PAPER – II : MODEL PAPER – 02

(BASED ON MARCH 2015**)**

MATHEMATICS & STATISTICS

COMMERCE

TIME : 1 HR 30 MIN **MARKS : 40**

Q4. Attempt any six of the following **(12)**

01. The ratio of number of boys and girls in a school is 3 : 2 . If 20 % of the boys and 30% of the girls are scholarship holders , find the percentage of students who are not scholarship holders

SOLUTION

Let boys = 3x & girls = 2x , Total = 5x

$$\text{Scholarship holders} = \frac{20}{100}(3x) + \frac{30}{100}(2x)$$

$$= \frac{3x}{5} + \frac{3x}{5}$$

$$= \frac{6x}{5}$$

$$\text{Non scholarship holders} = 5x - \frac{6x}{5} = \frac{19x}{5}$$

Hence percentage number of students who are not scholarship holders

$$= \frac{\frac{19x}{5}}{5x} \times 100$$

$$= \frac{19}{25} \times 100 = 76\%$$

02. Calculate CDR for city A and city B and compare

Age Group (Years)	Population A		Population B	
	Population in'000	No. of Deaths	Population in'000	No. of Deaths
0 – 15	10	200	14	320
15 – 60	30	300	44	490
60 & above	20	400	21	462

$$CDR(A) = \frac{\Sigma D}{\Sigma P} = \frac{900}{60} = 15$$

(DEATHS PER 000)

$$CDR(B) = \frac{\Sigma D}{\Sigma P} = \frac{1272}{79} = 16.10$$

(DEATHS PER 000)

LOG CALC
3.1045
– 1.8976
AL 1.2069
16.10

COMMENT : CDR(A) < CDR(B) . HENCE POPULATION A IS HEALTHIER THAN POPULATION B

03. The coefficient of rank correlation between marks in two subjects obtained by a group of students is – 0.5 . If the sum of the squares of the difference in ranks is 126 , find the number of students in the group

SOLUTION

$R = -0.5$; $\Sigma d^2 = 126$

$$R = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$$

$$-0.5 = 1 - \frac{6(126)}{n(n^2-1)}$$

$$\frac{6(126)}{n(n^2-1)} = 1 + 0.5$$

$$\frac{6(126)}{n(n^2-1)} = 1.5$$

$$\frac{6(126)}{n(n^2-1)} = \frac{3}{2}$$

$$n(n^2-1) = \frac{6 \times 126 \times 2}{3}$$

$$n(n^2-1) = 2 \times 126 \times 2$$

$$(n-1).n.(n+1) = 7 \times 8 \times 9$$

On comparing , $n = 8$

04. a salesman receives 4% commission on the sales upto ₹ 10,000 and 5% commission on sales over ₹ 10,000 . Find his total income on the sale of ₹ 35,000

SOLUTION

Sale = ₹ 35,000

salesman receives 4% commission on the sales upto □ 10,000 and 5% commission on sales over □ 10,000

∴ His total income

$$= \frac{4}{100}(10{,}000) + \frac{5}{100}(35{,}000 - 10{,}000)$$

$$= \frac{4}{100}(10{,}000) + \frac{5}{100}(25{,}000)$$

$$= 400 + 1{,}250 \qquad = ₹\ 1{,}650$$

05. find the present worth of ₹ 2,320 due 4 years hence at 4% p.a. simple interest . Find the true discount

SOLUTION

STEP 1 :

F.V. = P.W.+ INT ON P.W. FOR 4 YEARS @ 4 % p.a.

$$2320 = \text{P.W.} + \text{P.W.} \times 4 \times \frac{4}{100}$$

$$2320 = \text{P.W.}\left(1 + \frac{16}{100}\right)$$

$$2320 = \text{P.W.}\ \frac{116}{100}$$

$$\text{P.W.} = \frac{2320 \times 100}{116}$$

P.W. = ₹ 2000

STEP 2 :

T.D. = F.V. – P.W.

T.D. = 2320 – 2000

T.D. = ₹ 320

06. Compute Age – Specific Death rate for the following data

Age Group	Population In '000	No. of deaths	AGE SDR = $\frac{D}{P}$
0 – 10	11	240	21.82
10 – 20	12	150	12.50
20 – 60	9	125	13.89
60 & above	2	90	45

07. The bivariate frequency distribution of weight (kg) and height of 60 students of SYJC as follows

Weight (in kg)	Height (in cm)			
	100 – 109	110 – 119	120 – 129	130 – 139
40 – 44	9	6	–	3
45 – 49	6	3	3	1
50 – 54	–	6	3	3
55 – 59	3	4	7	3

Find conditional distribution of weight when height lies between 110 – 119

SOLUTION

CONDITIONAL DISTRIBUTION OF WEIGHT WHEN HEIGHT LIES IN 110 – 119

CI	40 – 44	45 – 49	50 – 54	55 – 59	TOTAL
F	6	3	6	4	19

08. for a Binomial Distribution mean = 7/4 & sd = $\sqrt{21}/4$. Find n and p

SOLUTION

In Binomial Distribution , mean = np = 7/4 ……… (1)

Variance = npq = 21/16 ……… (2)

(1) ÷ (2)

$$\frac{npq}{np} = \frac{21/16}{7/4}$$

$$q = 3/4$$

$$p = 1 - q = 1 - 3/4 = 1/4$$

SUBS IN (1)

$$n \frac{1}{4} = \frac{7}{4}$$

$$n = 7$$

Q5. (A) Attempt any TWO of the following **(06)**

01. Find the sequence that minimizes total elapsed time (in hours) required to complete the following jobs on two machines M_1 and M_2 in the order M_1M_2 . Also find the minimum elapsed time and idle time for two machines

Job	A	B	C	D	E
M_1	3	7	4	5	7
M_2	6	2	7	3	4

STEP 1 FINDING THE OPTIMAL SEQUENCE

Min time = 2 on job B on machine M_2 .

				B

Next min time = 3 on job A on machine M_1 ,
on job D on machine M_2

A			**D**	**B**

Next min time = 4 on job C on machine M_1
on job E on machine M_2

A	**C**	**E**	**D**	**B**

OPTIMAL SEQUENCE

Step 2 : Work table

Job	A	C	E	D	B	**total process time**	
M_1	3	4	7	5	7	=	26 hrs
M_2	6	7	4	3	2	=	22 hrs

WORK TABLE

MACHINES

	M_1		**M_2**		**Idle time**
JOBS	**IN**	**OUT**	**IN**	**OUT**	**on M_2**
A	0	3	3	9	3
C	3	7	9	16	
E	7	14	16	20	
D	14	19	20	23	3
B	19	26	26	28	

Step 3 :

Total elapsed time T = 28 hrs

Idle time on M_1 = T – {sum of processing time of all jobs on M1}

= 28 – 26 = 2 hrs

Idle time on M_2 = T – {sum of processing time of all jobs on M2}

= 28 – 22 = 6 hrs (**CHECK** : 3 + 3 = 6)

02. A and B are partners in the company having capitals in the ratio 5 : 6 and the profits received by them are in the ratio 5 : 4 . If B invested the capital in the company for 20 months , determine the period of A's investment

SOLUTION

STEP 1 :

PARTNER's NAME	CAPITAL INVESTED	PERIOD OF INVESTMENT
A	□ 5k	n MONTHS
B	□ 6k	20 MONTHS

STEP 2 :

Profits will be shared in the

'RATIO OF PRODUCT OF CAPITAL INVESTED & PERIOD OF INVESTMENT'

A		B
5k x n	:	6k x 20

$= 5n : 120$

$= n : 24$

STEP 3 :

Profits is shared in the ratio 5 : 4 …… Given

$$\frac{n}{24} = \frac{5}{4}$$

$$n = \frac{5 \times 24}{4}$$

$$n = 30 \quad \text{MONTHS}$$

03. From a lot of 25 bulbs of which 5 are defective a sample of 5 bulbs was drawn at random with replacement . Find the probability that the sample will contain

a) exactly 1 defective bulb b) at least 1 defective bulb

SOLUTION :

5 bulbs was drawn at random with replacement, $n = 5$

for a trial : success - defective bulb

$$p = \frac{5}{25} = \frac{1}{5} \quad ; \quad q = 1 - p = \frac{4}{5}$$

r.v.x = number of successes = 0,1,2,..... , 5 $X \sim B(5, {}^{1}/_{5})$

a) P(exactly 1 defective bulb)

$= P(x = 1)$

$= {}^{5}C_{1} \left(\frac{1}{5}\right)^{1} \left(\frac{4}{5}\right)^{4}$

`

$= \dfrac{5\ (1)\ 256}{3125}$

$= \dfrac{256}{625}$

b) P(at least 1 defective bulb)

$= P(x \geq 1)$

$= P(1) + P(2) + \ldots\ldots\ldots$

$= 1 - P(0)$

$= 1 - {}^{5}C_{0} \left(\frac{1}{5}\right)^{0} \left(\frac{4}{5}\right)^{5}$

$= 1 - \dfrac{1024}{3125}$

$= \dfrac{2101}{3125}$

Q5. (B) Attempt any TWO of the following **(08)**

x : 60 61 62 63 64 65

l_x : 2000 1500 1000 540 120 0. Complete the life table

AGE x	lx	$dx = lx - lx+1$	$qx = \frac{dx}{lx}$	$px = 1 - qx$	$Lx = \frac{lx + lx+1}{2}$	Tx	$e_x^0 = \frac{Tx}{lx}$
60	2000	2000 – 1500 = 500	$\frac{500}{2000}$ = 0.25	1 – 0.25 = 0.75	1500 + 250 = 1750	4160	$\frac{4160}{2000}$ = 2.08
61	1500	1500 – 1000 = 500	$\frac{500}{1500}$ = 0.33	1 – 0.33 = 0.67	1000 + 250 = 1250	2410	$\frac{2410}{1500}$ = 1.609
62	1000	1000 – 540 = 460	$\frac{460}{1000}$ = 0.46	1 – 0.46 = 0.54	540 + 230 = 770	1160	$\frac{1160}{1000}$ = 1.16
63	540	540 – 120 = 420	$\frac{420}{540}$ = 0.78	1 – 0.77 = 0.22	120 + 210 = 330	390	$\frac{390}{540}$ = 0.7223
64	120	120 – 0 = 120	$\frac{120}{120}$ = 1	1 – 1 = 0	0 + 60 = 60	60	$\frac{60}{120}$ = 0.5
65	0	----	----	----	----	----	----

LOG CALCULATIONS FOR 'e_x^0' LOG 2410 – LOG 1500 LOG 390 – LOG 540

3.3820	2.5911
– 3.1761	– 2.7324
AL 0.2059	AL $\bar{1}.8587$
1.979	0.7223

02. from the following data about the sales and expenditure of a firm

	Sales (crores)	exp.(crores)
	X	Y
Mean	40	6
S.D	10	1.5

correlation coefficient = 0.9

Obtain regression lines to

a) estimate ~~sales~~ x for a proposed ~~adv.~~ y exp. of Rs 10 crores

b) estimate ~~adv.~~ y exp. for proposed ~~sales~~ x target of Rs 60 crore

SOLUTION

y on x	**x on y**
$byx = r \cdot \frac{\sigma y}{\sigma x}$	$bxy = r \cdot \frac{\sigma x}{\sigma y}$
$= 0.9 \times \frac{1.5}{10}$	$= 0.9 \times \frac{10}{1.5}$
$= \frac{1.35}{10}$	$= 9 \times \frac{10}{15}$
$= 0.135$	$= 6$
$y - \overline{y} = byx(x - \overline{x})$	$x - \overline{x} = bxy(y - \overline{y})$
$y - 6 = 0.135(x - 40)$	$x - 40 = 6(y - 6)$
$y - 6 = 0.135x - 5.4$	$x - 40 = 6y - 36$
$y = 0.135x - 5.4 + 6$	$x = 6y - 36 + 40$
$y = 0.135x + 0.6$	$x = 6y + 4$
Put $x = 60$	Put $y = 10$
$y = 0.135(60) + 0.6$	$x = 6(10) + 4$
$y = 8.1 + 0.6 = 8.7$	$x = 60 + 4 = 64$
adv exp. = ₹ 8.7 cr when Sales = ₹ 60 cr	sales = ₹ 64 cr when adv exp. = ₹ 10 cr

03. daily requirement of two vitamins V_1 , V_2 and the mineral M for a certain person is at least 30 units of V_1 ; 60 units of V_2 but not more than 40 units of M . He meets this requirement by taking two brands of tablets A and B

Tablet A has 3 units of V_1 , 4 units of V_2 and 1 unit of M

Tablet B has 1 unit of V_1 , 3 units of V_2 and 2 units of M

Tablet A costs ₹ 2 and B costs ₹ 1

Formulate this problem as LPP in order to determine the quantities of A & B he should take to minimize expenditure

SOLUTION

TABULATION :

INTAKE	TABLET A x	TABLET B y	REQUIREMENT
	CONTENTS/TABLET		
VITAMIN V_1	3	1	ATLEAST 30
VITAMIN V_2	4	3	ATLEAST 60
MINERAL M	1	2	ATMOST 40
COST / TABLET	2/–	1/–	

CONSTRAINT

1. Daily requirement of V_1 is at least 30 units , $3x + y \geq 30$
2. Daily requirement of V_2 is at least 60 units , $4x + 3y \geq 60$
3. Daily requirement of M is not more that 40 units , $x + 2y \leq 40$
4. Since x and y are no of tablets , cannot be –ve , $x , y \geq 0$

OBJECTIVE FUNCTION

Total cost $= 2x + y$ (in rupees)
$\therefore$ Minimize $z = 2x + y$

LPP MODEL

Minimize $z = 2x + y$,

Subject to
$3x + y \geq 30$, $4x + 3y \geq 60$, $x + 2y \leq 40$, $x , y \geq 0$

Q6. (A) Attempt any TWO of the following **(06)**

01. The equations given of the two regression lines are
$2x - y - 15 = 0$ & $3x - 4y + 25 = 0$ Find Correlation coefficient

SOLUTION

STEP 1

ASSUME

XON Y : $2x - y - 15 = 0$

$2x = y + 15$

$x = \frac{1}{2}y + \frac{15}{2}$

$bxy = \frac{1}{2}$

Y ON X : $3x - 4y + 25 = 0$

$4y = 3x + 25$

$y = \frac{3}{4}x + \frac{25}{4}$

$byx = \frac{3}{4}$

STEP 2

$r^2 = bxy \cdot byx$

$= \frac{1}{2} \times \frac{3}{4}$

$= \frac{3}{8}$

Since $0 \le r^2 \le 1$
Our assumptions are correct

$r = \pm\sqrt{\frac{3}{8}}$

$r = +\sqrt{\frac{3}{8}}$ (byx & bxy are + ve)

$\log r = \frac{1}{2}\left(\log 3 - \log 8\right)$

$\log r = \frac{1}{2}\left(0.4771 - 0.9031\right)$

$\log r = 0.2386 - 0.4516$

$\log r = \bar{1}.7870$

$r = AL(\bar{1}.7870)$

$r = 0.6124$

03. The number of complaints which a bank manager receives per day is a Poisson random variable with parameter m = 4 . Find the probability that the manager will receive at most two complaints on any given day $(e^{-4} = 0.0183)$

SOLUTION

m = average number of complaints a bank manager receives per day = 4

r.v $X \sim P(4)$

P(at most two complaints on any given day)

$= P(x \leq 2)$

$= P(0) + P(1) + P(2)$

$= \frac{e^{-4}.4^0}{0!} + \frac{e^{-4}.4^1}{1!} + \frac{e^{-4}.4^2}{2!}$ Using $P(x) = \frac{e^{-m}.m^x}{x!}$

$= e^{-4}.\left(\frac{1}{1} + \frac{4}{1} + \frac{16}{2}\right)$

$= 0.0183\ (1 + 4 + 8)$

$= 0.0183(13)$

$= 0.2379$

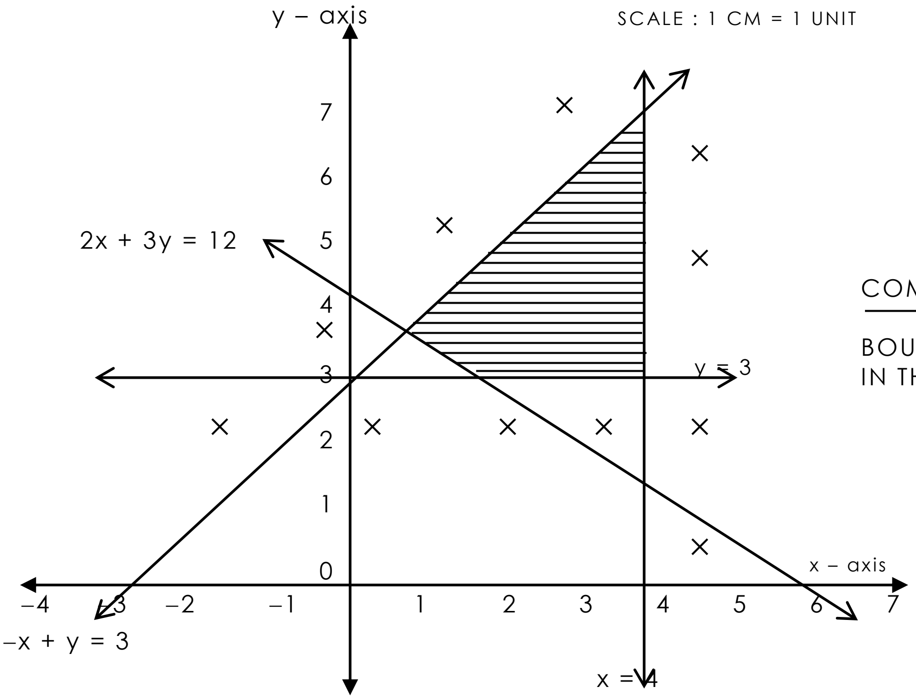
03. Find graphical solution for the following system of linear in equations

$$2x + 3y \geq 12 \quad ; \quad -x + y \leq 3 \quad ; \quad x \leq 4 \quad ; \quad y \geq 3$$

SOLUTION

$2x + 3y \geq 12$	$2x + 3y = 12$	Put (0,0) in $2x + 3y \geq 12$
	cuts x – axis at (6,0)	$0 \geq 12$
	cuts y – axis at (0,4)	(NOT SATISIFIED)
		SS : NON ORIGIN SIDE
$-x + y \leq 3$	$-x + y = 3$	Put (0,0) in $-x + y \leq 3$
	cuts x – axis at (–3,0)	$0 \leq 3$
	cuts y – axis at (0,3)	SS : ORIGIN SIDE
$x \leq 4$	$x = 4$	Put (0,0) in $x \leq 4$
	parallel to y – axis	$0 \leq 4$
	passing through (4,0)	SS : ORIGIN SIDE
$y \geq 3$	$y = 3$	Put (0,0) in $y \geq 3$
	parallel to x – axis	$0 \geq 3$
	passing through (0,3)	SS : NON ORIGIN SIDE

SCALE : 1 CM = 1 UNIT



COMMON SOLUTION SET

BOUNDED REGION AS SHOWN IN THE GRAPH

(B) Attempt any TWO of the following (08)

01. the value of godown of ₹ 40,000 contains stock worth ₹ 2,40,000 . They were insured for ₹ 25,000 and 80% of the stock respectively . Due to fire , stock worth ₹ 30,000 was completely destroyed while remaining was reduced to 60% of its value . The damage to the godown was ₹ 20,000 . What sum can be claimed under the policy

SOLUTION

GODOWN

Property value = ₹ 40,000

Insured value = ₹ 25,000

Loss = ₹ 20,000

Claim $= \dfrac{\text{insured val.}}{\text{Property val.}} \times \text{loss}$

$= \dfrac{25,000}{40,000} \times 20,000$

= ₹ 12,500

STOCK IN GODOWN

Value of stock = ₹ 2,40,000

Insured value = 80% of the stock

Loss

Note : Since the remainder was reduced to 60% of its value the loss on it is 40%

$= 30,000 + \dfrac{40}{100}(2,40,000 - 30,000)$

$= 30,000 + \dfrac{40}{100}(2,10,000)$

= 30, 000 + 84,000

= ₹ 1,14,000

Since 80% of the stock was insured

Claim = 80% of loss

$= \dfrac{80}{100} \times 1,14,000$

= ₹ 91,200

Hence

Total claim = 12, 500 + 91,200 = ₹ 1,03,700

02. X : 6 2 10 4 8

Y : 9 11 5 8 7

Calculate Karl Pearsons correlation coefficient

x	y	$x-\bar{x}$	$y-\bar{y}$	$(x-\bar{x})^2$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
6	9	0	1	0	1	0
2	11	−4	3	16	9	−12
10	5	4	−3	16	9	−12
4	8	−2	0	4	0	0
8	7	2	−1	4	1	−2
30 Σx	40 Σy	0 $\Sigma(x-\bar{x})$	0 $\Sigma(y-\bar{y})$	40 $\Sigma(x-\bar{x})^2$	20 $\Sigma(y-\bar{y})^2$	−26 $\Sigma(x-\bar{x})(y-\bar{y})$
$\bar{x} = 6$ $\bar{y} = 8$						

$$r = \frac{\Sigma(x-\bar{x}).(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}}.$$

$$r = \frac{-26}{\sqrt{40} \times \sqrt{20}}$$

$$r = \frac{-26}{\sqrt{40 \times 20}}$$

$$\text{let} \quad r' = \frac{26}{\sqrt{40 \times 20}} \quad (\text{Since } \log a,\ a > 0)$$

$$\log r' = \log 26 - \frac{1}{2}\left(\log 40 + \log 20\right)$$

$$\log r' = 1.4150 - \frac{1}{2}[1.6021 + 1.3010]$$

$$\log r' = 1.4150 - \frac{1}{2}(2.9031)$$

$$\log r' = 1.4150 - 1.4516$$

$$\log r' = \bar{1}.9634$$

$$r' = AL(\bar{1}.9634)$$

$$r' = 0.9191$$

$$r = -0.9191$$

03. A job production unit has four jobs A , B , C , D which can be manufactured on each of the four machines P , Q , R and S . The processing cost of each job is given in the following table

Jobs	Machines			
	P	Q	R	S
	Processing Cost (□)			
A	31	25	33	29
B	25	24	23	21
C	19	21	23	40
D	38	36	34	40

How should the jobs be assigned to the four machines so that the total processing cost is minimum .

$$\begin{array}{cccc} 6 & 0 & 8 & 4 \\ 4 & 3 & 2 & 0 \\ 0 & 2 & 4 & 5 \\ 4 & 2 & 0 & 6 \end{array}$$

Reducing the matrix using ROW MINIMUM

$$\begin{array}{cccc} 6 & \boxed{0} & 8 & 4 \\ 4 & 3 & 2 & \boxed{0} \\ \boxed{0} & 2 & 4 & 5 \\ 4 & 2 & \boxed{0} & 6 \end{array}$$

Allocation using SINGLE ZERO ROW – COLUMN METHOD

Since every row and every column contains an assigned zero ,

the ASSIGNMNET PROBLEM IS SOLVED .

OPTIMAL ASSIGNMENT : A – Q ; B – S ; C – P ; D – R

Min Cost = 25 + 21 + 19 + 34 = ₹ 99

DO NOT STOP

GET READY FOR NEXT