PAPER – I : MODEL PAPER – 03

(BASED ON MARCH 2015**)**

MATHEMATICS & STATISTICS

COMMERCE

TIME : 1 HR 30 MIN **MARKS : 40**

Q1. (A) Attempt any six of the following **(12)**

01. Express the following statement in symbolic form and write its truth value

"3 < 5 if and only if 3i is a real number"

SOLUTION

LET : $p \equiv 3 < 5$, TRUTH VALUE : T

$q \equiv$ 3i is a real number , TRUTH VALUE : F

3 < 5 if and only if 3i is a real number $\equiv p \leftrightarrow q$

$\equiv T \leftrightarrow F$

$= F$

02. $A = \begin{pmatrix} 3 & -1 \\ 2 & 4 \end{pmatrix}$; $B = \begin{pmatrix} 1 & 2 \\ -3 & 0 \end{pmatrix}$ Find the matrix X such that $2X + 3A - 4B = 0$

SOLUTION

$$2X + 3A - 4B = 0$$

$$2X = 4B - 3A$$

$$= 4\begin{pmatrix} 1 & 2 \\ -3 & 0 \end{pmatrix} - 3\begin{pmatrix} 3 & -1 \\ 2 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 8 \\ -12 & 0 \end{pmatrix} - \begin{pmatrix} 9 & -3 \\ 6 & 12 \end{pmatrix}$$

$$2X = \begin{pmatrix} -5 & 11 \\ -18 & -12 \end{pmatrix}$$

$$X = \frac{1}{2}\begin{pmatrix} -5 & 11 \\ -18 & -12 \end{pmatrix}$$

03. Find the value of k if the function

$$f(x) = \frac{\sin 9x}{4x} \quad ; \quad x \neq 0$$

$$= k \quad ; \quad x = 0 \quad \text{is continuous at } x = 0$$

SOLUTION

STEP 1: $\lim\limits_{x \to 0} f(x)$

$$= \lim_{x \to 0} \frac{\sin 9x}{4x}$$

$$= \lim_{x \to 0} \frac{9}{4} \frac{\sin 9x}{9x}$$

$$= 9/4$$

STEP 2 : Since f is continuous at x = 0 ; $f(0) = \lim\limits_{x \to 0} f(x)$

$$k = 9/4$$

04. Find dy/dx if $y = \sin^{-1}(x^2)$

SOLUTION

$$y = \sin^{-1}(x^2)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^4}} \frac{d}{dx}(x^2) = \frac{2x}{\sqrt{1 - x^4}}$$

05. The total cost for production of Q items is given $C = Q^3 - 600Q^2 + 1200Q$. Find the values of Q for which average cost is decreasing

SOLUTION

$C = Q^3 - 600Q^2 + 1200Q$

AVERAGE COST : $C_A = C/Q$

$$= Q^2 - 600Q + 1200$$

For average cost decreasing ; $\frac{dCA}{dQ} < 0$

$$2Q - 600 < 0$$

$$2Q < 600$$

$$Q < 300$$

06. Evaluate : $\int \dfrac{1}{x(3 + \log x)}\, dx$

SOLUTION let $3 + \log x = t$

$$\frac{1}{x}\, dx = dt$$

$$= \int \frac{1}{t}\, dt$$

$$= \log | t | + c$$

$$= \log | 3 + \log x | + c$$

07. $A = \begin{pmatrix} 4 & -3 \\ 2 & 1 \end{pmatrix}$ Show that : $A^2 - 5A + 10I = 0$

$A^2 - 5A + 10I$

$$= \begin{pmatrix} 4 & -3 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 4 & -3 \\ 2 & 1 \end{pmatrix} - 5 \begin{pmatrix} 4 & -3 \\ 2 & 1 \end{pmatrix} + 10 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 16 - 6 & -12 - 3 \\ 8 + 2 & -6 + 1 \end{pmatrix} - \begin{pmatrix} 20 & -15 \\ 10 & 5 \end{pmatrix} + \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}$$

$$= \begin{pmatrix} 10 & -15 \\ 10 & -5 \end{pmatrix} - \begin{pmatrix} 20 & -15 \\ 10 & 5 \end{pmatrix} + \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}$$

$$= \begin{pmatrix} 10 - 20 + 10 & -15 + 15 + 0 \\ 10 - 10 + 0 & -5 - 5 + 10 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = O = \text{RHS}$$

08. Evaluate : $\int x.\log x\, dx$

$$\int \log x \,.\, x\, dx$$

$$\log x \int x \ dx - \int \left(\frac{d}{dx} \log x \int x\, dx \right) dx$$

$$\log x \,.\, \frac{x^2}{2} - \int \frac{1}{x} \frac{x^2}{2}\, dx$$

$$\frac{x^2}{2}.\log x - \frac{1}{2} x \ dx$$

$$\frac{x^2}{2}.\log x - \frac{x^2}{4} + c$$

Q2. (A) Attempt any TWO of the following **(06)**

01. Prove : $\sim (p \leftrightarrow q) \equiv (\sim p \wedge q) \vee (p \wedge \sim q)$

					col 1			**col 2**
p	q	$\sim p$	$-q$	$p \leftrightarrow q$	$\sim (p \leftrightarrow q)$	$\sim p \wedge q$	$p \wedge \sim q$	$(\sim p \wedge q) \vee (p \wedge \sim q)$
T	T	F	F	T	F	F	F	F
T	F	F	T	F	T	F	T	T
F	T	T	F	F	T	T	F	T
F	F	T	T	T	F	F	F	F

Since truth values in col 1 & col 2 are identical , $\sim (p \leftrightarrow q) \equiv (\sim p \wedge q) \vee (p \wedge \sim q)$

02. Examine the continuity of the following function

$$f(x) = 5x - 3 \quad ; \quad 0 \le x < 1$$
$$= x^2 + 1 \quad ; \quad 1 \le x \le 2 \qquad \text{at } x = 1$$

SOLUTION

STEP 1:

$$\lim_{x \to 1-} f(x)$$
$$= \lim_{x \to 1} 5x - 3$$
$$= 5(1) - 3$$
$$= 2$$

STEP 2

$$\lim_{x \to 1+} f(x)$$
$$= \lim_{x \to 1} x^2 + 1$$
$$= 1 + 1$$
$$= 2$$

STEP 3 $f(1) = 1 + 1$

$= 2$

STEP 4

$$\lim_{x \to 1-} f(x) = \lim_{x \to 1+} f(x) = f(1)$$

f is CONTINUOUS at $x = 1$

03. Find $\dfrac{dy}{dx}$ if $y = \cot^{-1}\left[\dfrac{1 + 12x^2}{x}\right]$

SOLUTION $y = \tan^{-1}\left[\dfrac{x}{1 + 12x^2}\right]$

$$y = \tan^{-1}\left[\frac{4x - 3x}{1 + 4x.3x}\right]$$

$$y = \tan^{-1} 4x - \tan^{-1} 3x$$

Differentiating wrt x

$$\frac{dy}{dx} = \frac{1}{1 + 16x^2} \frac{d\,4x}{dx} + \frac{1}{1 + 9x^2} \frac{d\,3x}{dx}$$

$$= \frac{4}{1 + 16x^2} + \frac{3}{1 + 9x^2}$$

(B) Attempt any TWO of the following **(08)**

01. $A = \begin{pmatrix} 1 & 2 & -2 \\ 0 & -2 & 1 \\ -1 & 3 & 1 \end{pmatrix}$ Find A^{-1} by using elementary transformation

SOLUTION

$$|A| = 1(-0 - 3) - 2(0 + 1) - 2(0 - 2)$$
$$= 1(-3) - 2(1) - 2(-2)$$
$$= -3 - 2 + 4$$
$$= -1$$
$$\neq 0 \quad \text{Hence } A^{-1} \text{ exists}$$

$$AA^{-1} = I$$

$$\begin{pmatrix} 1 & 2 & -2 \\ 0 & -2 & 1 \\ -1 & 3 & 0 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$R_3 + R_1$

$$\begin{pmatrix} 1 & 2 & -2 \\ 0 & -2 & 1 \\ 0 & 5 & -2 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

R_{23}

$$\begin{pmatrix} 1 & 2 & -2 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$R_2 + 2\,R_3$

$$\begin{pmatrix} 1 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$R_1 - 2\,R_2$

$$\begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} A^{-1} = \begin{pmatrix} -1 & -4 & -2 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$R_3 + 2\,R_2$

$$\begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A^{-1} = \begin{pmatrix} -1 & -4 & -2 \\ 1 & 2 & 1 \\ 2 & 5 & 2 \end{pmatrix}$$

$R_1 + 2\,R_3$

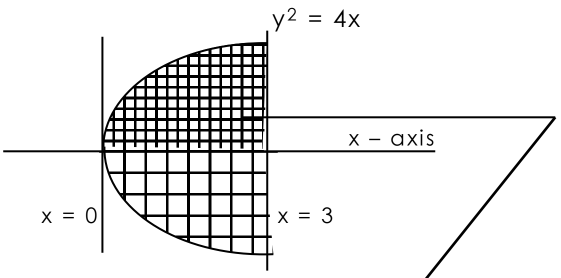
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A^{-1} = \begin{pmatrix} 3 & 6 & 2 \\ 1 & 2 & 1 \\ 2 & 5 & 2 \end{pmatrix}$$

$$I.A^{-1} = \begin{pmatrix} 3 & 6 & 2 \\ 1 & 2 & 1 \\ 2 & 5 & 2 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 3 & 6 & 2 \\ 1 & 2 & 1 \\ 2 & 5 & 2 \end{pmatrix}$$

02. Find the area of the region bounded by parabola $y^2 = 4x$ and $x = 3$

SOLUTION



$$A = 2 \int_0^3 y \; dx \quad \text{.... (BY SYMMETRY)}$$

$$= 2 \int_0^3 \sqrt{4x}\, dx$$

$$= 2 \int_0^3 2\sqrt{x}\, dx$$

$$= 4 \int_0^3 x^{1/2}\, dx$$

$$= 4 \left[\frac{x^{3/2}}{\frac{3}{2}} \right]_0^3$$

$$= \frac{8}{3} \left[x^{3/2} \right]_0^3$$

$$= \frac{8}{3} \left[3^{3/2} - 0^{3/2} \right]$$

$$= \frac{8}{3} \left[3\sqrt{3} \right]$$

$$= 8\sqrt{3} \text{ sq. units}$$

03. Demand function x , for a certain commodity is given as $x = 200 - 4p$, where p is the price . Find

i) elasticity of demand as function of p

ii) elasticity of demand when $p = 10$; $p = 30$. Interpret

iii) the price p for which elasticity of demand is equal to one

SOLUTION

STEP 1 : $x = 200 - 4p$

$$\frac{dx}{dp} = -4$$

STEP 2 :
$$\eta = \frac{-P}{D} \cdot \frac{dD}{dp}$$

$$= \frac{-p}{x} \cdot \frac{dx}{dp}$$

$$= \frac{-p}{200 - 4p} \cdot -4$$

$$= \frac{p}{50 - p}$$

STEP 3 :
$$\eta \Big|_{p = 10} = \frac{10}{50 - 10}$$

$$= \frac{10}{40}$$

$$= 0.25 < 1$$

Demand is relatively inelastic

STEP 4 :
$$\eta \Big|_{p = 30} = \frac{30}{50 - 30}$$

$$= \frac{30}{20}$$

$$= 1.5 > 1$$

Demand is relatively elastic

STEP 2 :
$$\eta = \frac{p}{50 - p}$$

$$1 = \frac{p}{50 - p}$$

$$50 - p = p$$

$$50 = 2p$$

$$p = 25$$

Q3. (A) Attempt any TWO of the following **(06)**

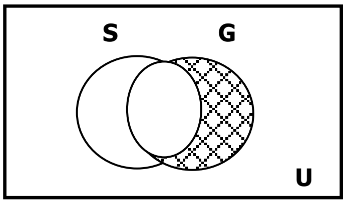
01. Express the truth of each of the following statements by Venn Diagram

a) Some graduates are not government servants

S ≡ set of all government servants

G ≡ set of all graduates

U ≡ set of all human beings

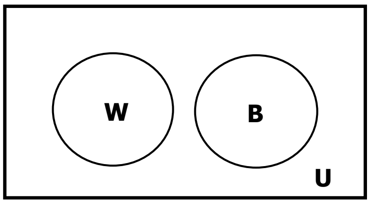


b) No wicket keeper is a bowler

W ≡ set of all wicket keepers

B ≡ set of all bowlers

U ≡ set of all players in a cricket team

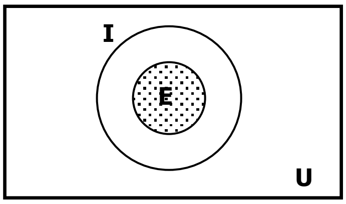


c) an equilateral triangle is an isosceles triangle

E ≡ set of all equilateral triangles

I ≡ set of all isosceles triangles

U ≡ set of all triangles



02. if f is continuous at x = 0 , then find f(0)

$$f(x) = \frac{10^x - 5^x - 2^x + 1}{\tan^2 x} \quad ; \quad x \neq 0$$

SOLUTION

$$\lim_{x \to 0} f(x)$$

$$= \lim_{x \to 0} \frac{10^x - 5^x - 2^x + 1}{\tan^2 x}$$

$$= \lim_{x \to 0} \frac{5^x\, 2^x - 5^x - 2^x + 1}{\tan^2 x}$$

$$= \lim_{x \to 0} \frac{5^x (2^x - 1) - 1(2^x - 1)}{\tan^2 x}$$

$$= \lim_{x \to 0} \frac{(5^x - 1)(2^x - 1)}{\tan^2 x}$$

$$= \lim_{x \to 0} \frac{\dfrac{(5^x - 1)(2^x - 1)}{x^2}}{\dfrac{\tan^2 x}{x^2}}$$

$$= \lim_{x \to 0} \frac{\dfrac{(5^x - 1)}{x}\dfrac{(2^x - 1)}{x}}{\left(\dfrac{\tan x}{x}\right)^2}$$

$$= \log 5 \,.\, \log 2$$

Since f is continuous at x = 0

$$f(0) = \lim_{x \to 0} f(x) = \log 5 \,.\, \log 2$$

03. find dy/dx if $x = e^{4t + 5}$; $y = e^{3t}$

SOLUTION

$$x = e^{4t+5}$$

$$\frac{dx}{dt} = e^{4t+5} \frac{d}{dt}(4t+5)$$

$$\frac{dx}{dt} = 4e^{4t+5}$$

$$y = e^{3t}$$

$$\frac{dy}{dt} = e^{3t} \frac{d}{dt}(3t)$$

$$\frac{dy}{dt} = 3e^{3t}$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$= \frac{3e^{3t}}{4e^{4t+5}}$$

$$= \frac{3}{4\ e^{t}}$$

(B) Attempt any TWO of the following **(08)**

01. Evaluate $\int \frac{2x - 1}{(x-1)(x+2)(x-3)} dx$

SOLUTION

$$\frac{2x - 1}{(x-1)(x+2)(x-3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x-3}$$

$$2x - 1 = A(x+2)(x-3) + B(x-1)(x-3) + C(x-1)(x+2)$$

Put $x = -2$

$$2(-2)-1 = B(-2-1)(-2-3)$$

$$-5 = B(-3)(-5)$$

$$-5 = B(15) \qquad \therefore\ B = -\frac{1}{3}$$

Put $x = 3$

$$2(3)-1 = C(3-1)(3+2)$$

$$6 - 1 = C(2)(5)$$

$$5 = C(10)$$

$$\therefore\ C = \frac{1}{2}$$

Put $x = 1$

$$2(1) - 1 = A(1+2)(1-3)$$

$$2 - 1 = A(3)(-2)$$

$$1 = A(-6)$$

$$\therefore\ A = -\frac{1}{6}$$

HENCE

$$\frac{2x - 1}{(x-1)(x+2)(x-3)} = \frac{\frac{-1}{6}}{x-1} - \frac{\frac{1}{3}}{x+2} + \frac{\frac{1}{2}}{x-3}$$

BACK IN THE SUM

$$= \int \frac{\frac{-1}{6}}{x-1} - \frac{\frac{1}{3}}{x+2} + \frac{\frac{1}{2}}{x-3} \, dx$$

$$= -\frac{1}{6}\log|x-1| \quad -\frac{1}{3}\log|x+2| + \frac{1}{2}\log|x-3| \quad c$$

02.

$$I = \int_3^9 \frac{\sqrt[3]{12-x}}{\sqrt[3]{x} + \sqrt[3]{12-x}}\, dx \quad \text{........(1)}$$

USING $\int_a^b f(x)dx = \int_b^b f(a+b-x)\,dx$

WE CHANGE 'x' TO '3 + 9 – x' i.e '12 – x'

$$I = \int_3^9 \frac{\sqrt[3]{12-(12-x)}}{\sqrt[3]{12-x} + \sqrt[3]{12-(12-x)}}\, dx$$

$$I = \int_3^9 \frac{\sqrt[3]{12-12+x}}{\sqrt[3]{12-x} + \sqrt[3]{12-12+x}}\, dx$$

$$I = \int_3^9 \frac{\sqrt[3]{x}}{\sqrt[3]{12-x} + \sqrt[3]{x}}\, dx \quad \text{........(2)}$$

(1) + (2)

$$I = \int_3^9 \frac{\sqrt[3]{12-x} + \sqrt[3]{x}}{\sqrt[3]{12-x} + \sqrt[3]{x}}\, dx$$

$$2I = \int_3^9 1\, dx$$

$$2I = \left(x\right)_3^9$$

$$2I = 9 - 3$$

$$2I = 6$$

$$I = 3$$

03. A firm wants to maximize its profit . The total cost function is $C = 370Q + 550$ and revenue $R = 730Q - 3Q^2$. Find the output for which profit is maximum and also find the profit amount at this output

SOLUTION

STEP 1 : Profit

$\pi = R - C$

$\pi = 730Q - 3Q^2 - (370Q + 550)$

$\pi = 730Q - 3Q^2 - 370Q - 550$

$\pi = 360Q - 3Q^2 - 550$

STEP 2 :

$$\frac{d\pi}{dQ} = 360 - 6Q$$

$$\frac{d^2\pi}{dQ^2} = -6$$

STEP 3 :

$$\frac{d\pi}{dQ} = 0$$

$360 - 6Q = 0$

$360 = 6Q$

$Q = 60$

STEP 4 :

$$\left.\frac{d^2\pi}{dQ^2}\right|_{Q = 60} = -6 < 0$$

Profit is maximum at Q = 60

STEP 5 :

Since profit is maximum at Q = 60

Maximum Profit

$= \pi \big|_{Q = 60}$

$= 360(60) - 3(60)^2 - 550$

$= 21600 - 3(3600) - 550$

$= 21600 - 10800 - 550$

$= 10800 - 550$

$= 10250$

DO NOT STOP

GET READY FOR NEXT

03. A firm wants to maximize its profit . The total cost function is $C = 370Q + 550$ and revenue $R = 730Q - 3Q^2$. Find the output for which profit is maximum and also find the profit amount at this output .

DO NOT STOP

GET READY FOR NEXT