# FUNCTIONS

## Q SET - 1

01) if $f(x) = lx - 4$ and $f(2) = 10$, find l  ans : $l = 7$

02) if $g(x) = 10 - 2px$ and $g(-1) = 4$. Find p  ans : $p = -3$

03) if $h(x) = px + q$, $h(0) = -3$ and $h(3) = 6$. Find p and q  ans : $p = 3$, $q = -3$

04) if $f(x) = x^2 + mx + n$, $f(0) = 6$ and $f(3) = 9$. Find m and n  ans : $m = -2$, $n = 6$

05) if $g(x) = ax^2 + bx + 1$, $g(1) = 15$ and $g(-1) = 3$. Find a and b  ans : $a = 8$, $b = 6$

## Q SET - 2

01) if $f(x) = x^2 - 3x + 5$. Solve the equation $f(x) = f(x + 1)$  ans : $x = 1$

02) if $f(x) = x^2 + 5x - 7$. Solve the equation $f(x) = f(x - 1)$  ans : $x = -2$

03) if $f(x) = x^2 + 4x + 5$. Solve the equation $f(x + 1) = f(x + 2)$  ans : $x = -7/2$

04) if $f(x) = x^2 + 3x - 2$. Solve the equation $f(x) = f(2x + 1)$  ans : $x = -4/3$, $-1$

05) if $f(x) = x^2 - 4x + 11$. Solve the equation $f(x) = f(3x - 1)$  ans : $x = 1/2$, $5/4$

## Q SET - 3

01) if $f(x) = \dfrac{3x + 2}{4x - 3}$ ; $x \neq 3/4$ ; Show that $f(f(x)) = x$

02) if $f(x) = \dfrac{4x + 3}{6x - 4}$ ; $x \neq 2/3$ ; Show that $f(f(x)) = x$

03) if $y = f(x) = \dfrac{5x - 1}{7x - 5}$ ; $x \neq 5/7$ ; Show that $f(y) = x$

04) if $f(x) = \dfrac{3x + 4}{5x - 7}$ and $g(x) = \dfrac{7x + 4}{5x - 3}$, Show that : $fog(x) = gof(x) = x$

05) if $f(x) = \dfrac{x + 3}{4x - 5}$ and $g(x) = \dfrac{3 + 5x}{4x - 1}$, Show that : $fog(x) = gof(x) = x$

06) if $f(x) = \dfrac{x + 1}{x - 1}$ and $g(x) = \dfrac{2x + 3}{3x - 2}$, find fog and gof **(MARCH – 2016)**

## Q SET - 4

01) $f(x) = x^2 + 6$ ; $g(x) = x - 4$ . Find fog & gof ans : $x^2 - 8x + 22$ ; $x^2 + 2$

02) $f(x) = 3x - 1$ ; $g(x) = x^2 + 1$ . Find fog & gof ans : $3x^2 + 2$ ; $9x^2 - 6x + 2$ **(MAR – 2013)**

03) $f(x) = x - 5$ ; $g(x) = x^2 - 1$ . Find fog & gof ans : $3x^2 + 2$ , $x^2 - 10x + 24$ **(MAR – 2014)**

04) $f(x) = 2x + 1$ ; $g(x) = 3x^2 - x + 4$ .Find fog & gof ans : $6x^2 - 2x + 9$ , $12x^2 + 10x + 6$

05) $f(x) = \sqrt{x}$ ; $g(x) = x^2 + 1$ . Find gof(x) . Also find value of x for which $g(x) = f(4)$
**(JAN – 2013)** ans : $x + 1$ , $\pm 1$

06) $f(x) = 8x^3$ ; $g(x) = \sqrt[3]{x}$ . Find fog & gof ans : $8x$ , $2x$

07) $f(x) = 256x^4$; $g(x) = \sqrt{x}$ . Find fog & gof ans : $256x^2$ , $16x^2$

08) $f(x) = \log\left(\frac{1 + x}{1 - x}\right)$ . Show that : $f\left(\frac{x + y}{1 + xy}\right) = f(x) + f(y)$ **(JAN – 2016)**

## Q SET - 5 : Find the range of the function

01) $f(x) = 5x - 3$ ; $-5 \leq x \leq 1$ ans : Range of f is $[-28 , 2]$

02) $f(x) = 2x + 6$ ; $-1 \leq x \leq 5$ ans : Range of f is $[4 , 16]$

03) $f(x) = 3 - 4x$ ; $-4 \leq x \leq 2$ ans : Range of f is $[-5 , 19]$

04) $f(x) = -2 - 7x$ ; $-2 \leq x \leq 4$ ans : Range of f is $[-30 , 12]$

05) $f(x) = 3x^2 + 5$ ; $-3 \leq x \leq 4$ ans : Range of f is $[5 , 53]$

06) $f(x) = 2x^2 - 4$ ; $1 \leq x \leq 4$ ans : Range of f is $[-2 , 28]$

07) $f(x) = 1 - 4x^2$ ; $-2 \leq x \leq 2$ ans : Range of f is $[-15 , 1]$

08) $f(x) = 7 - 2x^2$ ; $2 \leq x \leq 4$ ans : Range of f is $[-25 , -1]$

09) $f(x) = 2 - 5x^2$ ; $-1 \le x \le 3$ Also find x for which $f(x) = f(x + 1)$ **(JAN – 2015)**

ans : Range of f is $[-43 , 2]$ , $x = -1/2$

10) $f(x) = 9 - 2x^2$ ; $-2 \le x \le 1$ Also find x for which $f(x + 1) = f(x + 2)$ **(JAN – 2016)**

ans : Range of f is $[1 , 9]$ , $x = -3/2$

11) $f(x) = 9 - 2x^2$ ; $-5 \le x \le 3$ **(MAR – 2014)** ans : Range of f is $[-41 , 9]$

12) Find range of the $f(x) = 7 - 8x^2$ , $-4 \le x \le 2$ . Also find x for which $f(x) = f(-1)$ **(JAN – 2014)**

ans : Range of f is $[-121 , 7]$ , $x = \pm 1$

13) $f(x) = x^2 + 4x + 5$ , $x \in R$ ans : Range of f is $[1 , \infty )$

14) $f(x) = x^2 - 8x + 10$ , $x \in R$ ans : Range of f is $[-6 , \infty )$

15) $f(x) = 4x^2 - 4x + 7$ , $x \in R$ ans : Range of f is $[6 , \infty )$

16) Find range of the $f(x) = x^2 - 4x + 7$ , $x \in R$. Also find $f(-1) + f(1-x)$ **(JAN – 2017)**

ans : Range of f is $[3, \infty )$ , $x^2 + 2x + 16$

## Q SET - 6 : Find the inverse of the function

01) $f(x) = 2x + 5$ ans : $f^{-1}(x) = \frac{1}{2}(x - 5)$

02) $f(x) = \frac{2x}{3} + 5$ ans : $f^{-1}(x) = \frac{3}{2}(x - 5)$

03) $f(x) = \frac{3x}{4} - 7$ ans : $f^{-1}(x) = \frac{4}{3}(x + 7)$

04) Find range of the function $fog^{-1}(x)$ where $f(x) = 3 + 4x^2$ and $g(x) = x + 2$ **(JAN – 2014)**

ans : Range of $fog^{-1}(x)$ is $[3 , \infty)$

05) Find the inverse function $f^{-1}$ of the function $f: R \to R$ given by $f(x) = \frac{x}{3} + \frac{2}{5.}$ .

Also find $(f^{-1}of)(2x - 3)$ **(JAN – 2017)**

# SOLUTION TO Q SET – 1

01) if $f(x) = lx - 4$ and $f(2) = 10$, find l

**SOLUTION :**

$f(x) = lx - 4$

$f(2) = 10$

$l(2) - 4 = 10$

$2l = 14 \quad \therefore l = 7$

02) if $g(x) = 10 - 2px$ and $g(-1) = 4$. Find p

**SOLUTION :**

$g(x) = 10 - 2px$

$g(-1) = 4$

$10 - 2p(-1) = 4$

$10 + 2p = 4$

$2p = -6 \quad \therefore p = -3$

03) if $h(x) = px + q$, $h(0) = -3$ and $h(3) = 6$. Find p and q

**SOLUTION :**

$h(x) = px + q$

$h(0) = -3$

$p(0) + q = -3$

$q = -3$ →

$h(3) = 6$

$p(3) + q = 6$

$3p + q = 6$

$3p - 3 = 6$

$3p = 9 \quad \therefore p = 3$

04) if $f(x) = x^2 + mx + n$, $f(0) = 6$ and $f(3) = 9$. Find m and n

**SOLUTION :**

$f(x) = x^2 + mx + n$

$f(0) = 6$

$0^2 + m(0) + n = 6$

$n = 6$ →

$f(3) = 9$

$3^2 + m(3) + n = 9$

$9 + 3m + n = 9$

$3m + 6 = 0$

$3m = -6 \quad \therefore m = -2$

05) if $g(x) = ax^2 + bx + 1$, $g(1) = 15$ and $g(-1) = 3$. Find a and b

**SOLUTION :**

$g(x) = ax^2 + bx + 1$

$g(1) = 15$

$a(1)^2 + b(1) + 1 = 15$

$a + b = 14$ …….(1)

$g(-1) = 3$

$a(-1)^2 + b(-1) + 1 = 3$

$a - b = 2$ ……… (2)

Solving (1) & (2) : $a = 8$ & $b = 6$

# SOLUTION TO Q SET - 2

01) if $f(x) = x^2 - 3x + 5$ . Solve the equation $f(x) = f(x + 1)$

**SOLUTION :**

$$f(x) = f(x+1)$$
$$x^2 - 3x + \cancel{5} = (x+1)^2 - 3(x+1) + \cancel{5}$$
$$\cancel{x^2} - \cancel{3x} = \cancel{x^2} + 2x + 1 - \cancel{3x} - 3$$
$$0 = 2x - 2$$
$$2x = 2 \qquad \therefore\ x = 1$$

02) if $f(x) = x^2 + 5x - 7$ . Solve the equation $f(x) = f(x - 1)$

**SOLUTION :**

$$f(x) = f(x-1)$$
$$x^2 + 5x - \cancel{7} = (x-1)^2 + 5(x-1) - \cancel{7}$$
$$\cancel{x^2} + \cancel{5x} = \cancel{x^2} - 2x + 1 + \cancel{5x} - 5$$
$$0 = -2x - 4$$
$$2x = -4 \qquad \therefore\ x = -2$$

03) if $f(x) = x^2 + 4x + 5$ . Solve the equation $f(x + 1) = f(x + 2)$

**SOLUTION :**

$$f(x+1) = f(x+2)$$
$$(x+1)^2 + 4(x+1) + \cancel{5} = ((x+2)^2 + 4(x+2) + \cancel{5}$$
$$\cancel{x^2} + 2x + 1 + \cancel{4x} + \cancel{4} = \cancel{x^2} + \cancel{4x} + \cancel{4} + 4x + 8$$
$$2x + 1 = 4x + 8$$
$$-2x = 7 \qquad \therefore\ x = -\ 7/2$$

04) if $f(x) = x^2 + 3x - 2$ . Solve the equation $f(x) = f(2x + 1)$

**SOLUTION :**

$$f(x) = f(2x+1)$$
$$x^2 + 3x - \cancel{2} = (2x+1)^2 + 3(2x+1) - \cancel{2}$$
$$x^2 + 3x = 4x^2 + 4x + 1 + 6x + 3$$
$$x^2 + 3x = 4x^2 + 10x + 4$$
$$3x^2 + 7x + 4 = 0$$
$$3x^2 + 3x + 4x + 4 = 0$$
$$3x(x+1) + 4(x+1) = 0$$
$$(3x+4)(x+1) = 0 \qquad \therefore \qquad x = -4/3\ ,\ -1$$

05) if $f(x) = x^2 - 4x + 11$ . Solve the equation $f(x) = f(3x - 1)$

**SOLUTION :**

$$f(x) = f(3x - 1)$$

$$x^2 - 4x + 11 = (3x - 1)^2 - 4(3x - 1) + 11$$

$$x^2 - 4x = 9x^2 - 6x + 1 - 12x + 4$$

$$x^2 - 4x = 9x^2 - 18x + 5$$

$$8x^2 - 14x + 5 = 0$$

$$8x^2 - 4x - 10x + 5 = 0$$

$$4x(2x - 1) - 5(2x - 1) = 0$$

$$(2x - 1)(4x - 5) = 0 \qquad \therefore\ x = 1/2 , 5/4$$

## SOLUTION TO Q SET - 3

01) if $f(x) = \dfrac{3x + 2}{4x - 3}$ ; $x \neq 3/4$ ; Show that $f(f(x)) = x$

**SOLUTION :**

$$f(f(x)) = \frac{3f(x) + 2}{4f(x) - 3}$$

$$= \frac{3\left(\dfrac{3x + 2}{4x - 3}\right) + 2}{4\left(\dfrac{3x + 2}{4x - 3}\right) - 3}$$

$$= \frac{\dfrac{9x + 6 + 8x - 6}{4x - 3}}{\dfrac{12x + 8 - 12x + 9}{4x - 3}}$$

$$= \frac{17x}{17} \qquad = x = \text{RHS}$$

02) if $f(x) = \dfrac{4x + 3}{6x - 4}$ ; $x \neq 2/3$ ; Show that $f(f(x)) = x$

**SOLUTION :**

$$f(f(x)) = \frac{4f(x) + 3}{6f(x) - 4}$$

$$= \frac{4\left(\dfrac{4x + 3}{6x - 4}\right) + 3}{6\left(\dfrac{4x + 3}{6x - 4}\right) - 4}$$

$$= \frac{\dfrac{24x + 12 + 18x - 12}{6x - 4}}{\dfrac{24x + 18 - 24x + 16}{4x - 3}}$$

$$= \frac{34x}{34} = x = \text{RHS}$$

03) if $y = f(x) = \dfrac{5x-1}{7x-5}$ ; $x \neq 5/7$ ; Show that $f(y) = x$

**SOLUTION :**

$$f(y) = \frac{5y-1}{7y-5}$$

$$= \frac{5\left(\dfrac{5x-1}{7x-5}\right) - 1}{7\left(\dfrac{5x-1}{7x-5}\right) - 5}$$

$$= \frac{\dfrac{25x-5-7x+5}{7x-5}}{\dfrac{35x-7-35x+25}{4x-3}}$$

$$= \frac{18x}{18}$$

$$= x = \text{RHS}$$

04) if $f(x) = \dfrac{3x+4}{5x-7}$ and $g(x) = \dfrac{7x+4}{5x-3}$ , Show that : $fog(x) = gof(x) = x$

**SOLUTION :**

$$fog(x) = f(g(x))$$

$$= \frac{3g(x)+4}{5g(x)-7}$$

$$= \frac{3\left(\dfrac{7x+4}{5x-3}\right) + 4}{5\left(\dfrac{7x+4}{5x-3}\right) - 7}$$

$$= \frac{\dfrac{21x+12+20x-12}{5x-3}}{\dfrac{35x+20-35x+21}{5x-3}}$$

$$= \frac{41x}{41}$$

$$= x$$

$$gof(x) = g(f(x))$$

$$= \frac{7f(x)+4}{5f(x)-3}$$

$$= \frac{7\left(\dfrac{3x+4}{5x-7}\right) + 4}{5\left(\dfrac{3x+4}{5x-7}\right) - 3}$$

$$= \frac{\dfrac{21x+28+20x-28}{5x-7}}{\dfrac{15x+20-15x+21}{5x-7}}$$

$$= \frac{41x}{41}$$

$$= x$$

05) if $f(x) = \dfrac{x+3}{4x-5}$ and $g(x) = \dfrac{3+5x}{4x-1}$ , Show that : $fog(x) = gof(x) = x$

**SOLUTION :**

$$fog(x) = f(g(x)) = \frac{g(x)+3}{4g(x)-5} = \frac{\left(\frac{3+5x}{4x-1}\right)+3}{4\left(\frac{3+5x}{4x-1}\right)-5} = \frac{\frac{3+5x+12x-3}{4x-1}}{\frac{12+20x-20x+5}{5x-3}} = \frac{17x}{17} = x$$

$$gof(x) = g(f(x)) = \frac{3+5(f(x))}{4f(x)-1} = \frac{3+5\left(\frac{x+3}{4x-5}\right)}{4\left(\frac{x+3}{4x-5}\right)-1} = \frac{\frac{12x-15+5x+15}{4x-5}}{\frac{4x+12-4x+5}{4x-5}} = \frac{17x}{17} = x$$

06) if $f(x) = \dfrac{x+1}{x-1}$ and $g(x) = \dfrac{2x+3}{3x-2}$ , find fog and gof **(MARCH – 2016)**

**SOLUTION :**

$$fog = f(g(x)) = \frac{g(x)+1}{g(x)-1} = \frac{\left(\frac{2x+3}{3x-2}\right)+1}{\left(\frac{2x+3}{3x-2}\right)-1} = \frac{\frac{2x+3+3x-2}{3x-2}}{\frac{2x+3-3x+2}{3x-2}} = \frac{5x+1}{5-x}$$

$$gof(x) = g(f(x)) = \frac{2f(x)+3}{3f(x)-2} = \frac{2\left(\frac{x+1}{x-1}\right)+3}{3\left(\frac{x+1}{x-1}\right)-2} = \frac{\frac{2x+2+3x-3}{x-1}}{\frac{3x+3-2x+2}{x-1}} = \frac{5x-1}{x+5}$$

# SOLUTION TO Q SET - 4

01) $f(x) = x^2 + 6$ ; $g(x) = x - 4$ . Find fog & gof

**SOLUTION** :

$fog(x) = f(g(x))$
$= g(x)^2 + 6$
$= (x - 4)^2 + 6$
$= x^2 - 8x + 16 + 6$
$= x^2 - 8x + 22$

$gof(x) = g(f(x))$
$= f(x) - 4$
$= x^2 + 6 - 4$
$= x^2 + 2$

02) $f(x) = 3x - 1$ ; $g(x) = x^2 + 1$ . Find fog & gof **(MARCH – 2013)**

**SOLUTION** :

$fog(x) = f(g(x))$
$= 3(g(x) - 1$
$= 3(x^2 + 1) - 1$
$= 3x^2 + 3 - 1$
$= 3x^2 + 2$

$gof(x) = g(f(x))$
$= (f(x))^2 + 1$
$= (3x - 1)^2 + 1$
$= 9x^2 - 6x + 1 + 1$
$= 9x^2 - 6x + 2$

03) $f(x) = x - 5$ ; $g(x) = x^2 - 1$ . Find fog & gof **(MARCH – 2014)**

**SOLUTION** :

$fog(x) = f(g(x))$
$= (g(x) - 5$
$= x^2 - 1 - 5$
$= x^2 - 6$
$= 3x^2 + 2$

$gof(x) = g(f(x))$
$= (f(x))^2 - 1$
$= (x - 5)^2 - 1$
$= x^2 - 10x + 25 - 1$
$= x^2 - 10x + 24$

04) $f(x) = 2x + 1$ ; $g(x) = 3x^2 - x + 4$ . Find fog & gof

**SOLUTION** :

$fog(x) = f(g(x))$
$= 2g(x) + 1$
$= 2(3x^2 - x + 4) + 1$
$= 6x^2 - 2x + 8 + 1$
$= 6x^2 - 2x + 9$

$gof(x) = g(f(x))$
$= 3(f(x))^2 - f(x) + 4$
$= 3(2x + 1)^2 - (2x + 1) + 4$
$= 3(4x^2 + 4x + 1) - 2x - 1 + 4$
$= 12x^2 + 12x + 3 - 2x + 3$
$= 12x^2 + 10x + 6$

05) $f(x) = \sqrt{x}$ ; $g(x) = x^2 + 1$ . Find gof(x) . Also find value of x for which g(x) = f(4)

**SOLUTION** : **(JAN – 2013)**

$$gof(x) = g(f(x)) = (f(x))^2 + 1 = (\sqrt{x})^2 + 1 = x + 1$$

$$g(x) = f(4)$$
$$x^2 + 1 = \sqrt{4}$$
$$x^2 + 1 = 2$$
$$x^2 = 1 \qquad x = \pm 1$$

06) $f(x) = 8x^3$ ; $g(x) = \sqrt[3]{x}$ . Find fog & gof

**SOLUTION** :

$$fog(x) = f(g(x)) = 8(g(x))^3 = 8\left(\sqrt[3]{x}\right)^3 = 8x$$

$$gof(x) = g(f(x)) = \sqrt[3]{f(x)} = \sqrt[3]{8x^3} = 2x$$

07) $f(x) = 256x^4$; $g(x) = \sqrt{x}$ . Find fog & gof

**SOLUTION** :

$$fog(x) = f(g(x)) = 256(g(x))^4 = 256\left(\sqrt{x}\right)^4 = 256\,x^2$$

$$gof(x) = g(f(x)) = \sqrt{f(x)} = \sqrt{256x^4} = 16x^2$$

08) $f(x) = \log\left(\frac{1 + x}{1 - x}\right)$ . Show that : $f\left(\frac{x + y}{1 + xy}\right) = f(x) + f(y)$ **(JAN – 2016)**

**SOLUTION** : $LHS = f\left(\frac{x + y}{1 - xy}\right)$

$$= \log\left(\frac{1 + \frac{x + y}{1 + xy}}{1 - \frac{x + y}{1 + xy}}\right)$$

$$= \log\left(\frac{\frac{1 + xy + x + y}{1 + xy}}{\frac{1 + xy - x - y}{1 + xy}}\right) = \log\left(\frac{1 + xy + x + y}{1 + xy - x - y}\right)$$

$$RHS = f(x) + f(y)$$

$$= \log\left(\frac{1+x}{1-x}\right) + \log\left(\frac{1+y}{1-y}\right)$$

$$= \log\left(\frac{1+x}{1-x}\,\frac{1+y}{1-y}\right)$$

$$= \log\left(\frac{1+y+x+xy}{1-y-x+xy}\right)$$

LHS = RHS

## SOLUTION TO Q - SET 5

01) $f(x) = 5x - 3 \;;\; -5 \le x \le 1$

**SOLUTION :**

$$-5 \le x \le 1$$
$$-25 \le 5x \le 5$$
$$-25 - 3 \le 5x - 3 \le 5 - 3$$
$$-28 \le f(x) \le 2$$

Range of f is [–28 , 2 ]

02) $f(x) = 2x + 6 \;;\; -1 \le x \le 5$

**SOLUTION :**

$$-1 \le x \le 5$$
$$-2 \le 2x \le 10$$
$$-2 + 6 \le 2x + 6 \le 10 + 6$$
$$4 \le f(x) \le 16$$

Range of f is [4 , 16 ]

03) $f(x) = 3 - 4x \;;\; -4 \le x \le 2$

**SOLUTION :**

$$-4 \le x \le 2$$
$$16 \ge -4x \ge -8$$
$$16 + 3 \ge 3 - 4x \ge -8 + 3$$
$$19 \ge f(x) \ge -5$$

Range of f is [–5 , 19]

04) $f(x) = -2 - 7x \;;\; -2 \le x \le 4$

**SOLUTION :**

$$-2 \le x \le 4$$
$$14 \ge -7x \ge -28$$
$$14 - 2 \ge -2 - 7x \ge -28 - 2$$
$$12 \ge f(x) \ge -30$$

Range of f is [–30 ,12]

05) $f(x) = 3x^2 + 5 \;;\; -3 \le x \le 4$

**SOLUTION :**

$$-3 \le x \le 4$$
$$0 \le x^2 \le 16$$
$$0 \le 3x^2 \le 48$$
$$0 + 5 \le 3x^2 + 5 \le 48 + 5$$
$$5 \le f(x) \le 53$$

Range of f is [5 , 53]

06) $f(x) = 2x^2 - 4 \;;\; 1 \le x \le 4$

**SOLUTION :**

$$1 \le x \le 4$$
$$1 \le x^2 \le 16$$
$$2 \le 2x^2 \le 32$$
$$2 - 4 \le 2x^2 - 4 \le 32 - 4$$
$$-2 \le f(x) \le 28$$

Range of f is [–2 ,28]

07) $f(x) = 1 - 4x^2 \; ; \; -2 \leq x \leq 2$

**SOLUTION :**

$$-2 \leq x \leq 2$$
$$0 \leq x^2 \leq 4$$
$$0 \leq 4x^2 \leq 16$$
$$0 \geq -4x^2 \geq -16$$
$$0 + 1 \geq 1 - 4x^2 \geq -16 + 1$$
$$1 \geq f(x) \geq -15$$

Range of f is [–15 , 1]

08) $f(x) = 7 - 2x^2 \; ; \; 2 \leq x \leq 4$

**SOLUTION :**

$$2 \leq x \leq 4$$
$$4 \leq x^2 \leq 16$$
$$8 \leq 2x^2 \leq 32$$
$$-8 \geq -2x^2 \geq -32$$
$$7 - 8 \geq 7 - 2x^2 \geq 7 - 32$$
$$-1 \geq f(x) \geq -25$$

Range of f is [–25 ,–1]

09) $f(x) = 2 - 5x^2 \; ; \; -1 \leq x \leq 3$

Also find x for which f(x) = f(x + 1)

**(JAN – 2015)**

**SOLUTION :**

$$-1 \leq x \leq 3$$
$$0 \leq x^2 \leq 9$$
$$0 \leq 5x^2 \leq 45$$
$$0 \geq -5x^2 \geq -45$$
$$0 + 2 \geq 2 - 5x^2 \geq -45 + 2$$
$$2 \geq f(x) \geq -43$$

Range of f is [–43 , 2]

PART – II Solving

$$f(x) = f(x + 1)$$
$$2 - 5x^2 = 2 - 5(x + 1)^2$$
$$-5x^2 = -5(x + 1)^2$$
$$x^2 = (x + 1)^2$$
$$x^2 = x^2 + 2x + 1$$
$$0 = 2x + 1$$
$$x = -1/2$$

10) $f(x) = 9 - 2x^2 \; ; \; -2 \leq x \leq 1$

Also find x for which f(x + 1) = f(x + 2)

**(JAN – 2016)**

**SOLUTION :**

$$-2 \leq x \leq 1$$
$$0 \leq x^2 \leq 4$$
$$0 \leq 2x^2 \leq 8$$
$$0 \geq -2x^2 \geq -8$$
$$0 + 9 \geq 9 - 2x^2 \geq 9 - 8$$
$$9 \geq f(x) \geq 1$$

Range of f is [1 , 9]

PART – II Solving

$$f(x + 1) = f(x + 2)$$
$$9 - 2(x + 1)^2 = 9 - 2(x + 2)^2$$
$$-2(x + 1)^2 = -2(x + 2)^2$$
$$(x + 1)^2 = (x + 2)^2$$
$$x^2 + 2x + 1 = x^2 + 4x + 4$$
$$2x + 1 = 4x + 4$$
$$-3 = 2x$$
$$x = -3/2$$

11) $f(x) = 9 - 2x^2 \; ; \; -5 \le x \le 3$

**(MAR – 2014)**

**SOLUTION :**

$$-5 \le x \le 3$$

$$0 \le x^2 \le 25$$

$$0 \le 2x^2 \le 50$$

$$0 \ge -2x^2 \ge -50$$

$$0 + 9 \ge 9 - 2x^2 \ge 9 - 50$$

$$9 \ge f(x) \ge -41$$

Range of f is $[-41 , 9]$

12) Find range of the $f(x) = 7 - 8x^2$ , $-4 \le x \le 2$ . Also find x for which $f(x) = f(-1)$

**(JAN – 2014)**

**SOLUTION :** PART – I

$$-4 \le x \le 2$$

$$0 \le x^2 \le 16$$

$$0 \le 8x^2 \le 128$$

$$0 \ge -8x^2 \ge -128$$

$$0 + 7 \ge 7 - 8x^2 \ge 7 - 128$$

$$7 \ge f(x) \ge -121$$

Range of f is $[-121 , 7]$

PART – II Solving

$$f(x) = f(-1)$$

$$7 - 8x^2 = 7 - 8(-1)^2$$

$$7 - 8x^2 = -1$$

$$8 = 8x^2$$

$$x^2 = 1 \qquad \therefore x = \pm 1$$

13) $f(x) = x^2 + 4x + 5$ , $x \in R$

**SOLUTION :**

$$f(x) = x^2 + 4x + 5$$
$$= x^2 + 4x + 4 + 1$$
$$= (x + 2)^2 + 1$$

Now ; $(x + 2)^2 \geq 0$

$(x + 2)^2 + 1 \geq 1$

$f(x) \geq 1$ Range of f is $[1 , \infty )$

14) $f(x) = x^2 - 8x + 10$ , $x \in R$

**SOLUTION :**

$$f(x) = x^2 - 8x + 10$$
$$= x^2 - 8x + 16 + 10 - 16$$
$$= (x - 4)^2 - 6$$

Now ; $(x - 4)^2 \geq 0$

$(x - 4)^2 - 6 \geq -6$

$f(x) \geq -6$

Range of f is $[-6 , \infty )$

15) $f(x) = 4x^2 - 4x + 7$ , $x \in R$

**SOLUTION :**

$$f(x) = 4x^2 - 4x + 7$$
$$= 4x^2 - 4x + 1 + 6$$
$$= (2x - 1)^2 + 6$$

Now ; $(2x - 1)^2 \geq 0$

$(2x - 1)^2 + 6 \geq 6$

$f(x) \geq 6$

Range of f is $[6 , \infty )$

16) Find range of the $f(x) = x^2 - 4x + 7$ , $x \in R$. Also find $f(-1) + f(1-x)$ **(JAN – 2017)**

**SOLUTION :**

$$f(x) = x^2 - 4x + 7$$
$$= x^2 - 4x + 4 + 7 - 4$$
$$= (x - 2)^2 + 3$$

Now ; $(x - 2)^2 \geq 0$

$(x - 2)^2 + 3 \geq 3$

$f(x) \geq 3$ Range : $[3, \infty )$

$$f(-1) + f(1-x)$$
$$= \left[(-1)^2-4(1)+7\right] + \left[(1-x)^2 - 4(1-x) + 7\right]$$
$$= 1 + 4 + 7 + \left[1 - 2x + x^2 - 4 + 4x + 7\right]$$
$$= x^2 + 2x + 16$$

## Q SET - 6 : Find the inverse of the function

01) $f(x) = 2x + 5$

$y = 2x + 5$

$y - 5 = 2x$

$x = \frac{1}{2}(y - 5)$

$\therefore f^{-1}(x) = \frac{1}{2}(x - 5)$

02) $f(x) = \frac{2x + 5}{3}$

$y = \frac{2x + 5}{3}$

$y - 5 = \frac{2x}{3}$

$x = \frac{3}{2}(y - 5)$

$f^{-1}(x) = \frac{3}{2}(x - 5)$

03) $f(x) = \frac{3x - 7}{4}$

$y = \frac{3x - 7}{4}$

$y + 7 = \frac{3x}{4}$

$x = \frac{4}{3}(y - 7)$

$f^{-1}(x) = \frac{4}{3}(x - 7)$

04) Find range of the function $fog^{-1}(x)$ where $f(x) = 3 + 4x^2$ and $g(x) = x + 2$ **(JAN – 2014)**

**SOLUTION :**

$g(x) = x + 2$

$y = x + 2$

$x = y - 2$

$g^{-1}(x) = x - 2$

$f(x) = 3 + 4x^2$

$fog^{-1}(x) = f[g^{-1}(x)]$

$= 3 + 4\, g^{-1}(x)^2$

$= 3 + 4(x - 2)^2$

Now ;

$(x - 2)^2 \geq 0$

$4(x - 2)^2 \geq 0$

$4(x - 2)^2 + 3 \geq 3$

$fog^{-1}(x) \geq 3$

Range of $fog^{-1}(x)$ is $[3 , \infty)$

05) Find the inverse function $f^{-1}$ of the function $f: R \to R$ given by $f(x) = \frac{x}{3} + \frac{2}{5}.$

Also find $(f^{-1} o f)(2x - 3)$ **(JAN – 2017)**

**SOLUTION :**

STEP 1

$$f(x) = \frac{x}{3} + \frac{2}{5}$$

$$y = \frac{x}{3} + \frac{2}{5}$$

$$\frac{x}{3} = y - \frac{2}{5}$$

$$x = 3\left(y - \frac{2}{5}\right)$$

$$f^{-1}(x) = 3\left(x - \frac{2}{5}\right)$$

STEP 2

$$f(x) = \frac{x}{3} + \frac{2}{5}$$

$$f(2x - 3) = \frac{2x - 3}{3} + \frac{2}{5}$$

$$= \frac{2x}{3} - 1 + \frac{2}{5}$$

$$= \frac{2x}{3} - \frac{3}{5}$$

STEP 3

$$f^{-1}(x) = 3\left(x - \frac{2}{5}\right)$$

Now

$(f^{-1} o f)(2x - 3)$

$$= f^{-1}\left(f(2x - 3)\right)$$

$$= f^{-1}\left(\frac{2x}{3} - \frac{3}{5}\right)$$

$$= 3\left(\frac{2x}{3} - \frac{3}{5} - \frac{2}{5}\right)$$

$$= 3\left(\frac{2x}{3} - 1\right)$$

$$= 2x - 3$$