PAPER - I : MODEL PAPER - 05

(BASED ON MARCH 2017**)**

MATHEMATICS & STATISTICS

COMMERCE

TIME : 1 HR 30 MIN **MARKS : 40**

Q1. (A) Attempt any six of the following (12)

01. Find x , y , z , w if $\begin{pmatrix} x+y & x-y \\ y+z+w & 2w-z \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 9 & 5 \end{pmatrix}$

SOLUTION :

$$\begin{matrix} x+y & x-y \\ y+z+w & 2w-z \end{matrix} = \begin{matrix} 2 & -1 \\ 9 & 5 \end{matrix}$$

By Equality of Matrices

$x + y = 2$

$x - y = -1$ On Solving $x = {}^{1}/_{2}$ & $y = {}^{3}/_{2}$

$$\begin{array}{rcl} y + z + w & = & 9 \\ 2w - z & = & 5 \\ \hline y + 3w & = & 14 \end{array}$$

sub $y = {}^{3}/_{2}$

$\frac{3}{2} + 3w = 14$

$3w = \frac{25}{2}$ $w = 25/6$

sub w =

$2w - z = 5$

$2\,\frac{25}{6} - z = 5$

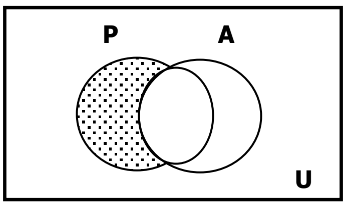
$\frac{25}{3} - 5 = z$ $z = 10/3$

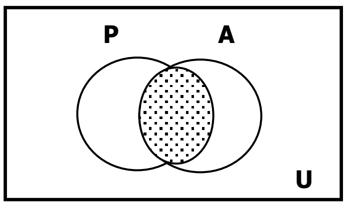
02. Express the truth of the following statements with the help of Venn Diagrams

a) there are politicians who are not actors

b) there are actors who are not politicians

P ≡ set of all politicians ; **A** ≡ set of all actors ; **U** ≡ set of all human beings





03. Find points of discontinuity , if any for the function $f(x) = \frac{x^2 - 9}{\sin x - 9}$

SOLUTION :

f is continuous for all $x \in R$, except at points where denominator

$$\sin x - 9 = 0$$

$$\sin x = 9$$

However $\sin x \neq 9$ since $-1 \leq \sin x \leq 1$

Hence f is continuous for all $x \in R$

04. Write negations of the following statements

a) Kiran is rich if and only if he is honest

Using : $\underline{\sim (P \leftrightarrow Q) \equiv (P \wedge \sim Q) \vee (Q \wedge \sim P)}$

Negation : Kiran is rich and he is not honest

OR

Kiran is honest and he is not rich

b) if $x \in A \cap B$, then $x \in A$ and $x \in B$

Using : $\underline{\sim (P \rightarrow Q) \equiv P \wedge \sim Q}$

Negation : $x \in A \cap B$ and $x \notin A$ OR $x \notin B$

05. Evaluate : $\int \tan^2 4x \, dx$

SOLUTION :

$$1 + \tan^2 x = \sec^2 x$$

$$\tan^2 x = \sec^2 x - 1$$

$$\tan^2 4x = \sec^2 4x - 1$$

$$\int \sec^2 4x - 1 \, dx$$

$$= \frac{\tan 4x}{4} - x + c$$

06. Find $\frac{d^2y}{dx^2}$ if $y = \log x$

SOLUTION : $y = \log x$

$$\frac{dy}{dx} = \frac{1}{x}$$

$$\frac{d^2y}{dx^2} = \frac{-1}{x^2}$$

07. Evaluate : $\int \frac{10x^9 + 10^x \log 10}{x^{10} + 10^x} dx$

SOLUTION :

$$x^{10} + 10^x = t$$

$$10x^9 + 10^x \log 10 \ dx = dt$$

$$\int \frac{1}{t} dt$$

$$= \log | t | + c$$

$$= \log | x^{10} + 10^x | + c$$

08. Find $\frac{dy}{dx}$, if $x^3 + y^2 + xy = 10$

SOLUTION : $x^3 + y^2 + xy = 10$

Differentiating wrt x

$$3x^2 + 2y \frac{dy}{dx} + x\frac{dy}{dx} + y = 0$$

$$3x^2 + y + (2y + x) \frac{dy}{dx} = 0$$

$$(2y + x) \frac{dy}{dx} = -(3x^2 + y)$$

$$\frac{dy}{dx} = \frac{-(3x^2 + y)}{2y + x}$$

Q2. (A) Attempt any TWO of the following **(06)**

01. $\begin{pmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{pmatrix}$ Find inverse of the matrix by ADJOINT METHOD

SOLUTION

COFACTOR'S

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix}$$

$$= 1(4 - 4) = 0$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} -1 & 2 \\ 1 & 4 \end{vmatrix}$$

$$= -1(-4 - 2) = 6$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} -1 & 1 \\ 1 & 2 \end{vmatrix}$$

$$= 1(-2 - 1) = -3$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix}$$

$$= -1(8 - 6) = -2$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix}$$

$$= 1(4 - 3) = 1$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix}$$

$$= -1(2 - 2) = 0$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix}$$

$$= 1(4 - 3) = 1$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 3 \\ -1 & 2 \end{vmatrix}$$

$$= -1(2 + 3) = -5$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix}$$

$$= 1(1 + 2) = 3$$

COFACTOR MATRIX OF A

$$= \begin{pmatrix} 0 & 6 & -3 \\ -2 & 1 & 0 \\ 1 & -5 & 3 \end{pmatrix}$$

ADJ A

= TRANSPOSE OF THE COFACTOR MATRIX

$$= \begin{pmatrix} 0 & -2 & 1 \\ 6 & 1 & -5 \\ -3 & 0 & 3 \end{pmatrix}$$

|A|

$$= 1(4 - 4) - 2(-4 - 2) + 3(-2 - 1)$$

$$= 1(0) - 2(-6) + 3(-3)$$

$$= 0 + 12 - 9$$

$$= 3$$

$$\mathbf{A^{-1}} = \frac{1}{|A|} . \text{adj } A$$

$$= \frac{1}{3} \begin{pmatrix} 0 & -2 & 1 \\ 6 & 1 & -5 \\ -3 & 0 & 3 \end{pmatrix}$$

02. $f(x) = \dfrac{x^7 - 128}{x^5 - 32}$; $x \neq 2$

$= k$; $x = 2$

Find k if f is continuous at x = 2

SOLUTION :

STEP 1

$$\lim_{x \to 2} f(x)$$

$$= \lim_{x \to 2} \frac{x^7 - 128}{x^5 - 32}$$

$$= \lim_{x \to 2} \frac{x^7 - 2^7}{x^5 - 2^5}$$

Divide Numerator & denominator by x – 2 ;

$x \to 2$; $x \neq 2$ $\therefore$ $x - 2 \neq 0$

$$= \lim_{x \to 2} \frac{\dfrac{x^7 - 2^7}{x - 2}}{\dfrac{x^5 - 2^5}{x - 2}}$$

$$= \frac{7(2)^{7-1}}{5(2)^{5-1}}$$

$$= \frac{7(2)^6}{5(2)^4}$$

$$= \frac{7}{5}(2)^2$$

$$= \frac{28}{5}$$

STEP 2 :

f(2) = k ……… given

STEP 3 :

Since *f* is continuous at x = 2

$f(2) = \lim_{x \to 2} f(x)$;

$k = 28/5$

03. if the demand function is $D = \dfrac{p + 6}{p - 3}$;

find elasticity of demand at p = 4

SOLUTION :

STEP 1 : $D = \dfrac{p + 6}{p - 3}$

$$\frac{dD}{dp} = \frac{(p-3)\frac{d}{dp}(p+6) - (p+6)\frac{d}{dp}(p-3)}{(p-3)^2}$$

$$= \frac{(p-3).1 - (p+6)(1)}{(p-3)^2}$$

$$= \frac{p - 3 - p - 6}{(p-3)^2}$$

$$= \frac{-9}{(p-3)^2}$$

STEP 2 : $\eta = \dfrac{-P}{D} . \dfrac{dD}{dp}$

$$= \frac{-\mathbf{p}}{\dfrac{\mathbf{p+6}}{\cancel{\mathbf{p-3}}}} . \frac{-\mathbf{9}}{(\mathbf{p-3})^{\cancel{\mathbf{2}}}}$$

$$= \frac{9p}{(p+6)(p-3)}$$

STEP 3 : $\eta \Big|_{p=4} = \dfrac{9(4)}{(4+6)(4-3)}$

$$= \frac{36}{10}$$

$$= 3.6 > 1$$

Demand is relatively elastic

(B) Attempt any TWO of the following **(08)**

01. Using truth table verify that : $(p \vee q) \to r \equiv (p \to r) \wedge (q \to r)$

SOLUTION

p	q	r	$q \vee r$	$p \vee q$	$(p \vee q) \to r$ (COL A)	$p \to r$	$q \to r$	$(p \to r) \wedge (q \to r)$ (COL B)
T	T	T	T	T	T	T	T	T
T	T	F	T	T	F	F	F	F
T	F	T	T	T	T	T	T	T
T	F	F	F	T	F	F	T	F
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	T	F	F
F	F	T	T	F	T	T	T	T
F	F	F	F	F	T	T	T	T

Since truth values in col A and col B are identical $(p \vee q) \to r \equiv (p \to r) \wedge (q \to r)$

02. If Mr. Rao orders x cupboards , with demand function as $p = 2x + \frac{32}{x^2} - \frac{5}{x}$

How many cupboards should he order for the most economical deal

SOLUTION

STEP 1 : COST

$$C = p.x$$

$$= \left(2x + \frac{32}{x^2} - \frac{5}{x}\right).x$$

$$= 2x^2 + \frac{32}{x} - 5$$

STEP 2 :

$$\frac{dC}{dx} = 4x - \frac{32}{x^2} =$$

$$= 4x - 32x^{-2}$$

$$\frac{d^2C}{dx^2} = 4 + 64x^{-3}$$

$$= 4 + \frac{64}{x^3}.$$

STEP 3 :

$$\frac{dC}{dx} = 0$$

$$4x - \frac{32}{x^2} = 0$$

$$4x = \frac{32}{x^2}$$

$$4x^3 = 32$$

$$x^3 = 8 \quad \therefore\ x = 2$$

STEP 4 :

$$\left.\frac{d^2C}{dx^2}\right|_{x = 2} = 4 + \frac{64}{2^3} > 0$$

Cost is minimum at x = 2

Mr. Rao must order 2 cupboards

03. $$\int_0^1 \frac{x.(\sin^{-1}x)^2}{\sqrt{1-x^2}}\,dx$$

$\sin^{-1}x = t \qquad x = \sin t$	when $x = 0$, $t = \sin^{-1}0 = 0$
$\frac{1}{\sqrt{1-x^2}}\,dx = dt$	$x = 1$, $t = \sin^{-1}1 = \pi/2$

$$= \int_0^{\pi/2} t^2.\sin t \quad dx$$

$$= \left\{ t^2 \int \sin t\, dt - \int \left(\frac{d}{dt} t^2 \int \sin t\, dt \right) dt \right\}_0^{\pi/2}$$

$$= \left\{ t^2 . -\cos t - \int 2t\, . -\cos t \;\, dt \right\}_0^{\pi/2}$$

$$= \left\{ -t^2.\; \cos t + 2 \int t\, .\, \cos t\;\, dt \right\}_0^{\pi/2}$$

$$= \left\{ -t^2.\; \cos t + 2 \left(t \int \cos t\; dt - \int \left(\frac{d}{dt}\; t \int \cos t\; dt \right) dt \right) \right\}_0^{\pi/2}$$

$$= \left\{ -t^2.\; \cos t + 2 \left(t\, .\, \sin t \;\; - \int 1.\sin t \;\; dt \right) \right\}_0^{\pi/2}$$

$$= \left\{ -t^2.\; \cos t + 2 \left(t\, .\, \sin t \;\; - \int \sin t \;\; dt \right) \right\}_0^{\pi/2}$$

$$= \left\{ -t^2.\; \cos t + 2 \left(t\, .\, \sin t \;\; + \cos t \right) \right\}_0^{\pi/2}$$

$$= \left\{ -t^2.\; \cos t \;+\; 2t.\sin t \;+\; 2\cos t \right\}_0^{\pi/2}$$

$$= \left(\frac{-\pi^2}{4} . \cos \frac{\pi}{2} + \; 2\,\frac{\pi}{2} . \sin \frac{\pi}{2} \;\; +2\cos \;\frac{\pi}{2} \right) \;\; - \left(-\,0\,.\, \cos 0 + 2(0).\sin 0 \; + 2\cos 0 \right)$$

$$= \left(0 \;+\; \pi\,(1) \;+\; 0\right) \quad - \left(0 \;+\; 0 \;+\; 2(1)\right)$$

$$= \quad \pi \;-\; 2$$

Q3. (A) Attempt any TWO of the following **(06)**

01. Solve the following equations by reduction method

$x + 3y + 3z = 16$;

$x + 4y + 4z = 21$;

$x + 3y + 4z = 19$

$AX = B$

$$\begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 4 \\ 1 & 3 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 16 \\ 21 \\ 19 \end{pmatrix}$$

$R_2 - R_1$

$$\begin{pmatrix} 1 & 3 & 3 \\ 0 & 1 & 1 \\ 1 & 3 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 16 \\ 5 \\ 19 \end{pmatrix}$$

$R_3 - R_1$

$$\begin{pmatrix} 1 & 3 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 16 \\ 5 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} x + 3y + 3z \\ y + z \\ z \end{pmatrix} = \begin{pmatrix} 16 \\ 5 \\ 3 \end{pmatrix}$$

BY EQUALITY OF TWO MATRICES

$z = 1$ .... (1)

$y + z = 3$

$y + 1 = 3$

$y = 2$ .... (2)

$x + 3y + 3z = 12$

$x + 3(2) + 3(1) = 12$

$x + 6 + 3 = 12$

$x + 9 = 12$

$x = 3$ .... (3)

SS { 3 , 2 , 1 }

02. $f(x) = x^2 . \sin\left(\frac{1}{x}\right)$; $x \neq 0$

$= 1$; $x = 0$

Discuss continuity at $x = 0$

SOLUTION

STEP 1

$\lim\limits_{x \to 0} f(x)$

$= \lim\limits_{x \to 0} x^2 . \sin\left(\frac{1}{x}\right)$

Since

$-1 \leq \sin\theta \leq 1$

$\therefore -1 \leq \sin\left(\frac{1}{x}\right) \leq 1$

$= \lim\limits_{x \to 0} x^2 . k$ where $k = \sin\left(\frac{1}{x}\right)$;

$-1 \leq k \leq 1$

$= 0.k$

$= 0$

STEP 2 :

$f(0) = 1$ .... given

STEP 3 :

$f(0) \neq \lim\limits_{x \to 0} f(x)$

$\therefore$ f is discontinuous at $x = 0$

STEP 4 :

Removable Discontinuity

f can be made continuous at $x = 0$ by redefining it as

$f(x) = x^2 . \sin\left(\frac{1}{x}\right)$; $x \neq 0$

$= 0$; $x = 0$

03. Find the values of x for which : $f(x) = 4x^3 - 12x^2 - 36x + 1$ is decreasing

SOLUTION

For f(x) decreasing ,

$f'(x) < 0$

$12x^2 - 24x - 36 < 0$

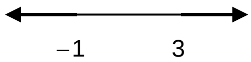
$x^2 - 2x - 3 < 0$

$(x - 3)(x + 1) < 0$

CASE 1 :

$x - 3 > 0$ & $x + 1 < 0$

$x > 3$ & $x < -1$

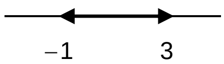


NOT POSSIBLE SO DISCARD

CASE 2 :

$x - 3 < 0$ & $x + 1 > 0$

$x < 3$ & $x > -1$



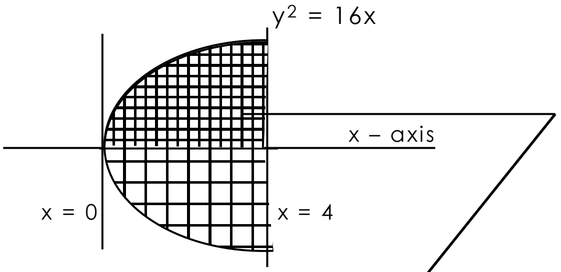
$-1 < x < 3$

$x \in (-1, 3)$

f **is decreasing for** $\mathbf{x \in (-1, 3)}$

(B) Attempt any TWO of the following **(08)**

01. Find the area of the region bounded by parabola $y^2 = 16x$ and $x = 4$



$$A = 2\int_0^4 y\,dx \quad \text{.... (BY SYMMETRY)}$$

$$= 2\int_0^4 \sqrt{16x}\,dx$$

$$= 2\int_0^4 4\sqrt{x}\,dx$$

$$= 8\int_0^4 x^{1/2}\,dx$$

$$= 8\left[\frac{x^{3/2}}{\frac{3}{2}}\right]_0^4$$

$$= \frac{16}{3}\left(x^{3/2}\right)_0^4$$

$$= \frac{16}{3}\left(4^{3/2} - 0^{3/2}\right)$$

$$= \frac{16}{3}\left(2^{2.3/2}\right)$$

$$= \frac{16}{3}\left(2^3\right)$$

$$= \frac{16}{3}(8)$$

$$= \frac{128}{3} \text{ sq. units}$$

02. if $x^3.y^5 = (x + y)^8$ then show that $\frac{dy}{dx} = \frac{y}{x}$

SOLUTION

$$x^3.y^5 = (x + y)^8$$

$$3\log x + 5\log y = 8\log(x + y)$$

$$3\,\frac{1}{x} + 5\,\frac{1}{y}\,\frac{dy}{dx} = 8\,\frac{1}{x + y}\,\frac{d}{dx}(x + y)$$

$$\frac{3}{x} + \frac{5}{y}\,\frac{dy}{dx} = \frac{8}{x + y}\left(1 + \frac{dy}{dx}\right)$$

$$\frac{3}{x} + \frac{5}{y}\,\frac{dy}{dx} = \frac{8}{x + y} + \frac{8}{x + y}\,\frac{dy}{dx}$$

$$\left(\frac{5}{y} - \frac{8}{x + y}\right)\frac{dy}{dx} = \frac{8}{x + y} - \frac{3}{x}$$

$$\frac{5x + 5y - 8y}{y(x + y)}\,\frac{dy}{dx} = \frac{8x - 3x - 3y}{x(x + y)}$$

$$\frac{5x - 3y}{y(x + y)}\,\frac{dy}{dx} = \frac{5x - 3y}{x\,(x + y)}$$

$$\frac{dy}{dx} = \frac{y}{x}$$

03. Evaluate $\displaystyle\int \frac{1 + \log x}{x(2 + \log x)(3 + \log x)} dx$

SOLUTION

$$\log x = t \qquad \therefore \frac{1}{x} . dx = dt$$

$$= \int \frac{1 + t}{(2 + t)(3 + t)} dt$$

$$\frac{1 + t}{(2 + t)(3 + t)} = \frac{A}{2 + t} + \frac{B}{3 + t}$$

$$1 + t = A(3 + t) + B(2 + t)$$

Put t = – 3

$$1 - 3 = B(2 - 3)$$

$$-2 = B(-1) \qquad \therefore B = 2$$

Put t = – 2

$$1 - 2 = A(3 - 2)$$

$$-1 = A(1) \qquad \therefore A = -1$$

HENCE

$$\frac{1 + t}{(2 + t)(3 + t)} = \frac{-1}{2 + t} + \frac{2}{3 + t}$$

BACK IN THE SUM

$$= \int \frac{-1}{2 + t} + \frac{2}{3 + t} dt$$

$$= -\log|2 + t| + 2\log|3 + t| + c$$

RESUBS. $= -\log|2 + \log x| + 2\log|3 + \log x| + c$

DO NOT STOP

GET READY FOR NEXT