIMAL SOLUTIONS)**

DO NOT STOP

GET READY FOR NEXT