PAPER – II : MODEL PAPER – 04

(BASED ON MARCH 2017**)**

MATHEMATICS & STATISTICS

COMMERCE

TIME : 1 HR 30 MIN **MARKS : 40**

Q4. Attempt any six of the following **(12)**

01. If the numerator of the fraction is increased by 20% and its denominator be diminished by 10% , the value of the fraction is 16⁄21 . Find the original fraction

SOLUTION

Let the numerator of the fraction = x

Denominator of fraction = y

As per the given condition :

$$\frac{x + \frac{20x}{100}}{y - \frac{10y}{100}} = \frac{16}{21}$$

$$\frac{\frac{120x}{100}}{\frac{90y}{100}} = \frac{16}{21}$$

$$\frac{120x}{90y} = \frac{16}{21}$$

$$\frac{4x}{3y} = \frac{16}{21}$$

$$\frac{x}{y} = \frac{4}{7}$$

02. in a complete life table $l_4 = 60$ and $L_4 = 45$. Find the value of p_4 .

STEP 1 :

$$L_x = \frac{l_x + l_{x+1}}{2}$$

$$L_4 = \frac{l_4 + l_5}{2}$$

$$45 = \frac{60 + l_5}{2}$$

$$90 = 60 + l_5$$

$$\mathbf{l_5 = 30}$$

STEP 2 :

$$p_x = \frac{l_{x+1}}{l_x}$$

$$p_4 = \frac{l_5}{l_4}$$

$$= \frac{30}{60}$$

$$\mathbf{p_4 = 0.5}$$

03.

If the present worth of a bill due six months hence is ₹ 2,500 at 10% p.a. , what is the sum due

SOLUTION

F.V. = P.W.+ INT ON P.W. FOR 6 MONTHS @10% p.a.

$$F.V. = 2500 + 2500 \times \frac{6}{12} \times \frac{10}{100}$$

$$F.V. = 2500 + 125$$

$$F.V. = ₹\ 2,625$$

04. The following data give the marks of 20 students in Mathematics (X) and Statistic (Y) each out of 10 , expressed as (x,y) . Construct ungrouped frequency distribution

(2,7) ; (3,8) ; (4,9) ; (2,8) ; (2,8) ; (5,6) ; (5,7) ; (4,9) ; (3,8) ; (4,8) ; (2,9) ; (3,8) ; (4,8) ; (5,6) ; (4,7) ; (4,7) ; (4,6) ; (5,6) ; (5,7) ; (4,6)

BIVARIATE FREQUENCY DISTRIBUTION TABLE

MARKS IN STATISTICS Y	MARKS IN MATH (X)				
	2	3	4	5	TOTAL
6			|| 2	||| 3	5
7	| 1		|| 2	|| 2	5
8	|| 2	||| 3	|| 2		7
9	| 1		|| 2		3
TOTAL	4	3	8	5	N = 20

05.

A college book store is given 15% trade discount & 5% cash discount by publisher . Find the total amount of discount if the store purchases books worth ₹ 8,000 , according to the price list from the publisher

SOLUTION

List Price	= ₹	8,000
Less 15% T.D.	–	1,200
Invoice Price	= ₹	6,800
Less 5% C.D.	–	340
Net Selling Price	= ₹	6,460

Total Discount = 1200 + 340 = ₹ 1,540

06. r.v. X = The time (in minutes) for a lab assistant to prepare the equipment

$$f(x) = \frac{1}{10} \; ; \quad 25 \le x \le 35$$
$$= 0 \; ; \quad \text{otherwise}$$

P(preparation time exceed 33 min)

$= P(x > 33)$

$$= \int_{33}^{35} \frac{1}{10}\, dx$$

$$= \left(\frac{x}{10}\right)_{33}^{35}$$

$$= \left(\frac{35}{10}\right) - \left(\frac{33}{10}\right)$$

$$= \frac{2}{10}$$

$$= \frac{1}{5}$$

07. Solve the following minimal assignment problem

	Jobs		
Machines	I	II	III
M_1	1	4	5
M_2	4	2	7
M_3	7	8	3

SOLUTION

0	3	4	Reducing the matrix using ROW MINIMUM
2	0	5	
4	5	0	

[0]	3	4	Allocation using SINGLE ZERO ROW COLUMN method
2	[0]	5	
4	5	[0]	

Since every row and every column contains an ASSIGNED ZERO ,
the ASSIGNMENT PROBLEM is solved

OPTIMAL ASSIGNMENT : M1 - I , M2 – II , M3 – III

Minimum value = 1 + 2 + 3 = 6 units

08 If X has Poisson Distribution with variance 2 , find P(X = 4) (Given : $e^{-2} = 0.1353$)

SOLUTION

In Poisson Distribution ; mean = variance = 2

$m = 2$

P(x = 4)

$$= \frac{e^{-2}.2^4}{4!} \quad \text{Using} \quad P(x) = \frac{e^{-m}.m^x}{x!}$$

$$= \frac{0.1353.\ (16)}{24} = \frac{0.1353 \times 2}{3}$$

$$= 0.0451 \times 2$$

$$= 0.0902$$

Q5. (A) Attempt any TWO of the following **(06)**

01. Find the number of years for which an annuity of ₹ 200 is paid at the end of each year , if its accumulated amount works out to ₹ 662 with interest compounded at 10% p.a.

SOLUTION

C = ₹ 200 ; A = ₹ 662 ; i = 10% = 0.1

$$A = C\left[\frac{(1+i)^n - 1}{i}\right]$$

$$662 = 200\left[\frac{(1+0.1)^n - 1}{0.1}\right]$$

$$662 = 200\left[\frac{(1.1)^n - 1}{0.1}\right]$$

$$\frac{662}{200} = \frac{(1.1)^n - 1}{0.1}$$

$$3.31 = \frac{(1.1)^n - 1}{0.1}$$

$$0.331 = (1.1)^n - 1$$

$$1.331 = (1.1)^n \qquad \therefore n = 3$$

02. Complete the following life table

x	lx	dx	qx	px	Lx
4	9100	60	?	?	?
5	?	45	?	?	

SOLUTION

$d_x = l_x - l_{x+1}$

$d_4 = l_4 - l_5$

$60 = 9100 - l_5$

$l_5 = 9100 - 60$

$l_5 = 9040$

$q_x = \frac{d_x}{l_x}$

$$q_4 = \frac{d_4}{l_4} = \frac{60}{9100} = 0.0066$$

LOG CALC
1.7782
− 3.9590
AL $\overline{3}$.8192
0.00659

$$q_5 = \frac{d_5}{l_5} = \frac{45}{9040} = 0.0050$$

LOG CALC
1.6532
− 3.9562
AL $\overline{3}$.6970
0.00497

$p_x = 1 - q_x$

✓ $p_4 = 1 - q_4 = 1 - 0.0066 = 0.9934$

✓ $p_5 = 1 - q_5 = 1 - 0.0050 = 0.9950$

$L_x = \frac{l_x + l_{x+1}}{2}$

✓ $L_4 = \frac{l_4 + l_5}{2} = \frac{9100 + 9040}{2} = 9070$

03. r = 0.5 ; $\Sigma xy = 120$; SDy = 8 $\Sigma x^2 = 90$; where x and y are deviations from their respective means . Find n

SOLUTION

r = 0.5 ; $\Sigma(x - \bar{x})(y - \bar{y}) = 120$;

$\sigma y = 8$; $\Sigma(x - \bar{x})^2 = 90$

$$\sigma y = 8$$

$$\sqrt{\frac{\Sigma(y - \bar{y})^2}{n}} = 8$$

$$\sqrt{\Sigma(y - \bar{y})^2} = 8\sqrt{n}$$

NOW

$$r = \frac{\Sigma(x - \bar{x}).(y - \bar{y})}{\sqrt{\Sigma(x - \bar{x})^2}\sqrt{\Sigma(y - \bar{y})^2}.}$$

$$0.5 = \frac{120}{\sqrt{90}\,.\,8\sqrt{n}}$$

$$\frac{1}{2} = \frac{120}{\sqrt{90}\,.\,8\sqrt{n}}$$

$$\sqrt{n} = \frac{120 \times 2}{\sqrt{90}\,.\,8}$$

$$\sqrt{n} = \frac{30}{\sqrt{90}}$$

Squaring ;

$$n = 900/90 = 10$$

(B) Attempt any TWO of the following **(08)**

01. following is a pdf of r.v. X

$f(x) = kx(1-x) \; ; \quad 0 < x < 1$

$= 0 \quad ; \quad$ otherwise

SOLUTION

$$\int_0^1 kx(1-x)\,dx = 1$$

$$k\int_0^1 (x - x^2)dx = 1$$

$$k\left(\frac{x^2}{2} - \frac{x^3}{3}\right)_0^1 = 1$$

$$k\left(\frac{1}{2} - \frac{1}{3}\right) = 1$$

$$k\left(\frac{1}{6}\right) = 1$$

$$k = 6$$

Hence , pdf of the r.v.x is given as

$f(x) = 6x(1-x) \; ; \quad 0 < x < 1$

$= 0 \quad ; \quad$ otherwise

ii) $P(1/4 < x < 1/2)$

$$= \int_{1/4}^{1/2} 6x(1-x)\,dx$$

$$= \int_{1/4}^{1/2} (6x - 6x^2)dx$$

$$= \left(\frac{6x^2}{2} - \frac{6x^3}{3}\right)_{1/4}^{1/2}$$

$$= \left(3x^2 - 2x^3\right)_{1/4}^{1/2}$$

$$= \left(\frac{3}{4} - \frac{2}{8}\right) - \left(\frac{3}{16} - \frac{2}{64}\right)$$

$$= \left(\frac{3}{4} - \frac{1}{4}\right) - \left(\frac{3}{16} - \frac{1}{32}\right)$$

$$= \frac{2}{4} - \frac{6-1}{32}$$

$$= \frac{1}{2} - \frac{5}{32}$$

$$= \frac{16-5}{32}$$

$$= \frac{11}{32}$$

02. Minimize $z = 30x + 20y$, subject to

$x + y \le 8$, $6x + 4y \ge 12$, $5x + 8y \ge 20$, $x , y \ge 0$

STEP 1

$x + y \le 8$	$x + y = 8$ cuts x – axis at (8,0) cuts y – axis at (0,8)	Put (0,0) in $x + y \le 8$ $0 \le 8$ SS : ORIGIN SIDE
$6x + 4y \ge 12$	$6x + 4y = 12$ cuts x – axis at (2,0) cuts y – axis at (0,3)	Put (0,0)in $6x + 4y \ge 12$ $0 \ge 12$ (NOT SATISFIED) SS : NON-ORIGIN SIDE
$5x + 8y \ge 20$	$5x + 8y = 20$ cuts x – axis at (4,0) cuts y – axis at (0,2.5)	Put (0,0) in $5x + 8y \ge 20$ $0 \ge 20$ (NOT SATISFIED) SS : NON-ORIGIN SIDE
$x , y \ge 0$		SS : I QUADRANT

FOR C $2 \times$ $6x + 4y = 12$ ..... (1)

$5x + 8y = 20$ ..... (2)

$12x + 8y = 24$

$5x + 8y = 20$

$7x = 4$

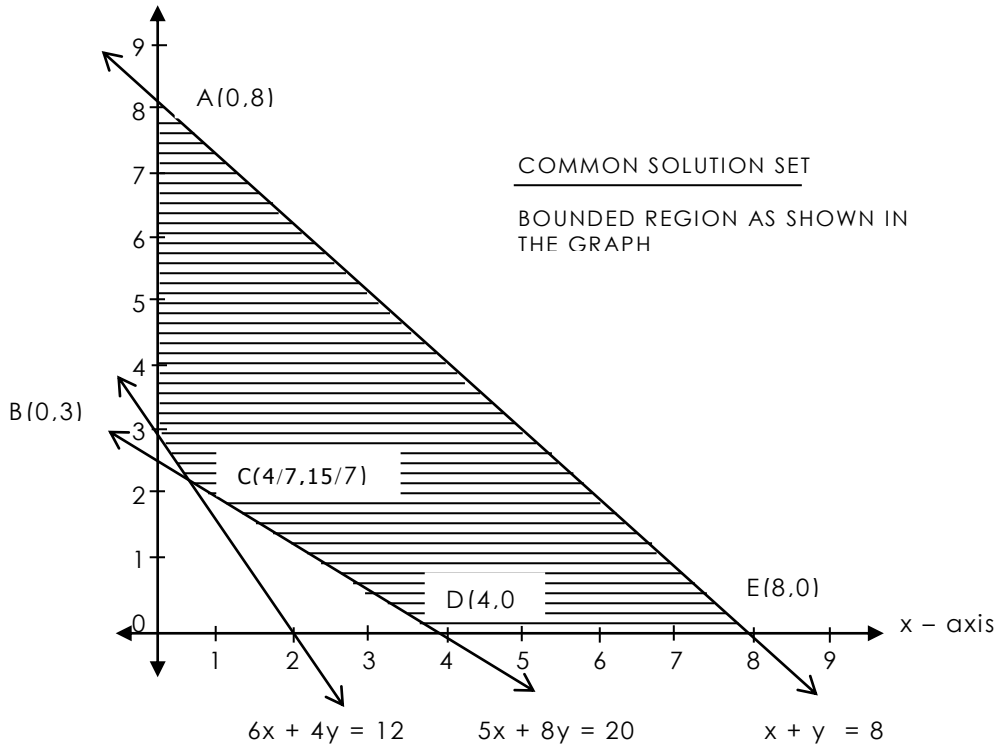
$x = 4/7$

subs in (1)

$y = 15/7$

$C \equiv (4/7, 15/7)$

Y – axis STEP 2 SCALE : 1 CM = 1 UNIT



COMMON SOLUTION SET

BOUNDED REGION AS SHOWN IN THE GRAPH

STEP 3 :

CORNERS	Z	=	$30x + 20y$		
A(0,2)	Z	=	$30(0) + 20(8)$	=	160
B(0.3)	Z	=	$30(0) + 20(3)$	=	60
C(4/7,15/7)	Z	=	$\frac{120}{7} + \frac{300}{7}$	= $\frac{420}{7}$	= 60
D(4.0)	Z	=	$30(4) + 20(0)$	=	120
E(8,0)	Z	=	$30(8) + 20(0)$	=	240

STEP 4

Zmin = 60 at all points on seg BC (INFINITE OPTIMAL SOLUTIONS)

02. Solve the MINIMAL ASSIGNMENT PROBLEM

		Topics				
		Linear Prog.	Queuing Theory	Dynamic Prog.	Regression Analysis	
	A	2	10	9	7	
Professors	B	15	4	14	8	TIME IN HRS
	C	13	14	16	11	
	D	4	15	13	9	

SOLUTION

0	8	7	5	Reducing the matrix using ROW MINIMUM
11	0	10	4	
2	3	5	0	
0	11	9	5	

0	8	2	5	Reducing the matrix using COLUMN MINIMUM
11	0	5	4	
2	3	0	0	
0	11	4	5	

[0]	8	2	5	Allocation using SINGLE ZERO ROW – COLUMN METHOD
11	[0]	5	4	
2	3	[0]	~~0~~	Allocation INCOMPLETE
~~0~~	11	4	5	

REVISE THE MATRIX

√	[0]	8	2	5
	~~1~~	~~0~~	~~5~~	~~4~~
	~~2~~	~~3~~	~~0~~	~~0~~
√	~~0~~	11	4	5
	√			

STEP 1 – Drawing minimum lines to cover ALL '0's

0	6	0	3
13	0	5	4
4	3	0	0
0	9	2	3

STEP 2 – REVISE THE MATRIX

reduce all the uncovered elements from its minimum and add the same at the intersection

~~0~~	6	[0]	3	allocation once again using
13	[0]	5	4	SINGLE ZERO ROW – COLUMN METHOD
4	3	0	[0]	
[0]	9	2	3	

Since every row and every column contains an assigned zero ,

the ASSIGNMNET PROBLEM IS SOLVED

OPTIMAL ASSIGNMENT : A – Dynamic Programming , B – Queuing theory ,

C – Regression Analysis , D – Linear Programming

min. time = 9 + 4 + 11 + 4 = 28 hours

Q6. (A) Attempt any TWO of the following **(06)**

01.

Age x	0	1	2
l_x	1000	900	700
T_x	---	---	11500

Find e_0^o ; e_1^o ; e_2^o

SOLUTION

$$\mathbf{L_x = \frac{l_x + l_{x+1}}{2}}$$

✓ $L_0 = \frac{l_0 + l_1}{2} = \frac{1000 + 900}{2} = 950$

✓ $L_1 = \frac{l_1 + l_2}{2} = \frac{900 + 700}{2} = 800$

$$\mathbf{T_{x+1} = T_x - L_x}$$

✓ $T_2 = T_1 - L_1$

$11500 = T_1 - 800$

$T_1 = 12300$

✓ $T_1 = T_0 - L_0$

$12300 = T_0 - 950$

$T_0 = 13250$

$$\mathbf{e_x^0 = \frac{T_x}{l_x}}$$

$e_0^0 = \frac{T_0}{l_0} = \frac{13250}{1000} = 13.25$

$e_1^0 = \frac{T_1}{l_1} = \frac{12300}{900} = 13.67$

$e_2^0 = \frac{T_2}{l_2} = \frac{11500}{700} = 16.43$

02. Two series of X and Y with 50 items each have standard deviations 4.5 and 3.5 respectively . If the sum of product of deviations of X and Y series from respective arithmetic means is 420 then find the correlation coefficient between X and Y

n = 50

$\sigma x = 4.5$; $\sigma y = 3.5$, $\Sigma(x - \bar{x})(y - \bar{y}) = 420$

$$r = \frac{cov(x,y)}{\sigma x \cdot \sigma y}$$

$$r = \frac{\frac{\Sigma(x - \bar{x})(y - \bar{y})}{n}}{\sigma x \cdot \sigma y}$$

$$r = \frac{\frac{420}{50}}{4.5 \times 3.5}$$

$$r = \frac{420}{50 \times 4.5 \times 3.5}$$

Taking log on both sides

$\text{Log } r = \log 420 - (\log 50 + \log 4.5 + \log 3.5)$

$\text{Log } r = 2.6232 - (1.6990 + 0.6532 + 0.5441)$

$\text{Log } r = 2.6232 - 2.8963$

$\text{Log } r = \bar{1}.7269$

$r = AL(\bar{1}.7269)$

$r = 0.5332$

03.

$2x + 3y - 6 = 0$ & $5x + 7y - 12 = 0$ are two regression lines .

Find 1) correlation coefficient 2) $\sigma x/\sigma y$

STEP 1

ASSUME

XON Y : $5x + 7y - 12 = 0$

$5x = -7y + 12$

$x = \frac{-7y}{5} + \frac{12}{5}$

$bxy = -\frac{7}{5}$

Y ON X : $2x + 3y - 6 = 0$

$3y = -2x + 6$

$y = -\frac{2}{3}y + \frac{6}{3}$

$byx = \frac{-2}{3}$

STEP 2

$r^2 = bxy \cdot byx$

$= \frac{-7}{4} \times \frac{-2}{3}$

$= \frac{14}{15}$

Since $0 \leq r^2 \leq 1$
Our assumptions are correct

$r = \pm\sqrt{\frac{14}{15}}$

$r = -\sqrt{\frac{14}{15}}$ (byx & bxy are –ve)

$\log r' = \frac{1}{2}\left(\log 14 - \log 15\right)$

$\log r' = \frac{1}{2}\left(1.1461 - 1.1761\right)$

$\log r' = \frac{1.1461}{2} - \frac{1.1761}{2}$

$\log r' = 0.5730 - 0.5880$

$\log r' = \bar{1}.9850$

$r' = AL(\bar{1}.9850)$

$r' = 0.9661$ $\quad r = -0.9661$

STEP 3

$bxy = r.\frac{\sigma x}{\sigma y}$

$-\frac{7}{5} = -0.9661\ \frac{\sigma x}{\sigma y}$

$\frac{\sigma x}{\sigma y} = \frac{7}{5 \times 0.9661}$

$\frac{\sigma x}{\sigma y} = \frac{7}{4.8305}$

$\frac{\sigma x}{\sigma y} = 1.449$

LOG CALC

0.8451
– 0.6840
AL 0.1611
1.449

(B) Attempt any TWO of the following **(08)**

01. P and Q started a business with capitals in the ratio 4 : 3 . After 9 months P withdrew 25% of his capital and Q put in an equal amount in addition to his earlier capital . If at the end of the year P's share in the profit was ₹ 15,450 , find the total profit and Q's share of profit

SOLUTION

PARTNER's NAME	CAPITAL INVESTED	PERIOD OF INVESTMENT
P	₹ 4k	9 MONTHS
	– 25%	
	₹ 3k	3 MONTHS
Q	₹ 3k + k	9 MONTHS
	₹ 4k	3 MONTHS

STEP 1 :

Profits will be shared in the
'RATIO OF PRODUCT OF CAPITAL INVESTED & PERIOD OF INVESTMENT'

	P		Q	
=	4k x 9 + 3k x 3	:	3k x 9 + 4k x 3	
=	36k + 9k	:	27k + 12k	
=	45k	:	39k	
=	15	:	13	TOTAL = 28

STEP 2 :

P share of profit = ₹ 15,450

P's share of profit $= \frac{15}{28} \times$ Total Profit

$15{,}450 = \frac{15}{28} \times$ Total Profit

$\text{Total Profit} = \frac{15450 \times 28}{15}$

= ₹ 28,840

Q's share of profit $= \frac{13}{28} \times 28{,}840$

= ₹ 13,390

02. A departmental store gives training to the salesmen in service followed by test . It is experienced that the performance regarding sales of any salesman is linearly related to the scores scored by him . The following data gives the test scores and the sales made by 9 salesmen during a fixed period

Test Scores (X)	16	22	28	24	29	25	16	23	24
Sales (Y) (in 00's)	35	42	57	40	54	51	34	47	45

a) Obtain the line of regression of Y on X b) Estimate Y when X = 17

SOLUTION

x	y	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$	$(x - \bar{x})(y - \bar{y})$
16	35	– 7	– 10	49	70
22	42	– 1	– 3	1	3
28	57	5	12	25	60
24	40	1	– 5	1	– 5
29	54	6	9	36	54
25	51	2	6	4	12
16	34	– 7	– 11	49	77
23	47	0	2	0	0
24	45	1	0	1	0
207	405	0	0	166	276–5 = 271
Σx	Σy	$\Sigma(x - \bar{x})$	$\Sigma(y - \bar{y})$	$\Sigma(x - \bar{x})^2$	$\Sigma(x - \bar{x})(y - \bar{y})$

$\bar{x} = \frac{207}{9} = 23$; $\bar{y} = \frac{405}{9} = 45$

$$byx = \frac{\Sigma(x - \bar{x})(y - \bar{y})}{\Sigma(x - x)^2}$$

$$= \frac{271}{166}$$

$$= 1.632$$

LOG CALC

2.4330
– 2.2201
AL 0.2129
1.632

$y - y = byx(x - x)$

$y - 45 = 1.632(x - 23)$

$y - 45 = 1.632x - 37.536$

$y = 1.632x + 7.464$

Put x = 17

$y = 1.632(17) + 7.464$

$= 27.744 + 7.464$

$= 35.208$

03. pmf for the random variable X is defined as

$$p(x) = k.x \quad ; \quad x = 1, 2, 3$$
$$= 0 \quad ; \quad \text{otherwise}$$

Find the mean and variance of X

SOLUTION

STEP 1 : $P(x) = kx \quad ; \quad x = 1, 2, 3$

$P(1) = k$

$P(2) = 2k$

$P(3) = 3k$

STEP 2 : $\Sigma p(x) = 1$

$k + 2k + 3k = 1$

$6k = 1$

$k = 1/6$

STEP 3 :

x	1	2	3	
p(x)	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	
pi.xi	$\frac{1}{6}$	$\frac{4}{6}$	$\frac{9}{6}$	$\Sigma pi.xi = \frac{14}{6}$
$pi.xi^2$	$\frac{1}{6}$	$\frac{8}{6}$	$\frac{27}{6}$	$\Sigma pi.xi^2 = \frac{36}{6}$

STEP 4 : $E(x) = \Sigma pi.xi = \frac{14}{6} = \frac{7}{3} = 2.33$

STEP 5 : $Var(x) = \Sigma\, pi.xi^2 - E(x)^2 = \frac{36}{6} - \frac{49}{9} = 6 - \frac{49}{9} = \frac{5}{9} = 0.55$

DO NOT STOP

GET READY FOR NEXT