

## PAPER - II : MODEL PAPER - 05

(BASED ON MARCH 2018)

MATHEMATICS & STATISTICS

COMMERCE

TIME : 1 HR 30 MIN

MARKS : 40

Q4. Attempt any six of the following

(12)

01. a person insures his shop valued ₹ 1 lakh for 80% of its value . He pays a premium of ₹ 4,000 . Calculate the rate of premium.

**SOLUTION :**

$$\text{Property value} = ₹ 1,00,000$$

$$\text{Insured Value} = \frac{80}{100} \times 1,00,000 = ₹ 80,000$$

$$\text{Premium} = ₹ 4000$$

$$\text{Rate of premium} = \frac{4000}{80000} \times 100 = 5\%$$

02. If for the following data CDR = 55 , find the value of x

Age Group	Population In '000	No. of deaths
0 – 25	25	1250
25 – 40	x	1000
40 – 70	28	1570
70 & above	15	1680

**SOLUTION :**

$$\text{CDR} = \frac{\sum D}{\sum P}$$

$$55 = \frac{5500}{68 + x}$$

$$68 + x = 100 \quad \therefore x = 32$$

03. if  $\sum d_i^2 = 42$  ,  $n = 8$  , find the rank correlation coefficient where  $d_i$  is the difference between the ranks of the  $i^{\text{th}}$  values

**SOLUTION :**

$$R = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$$

$$= 1 - \frac{6(42)}{8(63)}$$

$$= 1 - \frac{1}{2} = 0.5$$

04. if  $r(x,y) = -0.8$  ,  $\Sigma(x - \bar{x})(y - \bar{y}) = -120$  ,  $\sigma_x = 3$  ,  $\sigma_y^2 = 25$  . Find number of pairs of observation

**SOLUTION :**

$$r = \frac{\text{cov}(x,y)}{\sigma_x \cdot \sigma_y}$$

$$r = \frac{\frac{\Sigma(x - \bar{x})(y - \bar{y})}{n}}{\sigma_x \cdot \sigma_y}$$

$$-0.8 = \frac{-120}{\frac{n}{3 \cdot 5}}$$

$$12 = \frac{120}{n} \quad \therefore n = 10$$

05. The regression equation of Y on X is  $y = \frac{2}{9}x$  and  
 regression equation of X on Y is  $x = \frac{y}{2} + \frac{7}{6}$

Find a) correlation coefficient between X and Y b)  $\sigma_y^2$  if  $\sigma_x^2 = 4$

**SOLUTION :**

$$\text{Regression lines : } y \text{ on } x \quad y = \frac{2}{9}x \quad b_{yx} = \frac{2}{9}$$

$$x \text{ on } y \quad x = \frac{y}{2} + \frac{7}{6} \quad b_{xy} = \frac{1}{2}$$

$$r^2 = b_{yx} \cdot b_{xy}$$

$$= \frac{2}{9} \times \frac{1}{2}$$

$$= \frac{1}{9}$$

$$r = + \frac{1}{3} \dots\dots \text{since } b_{yx} \text{ \& } b_{xy} \text{ are + ve}$$

$$b_{yx} = r \frac{\sigma_y}{\sigma_x}$$

$$\frac{2}{9} = \frac{1}{3} \frac{\sigma_y}{2}$$

$$\sigma_y = \frac{4}{3} \quad \sigma_y^2 = \frac{16}{9}$$

06. Identify regression lines  $2x - y - 15 = 0$  &  $3x - 4y + 25 = 0$

**SOLUTION :**

assume  $x$  on  $y$  :  $2x - y - 15 = 0$

$$2x = y + 15$$

$$x = \frac{1}{2}y + \frac{15}{2} \quad b_{xy} = \frac{1}{2}$$

$y$  on  $x$  :  $3x - 4y + 25 = 0$

$$4y = 3x + 25$$

$$y = \frac{3}{4}x + \frac{25}{4} \quad b_{yx} = \frac{3}{4}$$

$$r^2 = b_{yx} b_{xy}$$

$$= \frac{3}{4} \times \frac{1}{2}$$

$$= \frac{3}{8}$$

Since  $0 \leq r^2 \leq 1$ , our assumptions are CORRECT

07. if a random variable  $X$  follows Poisson Distribution such that  $P(X = 1) = 2P(X = 2)$ ,

find the mean and the variance of the distribution

a random variable  $X$  follows Poisson Distribution such that

$$P(X = 1) = 2P(X = 2)$$

$$\frac{e^{-m}m^1}{1!} = 2 \cdot \frac{e^{-m}m^2}{2!}$$

$$m = m^2 \quad m = 1$$

In Poisson distribution ; mean = variance =  $m = 1$

08. three fair coins are tossed simultaneously . If  $X$  denotes the number of heads , find the probability distribution of  $X$

exp. : three fair coins are tossed ,  $n(S) = 8$

r.v.  $X$  : number of heads =  $0, 1, 2, 3$

x	outcomes	p(x)
0	TTT	1/8
1	HTT , THT , TTH	3/8
2	HHT , HTH , TTH	3/8
3	HHH	1/8

**Q5. (A) Attempt any TWO of the following****(06)**

01. A , B and C started a business by investing capitals in the ratio 4 : 5 : 6 . After 3 months B removed all his capitals and after 6 months C removed all his capitals from the business . At the end of the year A get ₹ 48,000 as profit . Find share of B and C in the profit

**SOLUTION :**

PARTNER's NAME	CAPITAL INVESTED	PERIOD OF INVESTMENT
A	₹ 4k	12 MONTHS
B	₹ 5k	3 MONTHS
C	₹ 6k	6 MONTHS

**STEP 1 :**

Profits will be shared in the

**'RATIO OF PRODUCT OF CAPITAL INVESTED & PERIOD OF INVESTMENT'**

	A	B	C
=	4k x 12	: 5k x 3	: 6k x 6
=	48k	: 15k	: 36k
=	48	: 15	: 36
=	16	: 5	: 12
			TOTAL = 33

**STEP 2 :**

A share of profit = ₹ 48,000

A's share of profit =  $\frac{16}{33} \times \text{Total Profit}$

48,000  $\frac{16}{33} \times \text{Total Profit}$

Total Profit =  $\frac{48000 \times 33}{16}$   
= ₹ 99,000

B's share of profit =  $\frac{5}{33} \times 99000$  = ₹ 15,000

C's share of profit =  $\frac{12}{33} \times 99000$  = ₹ 36,000

02.

	I	II	III	IV
A	2	10	9	7
B	13	2	12	2
C	3	4	6	1
D	4	15	4	9

SOLVE the MINIMAL ASSIGNMENT PROBLEM

**SOLUTION :**

0	8	7	5	Reducing the matrix using ROW MINIMUM
11	0	10	0	
2	3	5	0	
0	11	0	5	

<span style="border: 1px solid black;">0</span>	8	7	5	Allocation using SINGLE ZERO ROW – COLUMN METHOD
11	<span style="border: 1px solid black;">0</span>	10	<del>0</del>	
2	3	5	<span style="border: 1px solid black;">0</span>	
<del>0</del>	11	<span style="border: 1px solid black;">0</span>	5	

Since every row and every column contains an assigned zero ,

the ASSIGNMENT PROBLEM IS SOLVED

OPTIMAL ASSIGNMENT : A – I ; B – II ; C – IV ; D – III

$$\text{Min value} = 2 + 2 + 1 + 4 = 9$$

03. given :  $l_{26} = 9046$  ;  $l_{27} = 8898$  &  $T_{26} = 36,000$  . Find  $L_{26}$  ;  $T_{27}$  and  $e_{26}^0$ **SOLUTION :**

$$1) L_x = \frac{l_x + l_{x+1}}{2}$$

$$L_{26} = \frac{l_{26} + l_{27}}{2} = \frac{9046 + 8898}{2} = \frac{17944}{2} = 8972$$

$$2) T_{x+1} = T_x - L_x$$

$$T_{27} = T_{26} - L_{26} = 36000 - 8972 = 27028$$

$$3) e_x^0 = \frac{T_x}{l_x}$$

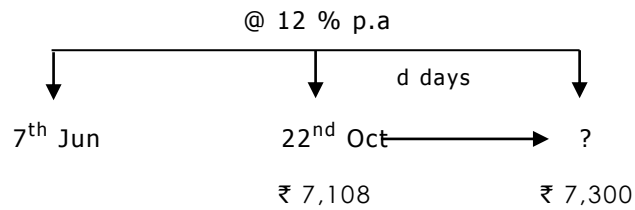
$$e_{26}^0 = \frac{T_{26}}{l_{26}} = \frac{36000}{9046} = 3.979$$

LOG CALC

$$\begin{array}{r} 4.5563 \\ - 3.9565 \\ \hline \text{AL } 0.5998 \\ 3.979 \end{array}$$

01. A bill of ₹ 7,300 drawn on 7<sup>th</sup> June 2007 was encashed for ₹ 7,108 on 22<sup>nd</sup> October 2007 .  
If the rate of interest was 12% p.a. Find the period of the bill

**SOLUTION :**



**STEP 1 :**

Let Unexpired period = d days

**STEP 2 :**

$$\begin{aligned}
 \text{B.D.} &= \text{F.V.} - \text{C.V.} \\
 &= 7,300 - 7,108 \\
 &= ₹ 192
 \end{aligned}$$

**STEP 3 :**

B.D. = Interest on F.V. for 'd' days @ 12% p.a.

$$\begin{aligned}
 192 &= 7300 \times \frac{d}{365} \times \frac{12}{100} \\
 d &= \frac{192 \times 5}{12} \\
 d &= 80 \text{ days}
 \end{aligned}$$

**STEP 4 :**

Legal Due date

$$= 22^{\text{nd}} \text{ Oct} + 80 \text{ days}$$

OCT NOV DEC JAN

$$= 9 + 30 + 31 + 10$$

$$= 10^{\text{th}} \text{ Jan , 2008}$$

– grace  
days – 3 days

$$= 7^{\text{th}} \text{ Jan , 2008 .....Nominal due date}$$

**STEP 5 :**

$$\begin{aligned}
 \text{Period of the bill} &= 7^{\text{th}} \text{ Jun 2007} - 7^{\text{th}} \text{ Jan, 2008} \\
 &\quad \quad \quad 7/06 \quad \quad \quad 7/01 \\
 &= 7 \text{ months}
 \end{aligned}$$

02. Find Karl Pearson's correlation coefficient for the following data

X : 3      2      1      5      4  
Y : 8      4      10      2      6

**SOLUTION :**

x	y	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$	$(y - \bar{y})^2$	$(x - \bar{x})(y - \bar{y})$
3	8	0	2	0	4	0
2	4	-1	-2	1	4	2
1	10	-2	4	4	16	-8
5	2	2	-4	4	16	-8
4	6	1	0	1	0	0
15 $\Sigma x$	30 $\Sigma y$	0 $\Sigma (x - \bar{x})$	0 $\Sigma (y - \bar{y})$	10 $\Sigma (x - \bar{x})^2$	40 $\Sigma (y - \bar{y})^2$	-14 $\Sigma (x - \bar{x})(y - \bar{y})$

$$\bar{x} = 3 \quad \bar{y} = 6$$

$$r = \frac{\Sigma (x - \bar{x}) \cdot (y - \bar{y})}{\sqrt{\Sigma (x - \bar{x})^2} \sqrt{\Sigma (y - \bar{y})^2}}$$

$$r = \frac{-14}{\sqrt{10} \times \sqrt{40}}$$

$$r = \frac{-14}{\sqrt{400}}$$

$$r = \frac{-14}{20}$$

$$r = -0.7$$

03. Solve the following using Graphical method

Minimize  $Z = 2x + y$

Subject to :

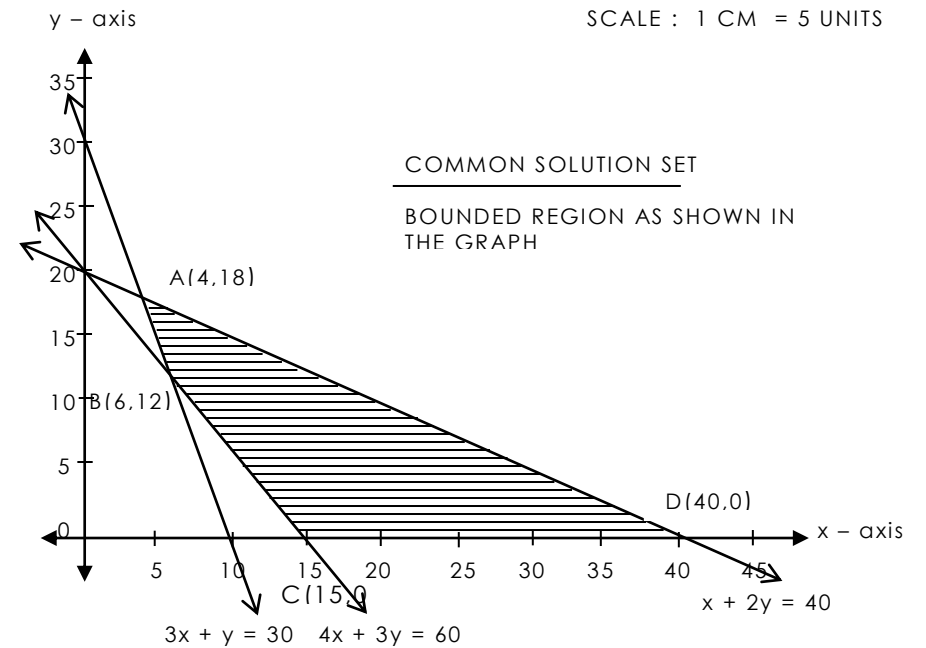
$3x + y \geq 30$  ,  $4x + 3y \geq 60$  ,  $x + 2y \leq 40$  ,  $x, y \geq 0$

**SOLUTION :**

STEP 1 :

$3x + y \geq 30$	$3x + y = 30$ cuts x- axis at (10,0) cuts y- axis at (0,30)	Put (0,0) in $3x + y \geq 30$ $0 \geq 30$ NOT SATISFIED SS : NON-ORIGIN SIDE
$4x + 3y \geq 60$	$4x + 3y = 60$ cuts x- axis at (15,0) cuts y- axis at (0,20)	Put (0,0) in $4x + 3y \geq 60$ $0 \geq 60$ NOT SATISFIED SS : NON-ORIGIN SIDE
$x + 2y \leq 40$	$x + 2y = 40$ cuts x- axis at (40,0) cuts y- axis at (0,20)	Put (0,0) in $x + 2y \leq 40$ $0 \leq 40$ SATISFIED SS : ORIGIN SIDE
$x, y \geq 0$		SS : I QUADRANT.

STEP 2



STEP 3 :

<u>CORNERS</u>	<u><math>Z = 2x + y</math></u>	
A(4,18)	$Z = 2(4) + 18$	= 26
B(6,12)	$Z = 2(6) + 12$	= 24
C(15,0)	$Z = 2(15) + 0$	= 30
D(40,0)	$Z = 2(40) + 0$	= 80

STEP 4 :

Optimal Solution :  $Z_{\min} = 24$  at (6,12)



**Q6. (A) Attempt any TWO of the following**

**(06)**

- 01.** x : years of service  
y : income (000's)

Find regression equation of **income** on the **years of service**

**SOLUTION :**

x	y	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$	$(y - \bar{y})^2$	$(x - \bar{x})(y - \bar{y})$
11	10	3	2	9		6
7	8	-1	0	1		0
9	6	1	-2	1		-2
5	5	-3	-3	9		9
8	9	0	1	0		0
6	7	-2	-1	4		2
10	11	2	3	4		6
56	56	0	0	28		21
$\Sigma x$ $\bar{x} = 7$	$\Sigma y$ $\bar{y} = 7$			$\Sigma(x - \bar{x})^2$	$\Sigma(y - \bar{y})^2$	$\Sigma(x - \bar{x})(y - \bar{y})$

$$b_{yx} = \frac{\Sigma(x - \bar{x})(y - \bar{y})}{\Sigma(x - \bar{x})^2}$$

$$= \frac{21}{28}$$

$$= \frac{3}{4}$$

$$= 0.75$$

REGRESSION EQUATION

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$y - 8 = 0.75(x - 8)$$

$$y - 8 = 0.75x - 6$$

$$y = 0.75x - 6 + 8$$

$$y = 0.75x + 2$$

- 02.** Sketch the graph on the real number line which shows solution set for the following inequation :  $-4 \leq -(5x + 3) < 8$

**SOLUTION :**

$$-4 \leq -(5x + 3) < 8$$

$$-4 \leq -5x - 3 < 8$$

$$-4 + 3 \leq -5x < 8 + 3$$

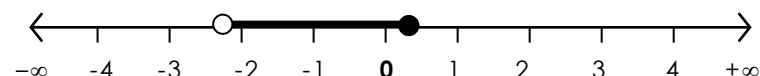
$$-1 \leq -5x < 11$$

$$1 \geq 5x > -11$$

$$0.2 \geq x > -2.2$$

SOLUTION INTERVAL :  $(-2.2, 0.2]$

SOLUTION GRAPH



03.  $x$  : 0 1 2 3 4 5  
 $l_x$  : 4000 3000 1000 200 40 0 . Construct the life table

**SOLUTION :**

AGE $x$	$l_x$	$dx = l_x - l_{x+1}$	$qx = \frac{dx}{l_x}$	$px = 1 - qx$	$Lx = \frac{l_x + l_{x+1}}{2}$	$T_x$	$e_x^0 = \frac{T_x}{l_x}$
0	4000	$4000 - 3000 = 1000$	$\frac{1000}{4000} = 0.25$	$1 - 0.25 = 0.75$	$3000 + 500 = 3500$	6240	$\frac{6240}{4000} = 1.56$
1	3000	$3000 - 1000 = 2000$	$\frac{2000}{3000} = 0.67$	$1 - 0.67 = 0.33$	$1000 + 1000 = 2000$	2740	$\frac{2740}{3000} = 0.9133$
2	1000	$1000 - 200 = 800$	$\frac{800}{1000} = 0.8$	$1 - 0.8 = 0.2$	$200 + 400 = 600$	740	$\frac{740}{1000} = 0.74$
3	200	$200 - 40 = 160$	$\frac{160}{200} = 0.8$	$1 - 0.8 = 0.2$	$40 + 80 = 120$	140	$\frac{140}{200} = 0.7$
4	40	$40 - 0 = 40$	$\frac{40}{40} = 1$	$1 - 1 = 0$	$0 + 20 = 20$	20	$\frac{20}{40} = 0.5$
5	0	----	----	----	----	----	----

**(B) Attempt any TWO of the following****(08)**

01. Find the probability of guessing correctly at most three of the seven answers in a True or False objective test

**SOLUTION :**

7 true – false Q's ,  $n = 7$

For a trial Success – a Q answered correctly

$p$  – probability of success =  $1/2$

$q$  – probability of failure =  $1 - 1/2 = 1/2$

r.v.  $X$  – no of successes =  $0, 1, 2, \dots, 7$  ;  **$X \sim B(7, 1/2)$**

$P(\text{guessing correctly at most three of the seven answers})$

=  $P(X \leq 3)$

=  $P(0) + P(1) + P(2) + P(3)$

$$= {}^7C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^7 + {}^7C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^6 + {}^7C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^5 + {}^7C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^4$$

$$= \frac{1 \cdot 1 \cdot 1 + 7 \cdot 1 \cdot 1 + 21 \cdot 1 \cdot 1 + 35 \cdot 1 \cdot 1}{2^7}$$

$$= \frac{64}{128} = \frac{1}{2}$$

02. machinery is expected to cost 25% more over its present cost of ₹ 6,96,000 after 20 years . The scrap value of the machinery will realize ₹ 1,50,000 . What sum should be set aside at the end of each year at 5% compound interest for 20 years to replace the machinery ( $1.05^{20} = 2.655$ )

**SOLUTION :**

Present cost = 6,96,000

Add 25% + 1,74,000

Expected cost

after 20 years 8,70,000

less scrap value – 1,50,000

7,20,000

$A = 7,20,000$  ,  $n = 20$  ,  $i = 0.05$

$$A = C \left[ \frac{(1+i)^n - 1}{i} \right]$$

$$7,20,000 = C \left[ \frac{(1+0.05)^{20} - 1}{0.05} \right]$$

$$7,20,000 = C \left[ \frac{(1.05)^{20} - 1}{0.05} \right]$$

$$7,20,000 = C \left[ \frac{2.655 - 1}{0.05} \right]$$

$$7,20,000 = C \left[ \frac{1.655}{0.05} \right]$$

$$7,20,000 = C \left[ \frac{165.5}{5} \right]$$

$$C = \frac{7,20,000}{33.1} \longrightarrow$$

$$= ₹ 21,750$$

LOG CALC

5.8573
– 1.5198
AL 4.3375
21750

		CITY			
		A	B	C	D
BRANCH MANAGER	P	11	11	9	9
	Q	13	16	11	10
	R	12	17	13	8
	S	16	14	16	12

MONTHLY BUSINESS (IN LACS)

Which manager should be appointed at which city so as to get maximum total monthly business .

**SOLUTION :**

6	6	8	8
4	1	6	7
5	0	4	9
1	3	1	5

Subtracting all the elements in the matrix from its largest element '17'

this matrix can now be solved as

'MINIMAL ASSIGNMENT PROBLEM'

0	0	2	2
3	0	5	6
5	0	4	9
0	2	0	4

Reducing the matrix using ROW MINIMUM

0	0	2	0
3	0	5	4
5	0	4	7
0	2	0	2

Reducing the matrix using COLUMN MINIMUM

0	×	2	×
3	0	5	4
5	×	4	7
×	2	0	2

Allocation using SINGLE ZERO ROW – COLUMN METHOD

**REVISE THE MATRIX**

0	×	2	×
✓ 3	0	5	4
✓ 5	×	4	7
×	2	0	2

STEP 1 – Drawing minimum lines to cover ALL '0's

0	3	2	0
0	0	2	1
2	0	1	4
0	5	0	2

STEP 2 – REVISE THE MATRIX

reduce all the uncovered elements by its minimum '3'  
and add the same at the intersection

<del>×</del>	3	2	<span style="border: 1px solid black;">0</span>	
<span style="border: 1px solid black;">0</span>	<del>×</del>	2	1	Allocation once again using
2	<span style="border: 1px solid black;">0</span>	1	4	SINGLE ZERO ROW – COLUMN METHOD
<del>×</del>	5	<span style="border: 1px solid black;">0</span>	2	

Since every row and every column contains an assigned zero ,

the ASSIGNMENT PROBLEM IS SOLVED

OPTIMAL ASSIGNMENT : P – D , Q – A , R – B , S – C ,

maximum business value

= 9 + 13 + 17 + 16

= 55 lacs

**DO NOT STOP**

**GET READY FOR NEXT**