# PAPER - I : MODEL PAPER - 02

**(BASED ON OCT – 2014)**

**MATHEMATICS & STATISTICS**

**COMMERCE**

**TIME : 1 HR 30 MIN** **MARKS : 40**

**Q1. (A) Attempt any six of the following (12)**

**01.** Write the following statements in symbolic form

**SOLUTION**

a) ABC is a triangle hence the points A, B & C are not collinear

P ≡ ABC is a triangle , Q ≡ points A, B & C are collinear

ABC is a triangle hence the points A, B & C are not collinear ≡ $P \rightarrow \sim Q$

b) It is not true that Subhash passed , then he is happy

P ≡ Subhash passed , Q ≡ Subhash is happy

It is not true that Subhash passed , then he is happy ≡ $\sim (P \rightarrow Q)$

**02** $A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \\ -1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 3 & 2 \\ 4 & -1 & -3 \end{pmatrix}$ Show that : AB is singular matrix

**SOLUTION**

$$AB = \begin{pmatrix} 1 & 2 \\ 3 & 2 \\ -1 & 0 \end{pmatrix} \begin{matrix} 1 & 3 & 2 \\ 4 & -1 & -3 \end{matrix}$$

$$= \begin{pmatrix} 1+8 & 3-2 & 2-6 \\ 3+8 & 9-2 & 6-6 \\ -1+0 & -3+0 & -2-0 \end{pmatrix}$$

$$= \begin{pmatrix} 9 & 1 & -4 \\ 11 & 7 & 0 \\ -1 & -3 & -2 \end{pmatrix}$$

$$|AB| = 9(-14+0) - 1(-22+0) - 4(-33+7)$$
$$= -126 + 22 + 104$$
$$= 0$$

Hence AB is a singular matrix

**03.** if $A = \begin{bmatrix} 7 & 1 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$; Verify : $|AB| = |A|.|B|$

**SOLUTION**

LHS

$$AB = \begin{bmatrix} 7 & 1 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 7+3 & 14-1 \\ 2+15 & 4-5 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 13 \\ 17 & -1 \end{bmatrix} \qquad |AB| = -10 - 221 = -231$$

RHS

$$|A|\,|B| = (35-2)(-1-6)$$

$$= 33(-7) \qquad = -231$$

HENCE $|AB| = |A|\,|B|$ .... Proved

**04.** Discuss continuity of the function at the given point . If the function is discontinuous then remove the discontinuity

$$f(x) = \frac{\tan^2 7x}{x^2} \quad ; \quad x \neq 0$$

$$= 49 \quad ; \quad x = 0$$

**STEP 1**

$$\lim_{x \to 0} f(x)$$

$$= \lim_{x\to 0} \frac{\tan^2 7x}{x^2}$$

$$= \lim_{x\to 0} \left(\frac{\tan 7x}{x}\right)^2$$

$$= \lim_{x\to 0} \left(7 \frac{\tan 7x}{7x}\right)^2$$

$$= (7\,.1)^2$$

$$= 49$$

**STEP 2 :**

$f(0) = 49$ ......... given

**STEP 3 :**

$$f(0) = \lim_{x\to 0} f(x)$$

$\therefore f$ is discontinuous at $x = 0$

**05.** Find the value of x for which the function is decreasing

$$f(x) = 4x^3 - 12x^2 - 36x + 1$$

**SOLUTION**

For f(x) decreasing ,

$f'(x) < 0$

$12x^2 - 24x - 36 < 0$

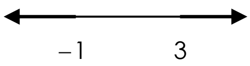
$x^2 - 2x - 3 < 0$

$(x - 3)(x + 1) < 0$

**CASE 1 :**

$x - 3 > 0$ & $x + 1 < 0$

$x > 3$ & $x < -1$




NOT POSSIBLE SO DISCARD

**CASE 2 :**

$x - 3 < 0$ & $x + 1 > 0$

$x < 3$ & $x > -1$


$-1 < x < 3$

$x \in (-1, 3)$

***f* is decreasing for** $\mathbf{x \in (-1, 3)}$

**06.** Differentiate : $\tan^{-1}(\cot 2x)$ wrt x

**SOLUTION**

$$y = \tan^{-1}(\cot 2x)$$

$$y = \tan^{-1}\tan\left(\pi/2 - 2x\right)$$

$$y = \pi/2 - 2x$$

$$\frac{dy}{dx} = -2$$

**07.** $f(x) = \dfrac{\sqrt{4+x} - 2}{3x}$ ; $x \neq 0$

$= {}^{1}/_{4}$ ; $x = 0$ Discuss continuity at $x = 0$

**SOLUTION**

**STEP 1**

$\lim\limits_{x \to 0} f(x)$

$$= \lim_{x \to 0} \frac{\sqrt{4+x} - 2}{3x}$$

$$= \lim_{x \to 0} \frac{\sqrt{4+x} - 2}{3x} \cdot \frac{\sqrt{4+x} + 2}{\sqrt{4+x} + 2}$$

$$= \lim_{x \to 0} \frac{4 + x - 4}{3x} \cdot \frac{1}{\sqrt{4+x} + 2}$$

$$= \lim_{x \to 0} \frac{\cancel{x}}{3\cancel{x}} \cdot \frac{1}{\sqrt{4+x} + 2} \qquad x \neq 0$$

$$= \lim_{x \to 0} \frac{1}{3} \cdot \frac{1}{\sqrt{4+x} + 2}$$

$$= \frac{1}{3} \cdot \frac{1}{\sqrt{4+0} + 2}$$

$$= \frac{1}{3} \cdot \frac{1}{2+2}$$

$$= \frac{1}{12}$$

**STEP 2 :**

$f(0) = 1/4$ ......... given

**STEP 3 :**

$f(0) \neq \lim\limits_{x \to 0} f(x)$ ; $f$ is discontinuous at $x = 0$

**STEP 4 :**

**Removable Discontinuity**

$f$ can be made continuous at $x = 0$ by redefining it as

$f(x) = \dfrac{\sqrt{4+x} - 2}{3x}$ ; $x \neq 0$

$= {}^{1}/_{12}$ ; $x = 0$

**08.** Evaluate : $\int e^x \frac{x+1}{(x+2)^2} dx$.

**SOLUTION**

$$= e^x \int \frac{x+2-1}{(x+2)^2} dx$$

$$= \int e^x \left(\frac{x+2}{(x+2)^2} - \frac{1}{(x+2)^2}\right) dx$$

$$= \int e^x \left(\frac{1}{x+2} + \frac{-1}{(x+2)^2}\right) dx$$

$$\frac{d}{dx} \frac{1}{x+2} = \frac{-1}{(x+2)^2}$$

HENCE THE SUM IS

$$= \int e^x \left[f(x) + f'(x)\right] dx$$

$$= e^x f(x) + c$$

$$= e^x \frac{1}{x+2} + c$$

$$= \frac{e^x}{x+4} + c$$

**Q2. (A) Attempt any TWO of the following** **(06)**

**01.** Without using the truth table , show that $\sim(p \vee q) \vee (\sim p \wedge q) \equiv \sim p$

**SOLUTION**

$\sim(p \vee q) \vee (\sim p \wedge q)$

$\equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q)$ ......... De Morgan's Law

$\equiv \sim p \wedge (\sim q \vee q)$ ......... Distributive Law

$\equiv \sim p \wedge t$ ......... Complement Law

$\equiv \sim p$ ......... Identity Law

**02.** $y = \tan^{-1}\left[\dfrac{1-\cos x}{\sin x}\right]$ . Find $dy/dx$

**SOLUTION**

$$y = \tan^{-1}\left[\frac{2\sin^2 {}^x/_2}{2\sin{}^x/_2.\cos{}^x/_2}\right]$$

$$y = \tan^{-1}\left[\frac{\sin{}^x/_2}{\cos{}^x/_2}\right]$$

$$y = \tan^{-1}(\tan {}^x/_2)$$

$$y = \frac{x}{2}$$

$$\frac{dy}{dx} = \frac{1}{2}$$

**03.** Evaluate $\displaystyle\int \frac{\tan x}{\sec x + \tan x}\, dx$

**SOLUTION**

$$\int \frac{\tan x}{\sec x + \tan x}\, dx$$

$$= \int \frac{\tan x}{\sec x + \tan x} \cdot \frac{\sec x - \tan x}{\sec x + \tan x}\, dx$$

$$= \int \frac{\sec x \tan x - \tan^2 x}{\sec^2 x - \tan^2 x}\, dx$$

$$= \int \sec x \tan x - (\sec^2 x - 1)\; dx$$

$$= \int \sec x \tan x - \sec^2 x + 1 \quad dx$$

$$= \sec x - \tan x + x + c$$

**(B) Attempt any TWO of the following** **(08)**

**01.** find a & b if f(x) is continuous at $x = 0$ & $f(1) = 2$ where ;

$$f(x) = \begin{cases} x^2 + a & ; \quad x \geq 0 \\ 2\sqrt{x^2+1} + b & ; \quad x < 0 \end{cases}$$

**SOLUTION :**

**STEP 1**

$$\lim_{x \to 0+} f(x)$$

$$= \lim_{x \to 0} x^2 + a$$

$$= 0^2 + a = a$$

**STEP 2**

$$\lim_{x \to 0-} f(x)$$

$$= \lim_{x \to 0} 2\sqrt{x^2+1} + b$$

$$= 2\sqrt{0^2+1} + b$$

$$= 2 + b$$

**STEP 3**

$$f(0) = 0^2 + a$$

$$= a$$

**STEP 4**

Since $f$ is continuous at $x = 0$

$$\lim_{x \to 0-} f(x) = \lim_{x \to 0+} f(x) = f(0)$$

$$2 + b = a = a$$

$$2 + b = a \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

**STEP 5**

$$f(1) = 2$$

$$1^2 + a = 2 \qquad \therefore \quad a = 1$$

Sub in (1)

$$2 + b = 1 \qquad \therefore \quad b = -1$$

**02.** the total cost function for producing a good x is given by

$$C = \frac{x^2}{4} + 7x + 100$$

where x is the output . Find the output of the good for which the average cost is minimum and the minimum average cost . Verify that at this output AC = MC

**SOLUTION**

**STEP 1 : AVERAGE COST**

$$C_A = \frac{C}{x}$$

$$= \frac{x}{4} + 7 + \frac{100}{x}$$

**STEP 2 :**

$$\frac{dC_A}{dx} = \frac{1}{4} - \frac{100}{x^2} = \frac{1}{4} - 100x^{-2}$$

$$\frac{d^2C_A}{dx^2} = 0 + 200x^{-3}$$

$$= \frac{200}{x^3}.$$

**STEP 3 :**

$$\frac{dC_A}{dx} = 0$$

$$\frac{1}{4} - \frac{100}{x^2} = 0$$

$$\frac{1}{4} = \frac{100}{x^2}$$

$$x^2 = 400$$

$x = 20$ (o/p cannot be – ve)

**STEP 4 :**

$$\left.\frac{d^2C_A}{dx^2}\right|_{x = 20} = \frac{200}{20^3} > 0$$

Average Cost is minimum at x = 20

**STEP 5 :**

$$\left.C_A\right|_{x = 40} = \frac{20}{4} + 7 + \frac{100}{20}$$

$$= 5 + 7 + 5 = 17$$

**STEP 6 : MARGINAL COST**

$$C_M = \frac{dC}{dx}$$

$$= \frac{2x}{4} + 7$$

$$\left.C_M\right|_{x = 20} = \frac{2(20)}{4} + 7$$

$$= 10 + 7$$

$$= 17$$

Hence $C_A = C_M$ at x = 20

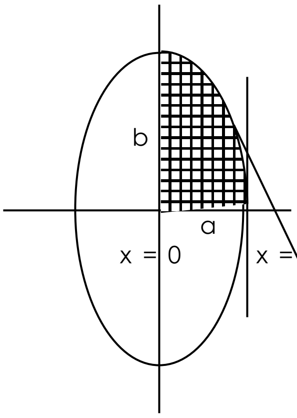
**03.** Find the area of the ellipse : $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$

**SOLUTION**

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

$a^2 = 4 \qquad b^2 = 9$

$a = 2 \qquad b = 3$




$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

$$\frac{y^2}{9} = 1 - \frac{x^2}{4}$$

$$\frac{y^2}{9} = \frac{4 - x^2}{4}$$

$$y^2 = \frac{9}{4}(4 - x^2)$$

$$y = \frac{3\sqrt{4 - x^2}}{2}$$

Area of Ellipse

$$= 4\int_0^2 y \, dx \quad \text{....... BY SYMMETRY}$$

$$= 4\int_0^2 \frac{3}{2}\sqrt{4 - x^2}\, dx$$

$$= 6\int_0^2 \sqrt{2^2 - x^2}\, dx$$

$$= 6\left[\frac{x}{2}\sqrt{2^2 - x^2} + \frac{2^2}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$$

$$= 6\left[\frac{x}{2}\sqrt{2^2 - x^2} + 2\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$$

$$= 6\left\{\left[\frac{2}{2}\sqrt{2^2 - 2^2} + 2\sin^{-1}\left(\frac{2}{2}\right)\right] - \left[\frac{0}{2}\sqrt{2^2 - 0^2} + 2\sin^{-1}\left(\frac{0}{2}\right)\right]\right\}$$

$$= 6\left[0 + 2\sin^{-1}(1)\right] - \left[0 + 2\sin^{-1}(0)\right]$$

$$= 6\left(2 \times \frac{\pi}{2}\right)$$

$= 6\pi$ sq. units

**Q3. (A) Attempt any TWO of the following (06)**

**01.** Examine whether the statement $(p \wedge \sim q) \leftrightarrow (p \to q)$ is a tautology or contradiction or neither of them

**SOLUTION**

| p | q | ~ q | $p \wedge \sim q$ | $p \to q$ | $(p \wedge \sim q) \leftrightarrow (p \to q)$ |
|---|---|---|---|---|---|
| T | T | F | F | T | F |
| T | F | T | T | F | F |
| F | T | F | F | T | F |
| F | F | T | F | T | F |

Since all the values in the last column are 'F' , given statement is **'Contradiction'**

**02.** Find $\frac{dy}{dx}$ ; if $x = \tan^{-1}\theta$ ; $y = \theta^3$

**SOLUTION**

$$\mathbf{x = \tan^{-1}\theta}$$

Diff wrt '$\theta$'

$$\frac{dx}{d\theta} = \frac{1}{1 + \theta^2}$$

$$\mathbf{y = \theta^3}$$

Diff wrt '$\theta$'

$$\frac{dy}{d\theta} = 3\theta^2$$

Now

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$= \frac{3\theta^2}{\frac{1}{1+\theta^2}}$$

$$= 3\theta^2.(1 + \theta^2)$$

**03.** Evaluate : $\int \frac{1}{x^2 - 10x - 39} dx$

**SOLUTION**

$$= \int \frac{1}{x^2 - 10x + 25 - 39 - 25} dx$$

$$= \int \frac{1}{(x - 5)^2 - 64} dx$$

$$= \int \frac{1}{(x - 5)^2 - 8^2} dx$$

$$= \frac{1}{2a} \log \left| \frac{x - a}{x + a} \right| + c$$

$$= \frac{1}{2(8)} \log \left| \frac{x - 5 - 8}{x - 5 + 8} \right| + c$$

$$= \frac{1}{16} \log \left| \frac{x - 13}{x + 3} \right|$$

**(B) Attempt any TWO of the following** **(08)**

**01.** Express the following equations in matrix form and solve them by method of inversion

$x - y + z = 4$ ; $2x + y - 3z = 0$ ; $x + y + z = 2$

**STEP 1 :**

$$\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix}$$

$AX = B$

$A^{-1}AX = A^{-1}B$

$IX = A^{-1}B$

$X = A^{-1}B$

**STEP 2 :**

$$A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}$$

**COFACTOR'S**

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 1 & -3 \\ 1 & 1 \end{vmatrix} = 1(1+3) = 4$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = -1(2+3) = -5$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1(2-1) = 1$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -1(-1-1) = 2$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1(1-1) = 0$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = -1(1+1) = -2$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix} = 1(3-1) = 2$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} = -1(-3-2) = 5$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 1(1+2) = 3$$

**COFACTOR MATRIX OF A**

$$\begin{pmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{pmatrix}$$

**ADJ A** = TRANSPOSE OF THE COFACTOR MATRIX

$$= \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{pmatrix}$$

**| A |**

$= 1(1+3) + 1(2+3) + 1(2-1)$

$= 1(4) + 1(5) + 1(1)$

$= 10$

$$\mathbf{A^{-1}} = \frac{1}{|A|} \cdot \text{adj } A$$

$$= \frac{1}{10} \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{pmatrix}$$

**STEP 3 :**

$$X = A^{-1}B$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{10}\begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{pmatrix}\begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{10}\begin{pmatrix} 16 + 0 + 4 \\ -20 + 0 + 10 \\ 4 + 0 + 6 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{10}\begin{pmatrix} 20 \\ -10 \\ 10 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

**BY EQUALITY OF TWO MATRICES**

$x = 2$ , $y = -1$ & $z = 1$ SS : $\{2, -1, 1\}$

**02.** The expenditure Ec of a person with income x is given by

$$E_C = 0.0006x^2 + 0.003x$$

Find the marginal propensity to consume & marginal propensity to save when x = 200 .

Also find the average propensity to consume and average propensity to save .

**SOLUTION**

$E_c = 0.0006x^2 + 0.003x$

$$\textbf{APC}\Big|_{x=200} = \frac{Ec}{x}$$

$$= \frac{0.0006x^2 + 0.003x}{x}$$

$$= 0.0006x + 0.003$$

$$= 0.0006(200) + 0.003$$

$$= 0.12 + 0.003$$

$$= \mathbf{0.123}$$

$$\textbf{APS}\Big|_{x=200} = 1 - \text{APC}\Big|_{x=200}$$

$$= 1 - 0.123$$

$$= \mathbf{0.877}$$

$$\textbf{MPC}\Big|_{x=200} = \frac{dEc}{dx}$$

$$= \frac{d}{dx}\,0.0006x^2 + 0.003x$$

$$= 0.0012x + 0.003$$

$$= 0.0012(200) + 0.003$$

$$= 0.24 + 0.003$$

$$= \mathbf{0.243}$$

$$\textbf{MPS}\Big|_{x=200} = 1 - \text{MPC}\Big|_{x=200}$$

$$= 1 - 0.243$$

$$= \mathbf{0.757}$$

**03.** Evaluate : $$\int_{\pi/5}^{3\pi/10} \frac{1}{1+\sqrt{\tan x}}\,dx$$

**SOLUTION**

$$I = \int_{\pi/5}^{3\pi/10} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx \quad \dots \textbf{(1)}$$

**USING** $$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$I = \int_{\pi/5}^{3\pi/10} \frac{\sqrt{\cos(\pi/2-x)}}{\sqrt{\cos(\pi/2-x)}+\sqrt{\sin(\pi/2-x)}}\,dx$$

$$I = \int_{\pi/5}^{3\pi/10} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx \quad \dots \textbf{(2)}$$

(1) + (2)

$$2I = \int_{\pi/5}^{3\pi/10} \frac{\sqrt{\cos x}+\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$$

$$2I = \int_{\pi/5}^{3\pi/10} 1\,dx$$

$$2I = \Big[x\Big]_{\pi/5}^{3\pi/10}$$

$$2I = 3\pi/10 - \pi/5$$

$$2I = 3\pi/10$$

$$I = \pi/20$$

DO NOT STOP

GET READY FOR NEXT