PAPER - II : MODEL PAPER - 03

(BASED ON MARCH 2016**)**

MATHEMATICS & STATISTICS

COMMERCE

TIME : 1 HR 30 MIN **MARKS : 40**

Q4. Attempt any six of the following **(12)**

01. Anandi and Rutuja invested ₹ 10,000 each in a business . Anandi withdrew her capital after 7 months . Rutuja continued for the year . After one year the profit earned by them was ₹ 5,700 . Find the profit by each person

SOLUTION

STEP 1 :

Profits will be shared in the

'RATIO OF PERIOD OF INVESTMENT'

ANANDI		RUTUJA	
7	:	12	TOTAL = 19

STEP 2 :

PROFIT = □ 5,700

Anandi's share of profit $= \frac{7}{19} \times 5700 =$ ₹ 2,100

Rutuja's share of profit $= \frac{12}{19} \times 5{,}700 =$ ₹ 3,600

02. Calculate age specific death (A-SDR) rates for the following data

Age Group	Population In '000	No. of deaths	SDR $= \frac{D}{P}$
Below 10	25	50	2
10 – 30	30	90	3
30 – 45	40	160	4
45 – 70	20	100	5

03. for a bivariate data $b_{yx} = -1.2$ and $b_{xy} = -0.3$. Find correlation coefficient

SOLUTION

$r^2 = byx \times bxy$

$r^2 = -1.2 \times -03$

$r^2 = 0.36$

$r = -0.6$ ... byx & bxy are negative

04. In the pmf of a random variable X , two entries were missing .

X = x	1	2	3	4	5
P(X = x)	$\frac{1}{20}$	$\frac{3}{20}$	a	b	$\frac{1}{20}$

Find a and b given that **b = 2a**

SOLUTION

$$\Sigma p(x) = 1$$

$$\frac{1}{20} + \frac{3}{20} + a + b + \frac{1}{20} = 1$$

$$a + b = 1 - \frac{5}{20}$$

$$a + 2a = \frac{15}{20} \quad \ldots\ldots \quad b = 2a \quad \ldots\text{GIVEN}$$

$$3a = \frac{15}{20} \qquad a = \frac{5}{20} \quad \& \quad b = \frac{10}{20}$$

05. from the regression equations :y = 4x – 5 and 3x = 2y + 5 .
find $\overline{x}$ and $\overline{y}$

SOLUTION

Regression lines : $y = 4x - 5$ and $3x = 2y + 5$.
Solving

$$3x = 2(4x - 5) + 5 .$$

$$3x = 8x - 10 + 5$$

$$3x = 8x - 5$$

$$5 = 5x \qquad \overline{x} = 1$$

subs in (1) $\quad y = 4 - 5 \qquad \overline{y} = -1$

06. the probability distribution function of continuous random variable X is given by

$$f(x) = \frac{x}{8} , \quad 0 < x < 4$$

$$= 0 , \quad \text{otherwise} \qquad \text{Find } P(X > 3)$$

SOLUTION

$$P(x > 3) = \int_3^4 \frac{x}{8}\, dx$$

$$= \left(\frac{x^2}{16}\right)_3^4$$

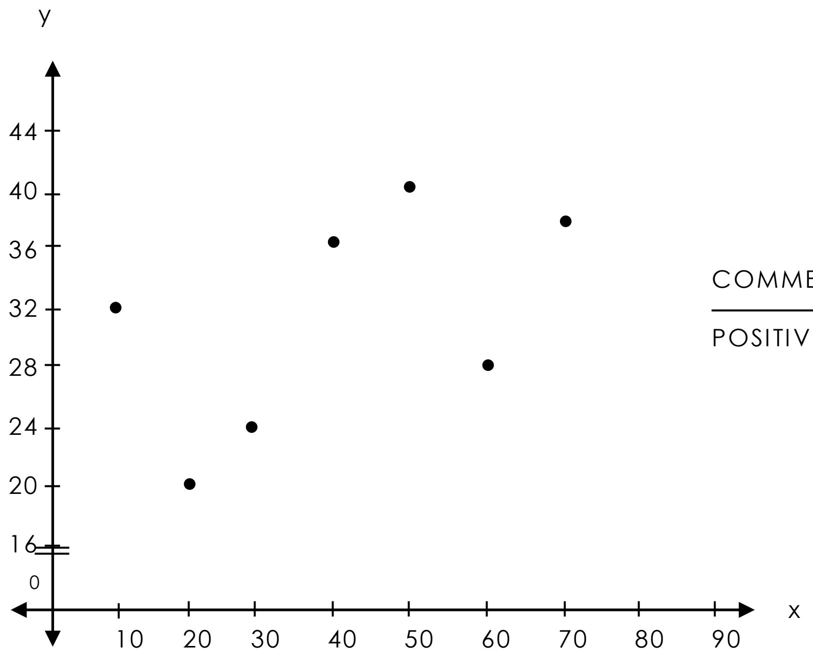
$$= \left(\frac{16}{16}\right) - \left(\frac{9}{16}\right)$$

$$= \frac{7}{16}$$

07. Draw scatter diagram for the following data and interpret it

x	10	20	30	40	50	60	70
y	32	20	24	36	40	28	38

SOLUTION



COMMENT :

POSITIVE CORRELATION BETWEEN THE TWO VARIABLES

08. $n = 100$; $\bar{x} = 62$; $\bar{y} = 53$; $\sigma x = 10$; $\sigma y = 12$

$\Sigma(x - \bar{x})(y - \bar{y}) = 8000$. Find correlation coefficient

SOLUTION

$$r = \frac{\text{cov}(x,y)}{\sigma x \,.\, \sigma y}$$

$$= \frac{\dfrac{\Sigma(x - \bar{x})(y - \bar{y})}{n}}{\sigma x \,.\, \sigma y}$$

$$= \frac{\dfrac{8000}{100}}{10.12}$$

$$= \frac{80}{10.12}$$

$$= \frac{2}{3}$$

$$= 0.67$$

Q5. (A) Attempt any TWO of the following **(06)**

01. Determine l_{92} ; l_{93} ; l_{94} given that $l_{91} = 97$; $d_{91} = 38$; $q_{92} = 27/59$; $p_{93} = 15/32$

SOLUTION

STEP 1 :

$$dx = lx - lx+1$$

$$d_{91} = l_{91} - l_{92}$$

$$38 = 97 - l_{92}$$

$$\boldsymbol{l_{92} = 59}$$

STEP 2 :

$$px = 1 - qx$$

$$p_{92} = 1 - q_{92}$$

$$= 1 - \frac{27}{59}$$

$$\boldsymbol{p_{92} = \frac{32}{59}}$$

STEP 3 :

$$px = \frac{lx+1}{lx}$$

$$p_{92} = \frac{l_{93}}{l_{92}}$$

$$\frac{32}{59} = \frac{l_{93}}{59}$$

$$\boldsymbol{l_{93} = 32}$$

02. if for a bivariate data $\overline{x} = 10$, $\overline{y} = 12$, $V(x) = 9$, $\sigma y = 4$ and $r = 0.6$ estimate y when x = 5

$$byx = r \frac{\sigma y}{\sigma x}$$

$$= 0.6 \ \frac{4}{3}$$

$$= 0.8$$

$$y - \overline{y} = byx\ (x - \overline{x})$$

$$y - 12 = 0.8(x - 10)$$

Put x = 5

$$y - 12 = 0.8(5 - 10)$$

$$y - 12 = 0.8(-5)$$

$$y - 12 = -4$$

$$y = 8 \quad \text{when } x = 5$$

03. Calculate CDR for the local population from the following data

Age Group	Population	No. of deaths
0 – 20	40000	350
20 – 65	65000	650
65 & above	15000	x

Find x if CDR = 13.4 per thousand

$$CDR = \frac{\Sigma D}{\Sigma P} \times 1000$$

$$13.4 = \frac{1000 + x}{1{,}20{,}000} \times 1000$$

$$13.4 = \frac{1000 + x}{120}$$

$$1608 = 1000 + x$$

$$x = 608$$

(B) Attempt any TWO of the following (08)

01. the coefficient of rank correlation of marks obtained by 10 students in English and Economics was found to be 0.5. It was later found that the difference in ranks in the two subjects obtained by one of the students was wrongly taken as 3 instead of 7 . Find the correct coefficient of rank correlation

SOLUTION

N = 10 , R = 0.5

Incorrect d = 3 while

correct d = 7

STEP – 1

$$\mathbf{R} = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

$$0.5 = 1 - \frac{6\Sigma d^2}{10(100 - 1)}$$

$$0.5 = 1 - \frac{6\Sigma d^2}{10(99)}$$

$$0.5 = 1 - \frac{\Sigma d^2}{165}$$

$$\frac{\Sigma d^2}{165} = 0.5$$

$$\Sigma d^2 = 82.5$$

STEP 2

$$\begin{array}{lll} \Sigma d^2 & = & 82.5 \\ -3^2 & & -9 \\ & & \quad +40 \\ +7^2 & & +49 \\ \hline \Sigma d^2_{\text{correct}} & = & 122.5 \end{array}$$

STEP 3

$$\mathbf{R}_{\text{correct}} = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

$$= 1 - \frac{6(122.5)}{10(100 - 1)}$$

$$= 1 - \frac{6(122.5)}{10(99)}$$

$$= 1 - \frac{245}{330}$$

$$= 1 - \frac{49}{66}$$

$$= \frac{17}{66}$$

$$= 0.2575$$

02. Solve the following minimal assignment problem and hence find the minimum time where '–' indicates that job cannot be assigned to the machine

	Processing time in hrs				
Machines	A	B	C	D	E
M1	9	11	15	10	11
M2	12	9	–	10	9
M3	–	11	14	11	7
M4	14	8	12	7	8

SOLUTION

$$\begin{matrix} 9 & 11 & 15 & 10 & 11 \\ 12 & 9 & \infty & 10 & 9 \\ \infty & 11 & 14 & 11 & 7 \\ 14 & 8 & 12 & 7 & 8 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

– Due to limited space , machine M_2 cannot be placed in C and M_3 cannot be placed in A . Hence '∞'

– Adding a dummy m/c M_5 with zero cost to balance the matrix

$$\begin{matrix} 0 & 2 & 6 & 1 & 2 \\ 3 & 0 & \infty & 1 & 0 \\ \infty & 4 & 7 & 4 & 0 \\ 7 & 1 & 5 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

Reducing the matrix using ROW MINIMUM

$$\begin{matrix} \boxed{0} & 2 & 6 & 1 & 2 \\ 3 & \boxed{0} & \infty & 1 & \cancel{0} \\ \infty & 4 & 7 & 4 & \boxed{0} \\ 7 & 1 & 5 & \boxed{0} & 1 \\ \cancel{0} & \cancel{0} & \boxed{0} & \cancel{0} & \cancel{0} \end{matrix}$$

Allocation using

SINGLE ZERO ROW – COLUMN METHOD

Since every row and every column contains an assigned zero ,

the ASSIGNMNET PROBLEM IS SOLVED

OPTIMAL ASSIGNMENT : M_1 – A , M_2 – B , M_3 – E , M_4 – D , M_5 – C (DUMMY)

Min Cost = 9 + 9 + 7 + 7 = 3200 hrs

03. There are five jobs , each of which is to be processed through three machines A , B and C in the order ABC . Processing time in hours are shown in the following table .Determine the optimal sequence for the five jobs and the minimum elapsed time . Also find the idle time for three machines

Job	1	2	3	4	5
A	3	8	7	5	4
B	4	5	1	2	3
C	7	9	5	6	10

SOLUTION :

STEP 1 : Min time on m/c A = 3 ;

Max time on m/c B = 5 Min (m/c C) $\geq$ Max (m/c B)

Min time on m/c C = 5 ……… condition satisfied to convert 3 m/c's to 2 m/c's

STEP 2 : CONVERTING TO 2 FICTITIOUS M/C'S G & H

G = A + B

H = B + C

Job	1	2	3	4	5
G	7	13	8	7	7
H	11	14	6	8	13

STEP 3 : OPTIMAL SEQUENCE

Min time = 6 on job 3 on machine H . Place the job at the end of the sequence

				3

Next min time = 7 on jobs 1 , 4 & 5 on machine G . Place them randomly at the start of the sequence

1	**4**	**5**		**3**

RANDOM

OPTIMAL SEQUENCE

1	**4**	**5**	**2**	**3**

STEP 4 : WORK TABLE

Job	1	4	5	2	3		TOTAL PROCESSING TIME
A	3	5	4	8	7	=	27 hrs
B	4	2	3	5	1	=	15 hrs
C	7	6	10	9	5	=	37 hrs

JOBS	M/c A		IDLE	M/c B		IDLE	M/c C		IDLE
	IN	OUT	TIME	IN	OUT	TIME	IN	OUT	TIME
						3			7
1	0	3	--	3	7	1	7	14	--
4	3	8	--	8	10	2	14	20	--
5	8	12	--	12	15	5	20	30	--
2	12	20	--	20	25	2	30	39	--
3	20	27	17	27	28	16	39	44	--

STEP 5 : Total elapsed time T = 44 hrs

Idle time on M/c A = T – (sum of processing time of all 5 jobs on M/c A)

= 44 – 27

= 17 hrs

Idle time on M/c B = T – (sum of processing time of all 5 jobs on M/c B)

= 44 – 15

= 29 hrs (CHECK – 3 + 1 + 2 + 5 + 2 + 16 = 29)

Idle time on M/c C = T – (sum of processing time of all 5 jobs on M/c C)

= 44 – 37

= 7 hrs

Q6. (A) Attempt any TWO of the following **(06)**

01. Find the true discount , banker's discount and banker's gain on a bill of ₹ 36,600 due 4 months hence discounted at 5% p.a.

SOLUTION

STEP 1 :

FV = PW+ Int on PW for 4 months @ 5% p.a.

$$36600 = PW + PW \times \frac{4}{12} \times \frac{5}{100}$$

$$36600 = PW + \frac{PW}{60}$$

$$36600 = \frac{61}{60} PW$$

$$PW = \frac{36600 \times 60}{61}$$

= ₹ 36,000

STEP 2 :

TD = Int on PW for 4 months @ 5% p.a.

$$= 36000 \times \frac{4}{12} \times \frac{5}{100}$$

= ₹ 600

STEP 3 :

BD = Int on FV for 4 months @ 5% p.a.

$$= 36600 \times \frac{4}{12} \times \frac{5}{100}$$

= ₹ 610

STEP 4 :

BG = BD – TD

= 610 – 600

= ₹ 10

02. Mr Anil wants to invest at most ₹ 60,000 in Fixed Deposit (F.D.) and Public Provident Fund (P.P.F.) He wants to invest at least ₹ 20,000 in F.D. and at least ₹ 15,000 in P.P.F. . The rate of interest on F.D. is 8% p.a. and that on P.P.F. is 10% p.a. Formulate the above problem as L.P.P to determine maximum yearly income

Let
Amt invested in FD = x
Amt invested in PPF = y

CONSTRAINTS

1) Since Mr Anil wants to invest at most Rs 60,000 in Fixed Deposit (F.D.) and Public Provident Fund (P.P.F.)

$$x + y \le 60000$$

2) Anil wants to invest at least Rs 20,000 in F.D. and at least Rs 15,000 in P.P.F

$$x \ge 20000 \quad \& \quad y \ge 15000$$

3) since x & y are amounts invested

$$x , y \ge 0$$

OBJECTIVE FUNCTION

The rate of interest on F.D. is 8% p.a. and that on P.P.F. is 10% p.a

Yearly income = $0.08x + 0.10y$

Maximize Z = $0.08x + 0.10y$

LPP MODEL

Maximize Z = $0.08x + 0.10y$
Subject to

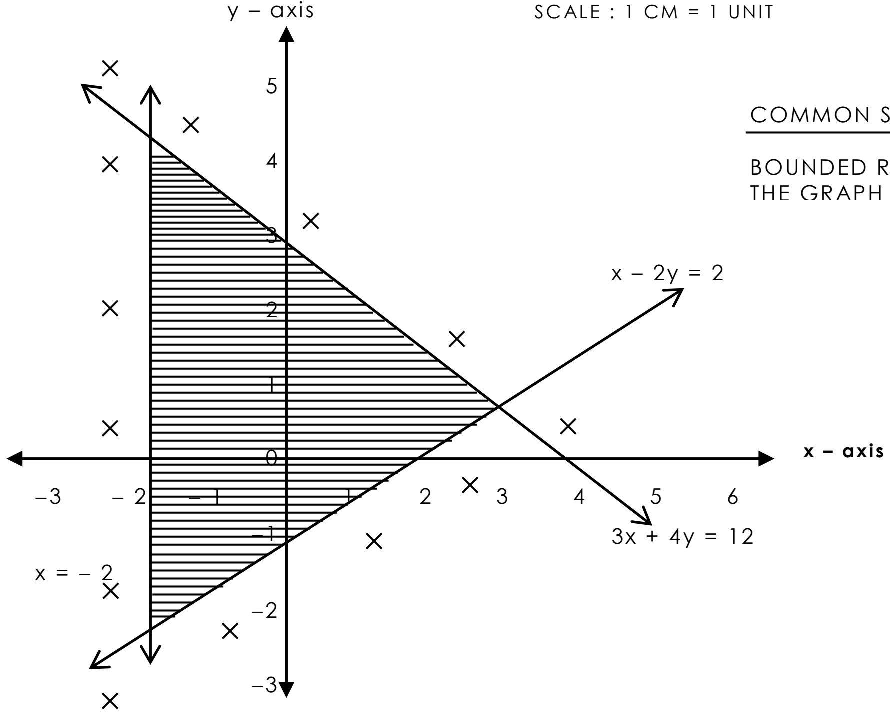
$$x + y \le 60000$$
$$x \ge 20000$$
$$y \ge 15000$$
$$x , y \ge 0$$

03. Find graphical solution for the following system of linear in equations

$3x + 4y \le 12$; $x - 2y \le 2$; $x \ge -2$

$3x + 4y \le 12$	$3x + 4y = 12$	Put (0,0) in $3x + 4y \le 12$
	cuts x – axis at (4,0)	$0 \le 12$
	cuts y – axis at (0,3)	SS : ORIGIN SIDE
$x - 2y \le 2$	$x - 2y = 2$	Put (0,0) in $x - 2y \le 2$
	cuts x – axis at (2,0)	$0 \le 2$
	cuts y – axis at (0,–1)	SS : ORIGIN SIDE
$x \ge -2$	$x = -2$	Put (0,0) in $x \ge -2$
	parallel to y – axis ,	$0 \ge -2$
	passing through (–2,0)	SS : ORIGIN SIDE

SCALE : 1 CM = 1 UNIT



COMMON SOLUTION SET

BOUNDED REGION AS SHOWN IN THE GRAPH

(B) Attempt any TWO of the following **(08)**

01. Mr. Rana plans to save for his son's higher inv studies . He wants to accumulate a sum of ₹ 2,00,000 at the end of 4 years . How much should he invest at the end of each year from now , if he can get interest compounded at 10% p.a. ($1.1^4 = 1.4641$)

SOLUTION :

A = ₹ 2,00,000 ; i = 10% = 0.1; n = 4

$$A = C\left(\frac{(1+i)^n - 1}{i}\right)$$

$$2,00,000 = C\left(\frac{(1+0.1)^4 - 1}{0.1}\right)$$

$$2,00,000 = C\left(\frac{(1.1)^4 - 1}{0.1}\right)$$

$$2,00,000 = C\left(\frac{1.4641 - 1}{0.1}\right)$$

$$2,00,000 = C\left(\frac{0.4641}{0.1}\right)$$

$$2,00,000 = C\left(\frac{4.641}{1}\right)$$

$$C = \frac{2,00,000}{4.641} \rightarrow$$

= ₹ 43,090

LOG CALC

	5.3010
–	0.6666
AL	4.6344
	43090

02. a car valued at ₹ 4,00,000 is insured for ₹ 2,50,000 . The rate of premium is 5% less 20% . How much loss does the owner bear including premium , if the value of the car is reduced to 60% of its original value

Solution

Value of car = ₹ 4,00,000

Insured value = ₹ 2,50,000

Rate of premium = 5 % less 20%.

Premium $= \frac{5}{100} \times 2,50,000$

= ₹ 12,500

less 20% disc – 2,500

Net Premium = ₹ 10,000

Since the value of the car is reduced to 60% of its original value , the loss on the car is 40%

Loss $= \frac{40}{100} \times 4,00,000$

= ₹ 1,60,000

Claim $= \frac{\text{insured val.}}{\text{Property val.}} \times \text{loss}$

$= \frac{2,50,000}{4,00,000} \times 1,60,000$

= ₹ 1,00,000

Loss	=	1,60,000
Less claim	–	1,00,000
Net loss	=	60,000
Add premium	+	10,000
Net loss Incl. premium	= ₹	70,000

03. The defects on a plywood sheet occurs at random with an average of one defect per 50 sq. ft. What is the probability that such sheet will have

(a) no defects (b) at least one defect

(Use e^{-1} = 0.3678)

SOLUTION

m = average number of defects in a ply wood sheet = 1

r.v $X \sim P(1)$

P(sheet will have no defects)

= P(0)

$= \frac{e^{-1}1^{0}}{0!}$ Using $P(x) = \frac{e^{-m}.m^{x}}{x!}$

$= e^{-1}.(1)$

= 0.3678

P(sheet will have at least 1 defect)

= $P(x \geq 1)$

= P(1) + P(2) + ...

= 1 – P(0)

= 1 – 0.3678

= 0.6322

DO NOT STOP

GET READY FOR NEXT